Unit 02

Force and Motion

STUDENT'S LEARNING OUTCOMES (SLO's)

After studying this unit, the students will be able to :

- Differentiate between scalar and vector quantities
- Represent a vector in 2-D as two perpendicular components.
- Describe the product of two vectors (dot and cross-product) along with their properties.
- Derive the equations of motion [For uniform acceleration cases only. Derive from the definitions of velocity and acceleration as well as graphically]
- Solve problems using the equations of motion [For the cases of uniformly accelerated motion in a straight line, including the motion of bodies falling in a uniform gravitational field without air resistance. This also includes situations where the equations of motion need to be resolved into vertical and horizontal components for 2-D motion]
- Evaluate and analyse projectile motion in the absence of air resistance
  [This includes solving problems making use of the below facts:
  (i) Horizontal component *(V_H)*of velocity is constant.
  (ii) Acceleration is in the vertical direction and is the same as that of a vertically free falling object.
  (iii) The horizontal motion and vertical motion are independent of each other. Situations may require students to determine for projectiles:
  - How high does it go?
  - How far would it go along the level land?
  - Where would it be after a given time?
  - How long will it remain in flight?

  Situations may also require students to calculate for a projectile launched from ground height the
  - launch angle that results in the maximum range.
  - relation between the launch angles that result in the same range.]
- Predict qualitatively how air resistance affects projectile motion. [This includes analysis of both the horizontal component and vertical component of velocity and hence predicting qualitatively the range of the projectile.]
- Apply the principle of conservation of momentum to solve simple problems [Including elastic and inelastic interactions between objects in both one and two dimensions.
  Knowledge of the concept of coefficient of restitution is not required.
  Examples of applications include:
  - karate chops to break a pile of bricks
  - car crashes
  - ball & bat
  - the motion under thrust of a rocket in a straight line considering short thrusts during which the mass remains constant]
- Predict and analyse motion for elastic collisions [This includes making use of the fact that for an elastic collision, total kinetic energy is conserved and the relative speed of approach is equal to the relative speed of separation]
- Justify how the momentum of a closed system is always conserved, some change in kinetic energy may take place.

2.1 SCALARS

Q1. Define scalar and vector quantities. Also give their examples.

Ans. Scalar Quantities:

Definition

The Physical quantities which can be described only by their numerical value without direction are called scalar quantities.

Examples: Mass, distance, speed, time, energy, temperature, etc.

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Vector Quantities

Definition

The physical quantities which can be described with both numerical value and direction are called vector quantities.

Examples: Displacement, velocity, acceleration, force, etc.

Q 2. How a vector is represented graphically? Given an example.

Ans. Graphical Representation of a Vector

A graphical representation of a vector shows the vector with an arrow in a coordinate system. The length of the line represents the magnitude (size) of the vector, and the direction of the arrow indicates the direction of the vector.

Example: Consider a vector $\vec{P}$ of magnitude 5 cm, which makes an angle θ equal to 45° along the x-axis. This vector $\vec{P}$ graphically can be represented as

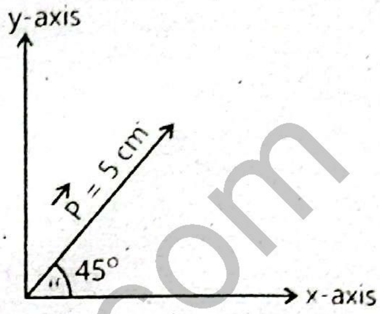

Fig. 1

2.2 VECTORS

Q 3. What are rectangular components of a vector? How a vector can be determined from its rectangular components?

Ans. Component of a Vector

Definition

A component of a vector is its effective value in a given direction.

Rectangular Components of a Vector

Definition

The components of a vector which are mutually perpendicular to each other are called its rectangular components.

Explanation

Consider a vector **A** represented by a line OP making an angle θ with the x-axis. Draw projection OM of vector **A** on x-axis and projection ON of vector **A** on y-axis. Projection OM being along x-axis is represented by $\mathbf{A_x}$ and projection ON along y-axis is represented by $\mathbf{A_y}$. By applying head to tail rule:

$$\mathbf{A} = \mathbf{A_x} + \mathbf{A_y}$$

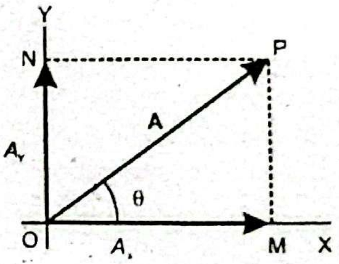

Fig. 2

Thus, $\mathbf{A_x}$ and $\mathbf{A_y}$ are the components of vector **A**. Since these are at right angles to each other, they are called rectangular components of vector **A**.

Considering the right angled triangle OMP, the magnitude of $\mathbf{A_x}$ or x-component of vector **A** is:

$$A_x = A\cos\theta$$

And the magnitude of $\mathbf{A_y}$ or y-component of vector **A** is:

$$A_y = A\sin\theta$$

Determination of a Vector from its Rectangular Components

If the rectangular components of a vector as shown in Fig. (2) are given, we can find out the magnitude of the vector by using Pythagorean theorem.

In the right angle ΔOMP:

$$(OP)^2 = (OM)^2 + (MP)^2$$

or $$A^2 = A_x^2 + A_y^2$$

or $$A = \sqrt{A_x^2 + A_y^2}$$

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The direction θ is given by

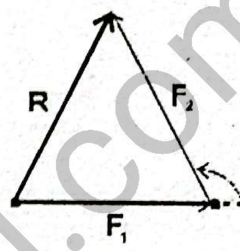
$$\tan\theta = \frac{MP}{OM} = \frac{A_y}{A_x}$$

or $$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$

Example 2.1: Find the angle between two forces of equal magnitude when the magnitude of their resultant is also equal to the magnitude of either of these forces.

Solution: Let θ be the angle between two forces F_1 and F_2, where F_1 is along the x-axis. Then x-component of their resultant will be:

$$R_x = F_1\cos 0^\circ + F_2\cos\theta$$
$$R_x = F_1 + F_2\cos\theta$$



And y-component of their resultant is

$$R_y = F_1\sin 0^\circ + F_2\sin\theta$$
$$R_y = F_2\sin\theta$$

The resultant R is given by

$$R^2 = R_x^2 + R_y^2$$

As $R = F_1 = F_2 = F$

Hence $F^2 = (F + F\cos\theta) + (F\sin\theta)^2$

$F^2 = F^2 + F^2\cos^2\theta + 2F^2\cos\theta + F^2\sin^2\theta$

or $0 = 2F^2\cos\theta + F^2(\cos^2\theta + \sin^2\theta)$

or $0 = 2F^2\cos\theta + F^2$, $0 = F^2(2\cos\theta + 1)$ $(\because \cos^2\theta + \sin^2\theta = 1)$

As $F \neq 0$

So $2\cos\theta + 1 = 0$

or $\cos\theta = -0.5$

or $\theta = \cos^{-1}(-0.5) = 120^\circ$ **Ans.**

1. **Mathematically, the unit vector is given by**
   (A) $\hat{A} = \vec{A}A$ (B) $\hat{A} = A/A$
   (C) $\hat{A} = \vec{A}/A$ ✓ (D) None of these
2. **A single vector having the same effect as all the original vectors taken together is called:**
   (A) Resultant vector ✓ (B) Position vector
   (C) Equal vector (D) Unit vector
3. **Which one is a vector quantity?**
   (A) Length (B) Velocity ✓
   (C) Volume (D) Work
4. **When two vectors are antiparallel, the angle between them is:**
   (A) Zero (B) 90°
   (C) 180° ✓ (D) 270°
5. **Which one is a vector quantity?**
   (A) Power (B) Entropy
   (C) Inertia (D) Tension ✓
6. **The magnitude of vector $\vec{A}$ will be:**
   (A) Zero (B) 1
   (C) A^2 (D) A ✓
7. **The unit vector in the direction of a vector $\vec{A}$ is:**
   (A) $A = \frac{A}{A}$ (B) $\hat{A} = \overrightarrow{AA}$
   (C) $\hat{A} = \frac{\vec{A}}{A}$ ✓ (D) $\vec{A} = \frac{A}{\vec{A}}$
8. **A vector in space has components:**
   (A) 2 (B) 3 ✓
   (C) 4 (D) 5
9. **The magnitude of rectangular components of a vector are equal if its angle with the x-axis is:**
   (A) 30° (B) 45° ✓
   (C) 60° (D) 9°
10. **If two unit vectors perpendicular to each other are added, the magnitude of the resultant is:**
    (A) 2 (B) $\sqrt{2}$ ✓
    (C) $\sqrt{\frac{1}{2}}$ (D) 4
11. **If a vector of magnitude 10N is along y- axis, its component along x-axis is:**
    (A) Zero ✓ (B) 5N
    (C) 8.66N (D) 10N

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12. If $|a + b| = |a - b|$, then the angle between $\vec{a}$ and $\vec{b}$ is:
(A) Zero (B) 45°
(C) 90° ✓ (D) 180°

13. The angle between rectangular components of a vector is:
(A) 60° (B) 90° ✓
(C) 180° (D) zero

14. The magnitude of a vector $\vec{A} = \vec{A}_x - \vec{A}_y$ is:
(A) $\sqrt{(Ax^2) + (-Ay)^2}$ ✓ (B) $(Ay)^2 - (Ay)^2$
(C) $(Ax)^2 + (Ay)$ (D) $\sqrt{(Ax)^2 - (Ay)^2}$

15. The resultant of two forces 30 N and 40 N acting parallel to each other and in the same direction is:
(A) 30N (B) 40N
(C) 70N ✓ (D) 10N

16. The resultant of two forces 3N and 4N acting at right angle to each other is:
(A) 5N ✓ (B) 6N
(C) 1N (D) 7N

17. The resultant of two forces 5N and 12N making an angle of 90° with each other is:
(A) 17N (B) 7N
(C) 13N ✓ (D) 15N

18. If a vector $\vec{A}$ makes an angle of 0° with x-axis, its x-component is equal to:
(A) $A \cos \theta$ (B) A^2
(C) A ✓ (D) $A \sin \theta$

19. Maximum number of components of a vector may be:
(A) One (B) Two
(C) Three (D) Infinite ✓

20. The resultant of two forces 3N and 4N acting parallel to each other in the same direction is:
(A) 7N ✓ (B) 1N
(C) 5N (D) 4N

21. If both components R_x and R_y of a resultant vector $\vec{R}$ are negative, then angle "θ" of $\vec{R}$ with x-axis will be:
(A) $0 = 270^\circ$ (B) $180^\circ < \theta > 270^\circ$
(C) $180^\circ < \theta < 270^\circ$ ✓ (D) $\theta \leq 270^\circ$

22. A force of 10N makes an angle of 30° with y-axis. The magnitude of x-components will be:
(A) 5N ✓ (B) 8.66N
(C) 10N (D) Zero

23. Magnitude of the resultant vector of 6N and 5N which are perpendicular to each other is:
(A) 14N (B) 10N ✓
(C) 20N (D) 2N

24. Minimum number of unequal forces whose vector sum is zero are:
(A) 5 (B) 4
(C) 3 (D) 2 ✓

25. Resultant of two perpendicular vectors each of magnitude A is:
(A) A (B) $\sqrt{2}A$ ✓
(C) $A\sqrt{2}$ (D) A^2

26. The vector $\vec{A}$ has components Ax and Ay, the magnitude of Ax is given by
(A) A-A_y (B) $(A$-$A_y)^{1/2}$
(C) $(A$-$A_y)^{1/2}$ (D) $[A^2$-$A_y]^{1/2}$ ✓

27. Which of the following pair of forces give the resultant force zero?
(A) 2N & 2N ✓ (B) 1N & 2N
(C) 2N & 5N (D) 1N & 2N

28. The sum of two perpendicular forces 8N and 6N is:
(A) 2N (B) $-14N$
(C) 10N ✓ (D) $-2N$

29. If $AB \sin \theta = AB \cos\theta$ then the angle between $\vec{A}$ and $\vec{B}$ is:
(A) 30° (B) 45° ✓
(C) 60° (D) 90°

30. If the resultant of two vectors, each of magnitude 'F', is also of magnitude 'F' then the angle between them will be:
(A) 30° (B) 60°
(C) 90° (D) 120° ✓

31. If a force of 10N is acting along x-axis, then its component along y-axis is:
(A) Zero ✓ (B) 5 N
(C) 10 N (D) 15 N

32. Rectangular components have angle between them:
(A) 30° (B) 45°
(C) 60° (D) 90° ✓

33. Two forces of magnitude 10N each, their resultant is equal to 20N. The angle between them is:
(A) 180° (B) 30°
(C) 90° (D) 0° ✓

34. If the two components of a vector are equal in magnitude, the vector making an angle with x-axis will be:
(A) 30° (B) 45° ✓
(C) 60° (D) 90°

35. The effective value of a vector in a given direction is called:
(A) Component of a vector ✓
(B) Horizontal vector
(C) Vertical Vector (D) None of these

36. A force of 15 N makes an angle of 90° with x-axis, its y-component is:
(A) 15 N (B) Zero N ✓
(C) 30 N (D) 45 N

37. A force of 10 N makes an angle of 30° with x-axis its x-component will be:
(A) 5 N ✓ (B) 866
(C) $\frac{10}{\sqrt{2}}N$ (D) $10\sqrt{2}\,N$

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38. **The resultant of two perpendicular vectors each of magnitude A is:**

(A) A (B) $2A$

(C) $\sqrt{2}A$ ✓ (D) A^2

39. **The component along y-axis is half of the maximum value for angle θ = ?**

(A) 0° (B) 30° ✓

(C) 60° (D) 90°

40. **The resultant vector of two mutually perpendicular unit vectors has magnitude:**

(A) Unity (B) Zero

(C) $\sqrt{2}$ ✓ (D) $\frac{1}{\sqrt{2}}$

2.3 PRODUCT OF TWO VECTORS

Q 4. Define and explain scalar product of two vectors. Write down its characteristics. Also give examples.

Ans. Scalar Product

Definition

If the product of two vectors gives a scalar, then that multiplication of vectors is called dot product or scalar product of two vectors.

Scalar product can be found by taking the component of one vector in the direction of the other vector and multiplying it with the magnitude of the other vector.

Explanation

The scalar product of two vectors **A** and **B** is written as **A.B** and is defined as:

$$\mathbf{A.B} = AB\cos\theta$$

where A and B are the magnitudes of vectors **A** and **B** and θ is the angle between them.

For physical interpretation of dot product of two vectors **A** and **B**, these are first brought to a common origin; Fig. 3(a) then,

A.B = A (projection of **B** on **A**)

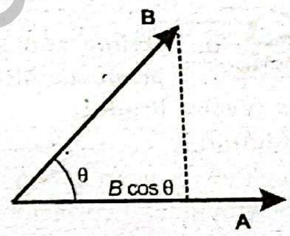

Fig. 3 (a)

Or **A.B** = **A** (magnitude of component of **B** in the direction of **A**); Fig. 3(b)

$= A(B\cos\theta) = AB\cos\theta$

Similarly **B.A** = $B(A\cos\theta) = BA\cos\theta$

This type of product when we consider the work done by a force **F** whose point of application moves a distance d in a direction making an angle θ with the line of action of **F**; Fig. 4.

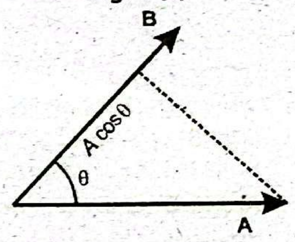

Fig. 3 (b)

Work done = (Effective component of force in the direction of motion) × Distance moved

$= (F\cos\theta)\,d = Fd\cos\theta$

Using vector notation:

F.d = $Fd\cos\theta$ = Work done

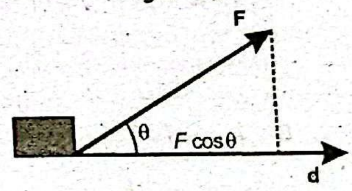

Fig. 4

Characteristics of Scalar Product

(i) Since **A.B** = $AB\cos\theta$ and **B.A** = $BA\cos\theta = AB\cos\theta$, hence, **A.B** = **B.A**. The order of multiplication is irrelevant. In other words, scalar product is commutative.

(ii) The scalar product of two mutually perpendicular vectors ($\theta = 90°$) is zero.

$$\mathbf{A.B} = AB\cos 90° = 0$$

(iii) The scalar product of two parallel vectors is equal to the product of their magnitudes. Thus, for parallel vectors ($\theta = 0°$).

$$\mathbf{A.B} = AB\cos 0° = AB$$

For antiparallel vectors ($\theta = 180°$)

$$\mathbf{A.B} = AB\cos 180° = -AB$$

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(iv) The self product of a vector **A** is equal to square of its magnitude.

$$\mathbf{A.A} = AA\cos 0^\circ = A^2$$

(v) Scalar product of two vectors **A** and **B** in terms of their rectangular components

$$\mathbf{A.B} = A_x B_x + A_y B_y + A_z B_z$$

Equation (2.6) can be used to find the angle between two vectors. Since,

$$\mathbf{A.B} = AB\cos\theta = A_x B_x + A_y B_y + A_z B_z$$

Therefore $$\cos\theta = \frac{A_xB_x + A_yB_y + A_zB_z}{AB}$$

Examples of Scalar Product

(i) Work done by a force

$$W = \vec{F}\cdot\vec{d} = Fd\cos\theta$$

If you push a box with a force of 10 N at an angle of 30° to the horizontal, and it moves 5 m, the work done is a scalar product of force and displacement.

(ii) Power

As $$P = \vec{F}\cdot\vec{v} = Fv\cos\theta$$

If a force 10 N acts on a body and it starts to move with a velocity of 5 ms^{-1} along the horizontal, the power is a scalar product of force and velocity.

Q 5. Define and explain vector product of two vectors. Discuss important characteristics of vector products. Also give examples.

Ans. Vector Product

Definition

If the product of two vectors gives a vector, then that multiplication of vectors is called cross-product or vector product of two vectors.

Explanation

The vector product of two vectors **A** and **B**, is a vector which is defined as:

$$\mathbf{A}\times\mathbf{B} = AB\sin\theta\,\hat{\mathbf{n}}$$

where $\hat{\mathbf{n}}$ is a unit vector perpendicular to the plane containing **A** and **B** as shown in Fig. 5(a). Its direction can be determined by right hand rule. For that purpose, place together the tail of vectors **A** and **B** to define the plane of vectors **A** and **B**. The direction of the product vector is perpendicular to this plane. Rotate the First vector **A** into **B** through the smaller of the two possible angles and curl the fingers of the right hand in the direction of rotation, keeping the thumb erect. The direction of the product vector will be along the erect thumb; Fig. 5(b). Because of this direction rule, **B** × **A** is a vector opposite in sign to **A** × **B**; Fig. 5(c). Hence,

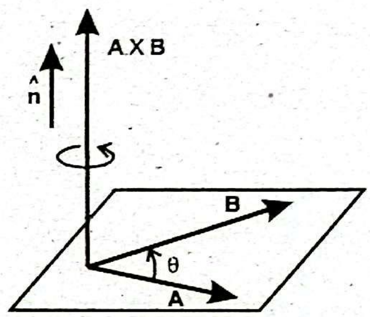

Fig. 5(a)

$$\mathbf{A}\times\mathbf{B} = -\mathbf{B}\times\mathbf{A}$$

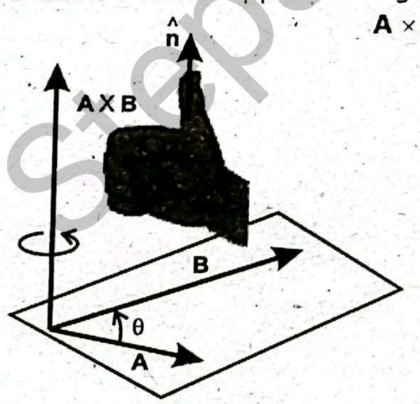

Fig. 5(b)

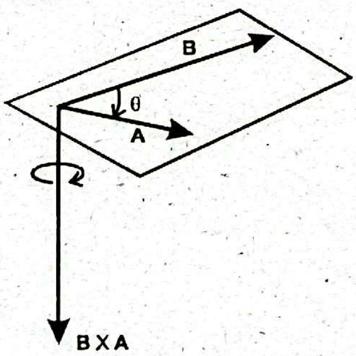

Fig. 5(c)

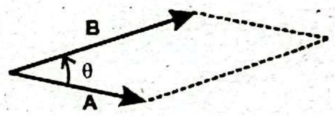

Fig. 5(d)

Characteristics of Cross Product

(i) Since **A** × **B** is not the same as **B** × **A**; the cross product is non commutative. so,

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$$\mathbf{A} \times \mathbf{B} \neq \mathbf{B} \times \mathbf{A}$$

(ii) The cross product of two perpendicular vectors ($\theta = 90^\circ$) has maximum magnitude.

$$\mathbf{A} \times \mathbf{B} = AB \sin 90^\circ\, \hat{\mathbf{n}} = AB\, \hat{\mathbf{n}}$$

(iii) The cross product of two parallel or anti-parallel vectors is a null vector, because for such vectors $\theta = 0^\circ$ or 180°. Hence,

$$\mathbf{A} \times \mathbf{B} = AB \sin 0^\circ\, \hat{\mathbf{n}} = 0 \quad \text{or} \quad \mathbf{A} \times \mathbf{B} = AB \sin 180^\circ\, \hat{\mathbf{n}} = 0$$

As a consequence $\mathbf{A} \times \mathbf{A} = \mathbf{0}$ ($\theta = 0^\circ$)

The magnitude of $\mathbf{A} \times \mathbf{B}$ is equal to the area of the parallelogram formed with **A** and **B** as two adjacent sides (Fig. 2.4-d).

Examples of Vector Product

(i) When a force **F** is applied on a rigid body at a point whose position vector is **r** from any point on the axis about which the body rotates, then the turning effect of the force called the torque τ is given by the vector product of **r** and **F**.

$$\text{Thus } \tau = \mathbf{r} \times \mathbf{F}$$

(ii) The force **F** on a particle of charge q and velocity **v** in a magnetic field of strength **B** is given by vector product of **v** and **B**.

$$\text{Thus } \mathbf{F} = q\,(\mathbf{v} \times \mathbf{B})$$

MCQs

1. **Dot product of a vector with itself is:**
(A) zero (B) $2A$
(C) A^2 ✓ (D) A

2. **It two non-zero vectors $\vec{A}$ and $\vec{B}$ are parallel to each other, then:**
(A) $\vec{A}.\vec{B} = 0$ (B) $\vec{A}.\vec{B} = AB$ ✓
(C) $|\vec{A} \times \vec{B}| = AB$ (D) $|\vec{A} \times \vec{B}| = \vec{A}.\vec{B}$

3. **If the magnitude of $\vec{A}.\vec{B} = \frac{1}{2} AB$, then the angle between $\vec{A}$ and $\vec{B}$ is:**
(A) 30° (B) 45°
(C) 60° ✓ (D) 90°

4. **If the angle between two vectors with the magnitude 12 and 6 is 60°, then their scalar product is:**
(A) 6 (B) 12
(C) 24 ✓ (D) None of these

5. **The scalar product of two vectors is maximum when they are:**
(A) Parallel ✓ (B) Perpendicular
(C) Null vector (D) Antiparallel

6. **Projection of $\vec{B}$ on $\vec{A}$ is:**
(A) A $\cos\theta$ (B) B $\cos\theta$
(C) A $\sin\theta$ (D) B $\sin\theta$ ✓

7. **The dot product of $\vec{A}$ with itself is equal to:**
(A) A (B) A^2 ✓
(C) Zero (D) 2A

8. **Scalar product of force and velocity is called:**
(A) Work ✓ (B) Power
(C) Energy (D) Acceleration

9. **Self dot product of a vector $\vec{A}$ is:**
(A) A (B) A^2 ✓
(C) Zero (D) B

10. **The projection of $\vec{A}$ in the direction of $\vec{B}$ is:**
(A) B $\cos\theta$ (B) AB $\cos\theta$ ✓
(C) A $\cos\theta$ (D) A $\sin\theta$

11. **Dot product of two antiparallel vectors A and B is:**
(A) AB $\cos\theta$ (B) AB
(C) 0 (D) −AB ✓

12. **Scalar product of two mutually perpendicular vectors $\vec{A}$ and $\vec{B}$ is:**
(A) AB $\cos\theta$ (B) 1
(C) AB $\sin\theta$ (D) 0 ✓

13. **Both the dot product and cross product of two vectors $\vec{A}$ and $\vec{B}$ is zero when:**
(A) $\vec{A}$ and $\vec{B}$ are parallel to each other
(B) $\vec{A}$ and $\vec{B}$ are ant parallel
(C) A and B are perpendicular to each other
(D) Either the vector is zero ✓

14. **The vector product ($A \times A$) is:**
(A) 0 (B) 1
(C) A (D) 0 ✓

15. **The magnitude of vector product of two non-zero vectors A and B making an angle θ with each other is:**
(A) $AB \sin\theta$ ✓ (B) $A + B \sin\theta$
(C) $AB \cos\theta$ (D) AB

16. **The magnitude of dot and cross product of two vectors are $6\sqrt{3}$ and 6 respectively. The angle between them is:**
(A) 0° (B) 30° ✓
(C) 45° (D) 60°

17. **The magnitude of dot and cross product of two vectors are equal when angle between them is:**
(A) Zero (B) 45° ✓
(C) 90° (D) 270°

18. **The cross product of a vector A with itself has the magnitude:**

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(A) A　　(B) 1
(C) A^2　　(D) Zero ✓

19. An area of parallelogram formed by A and B two adjacent sides is given as:
(A) $AB\sin\theta$ ✓　　(B) $AB\cos\theta$
(C) $AB\tan\theta$　　(D) $A \cdot B$

20. If $\vec{A}\times\vec{B} = (0)$, then angle between the vectors is:
(A) 90°　　(B) 450°
(C) 0° ✓　　(D) None of these

21. $AB\sin\theta\,\hat{n}\times AB\sin\theta\,\hat{n}$ is:
(A) $A^2B^2\sin^2\theta$　　(B) A^2B^2
(C) $A^2B^2\hat{n}$　　(D) $\vec{0}$ ✓

22. Three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ satisfy the relation $\vec{A}.\vec{B} = 0$ and $\vec{A}.\vec{C} = 0$, the vector $\vec{A}$ is parallel to:
(A) $\vec{B}$　　(B) $\vec{C}$
(C) $\vec{B}\cdot\vec{C}$　　(D) $\vec{B}\times\vec{C}$ ✓

23. If cross product of two vectors $\vec{A}\times\vec{B}$ points along positive z-axis, then the vectors $\vec{A}$ and $\vec{B}$ must lie in:
(A) yz-plane　　(B) xz-plane
(C) zy-plane ✓　　(D) No plane

24. If the magnitudes of scalar products and vector product of two vectors are $2\sqrt{3}$ and 2 respectively, the angle between the vectors is:
(A) 30° ✓　　(B) 60°
(C) 120°　　(D) 180°

25. If $\vec{A}\times\vec{B}$ is along y-axis, then $\vec{A}$ and $\vec{B}$ are in:
(A) xy-plane　　(B) yz-plane
(C) Space　　(D) xz-plane ✓

26. The cross product of two antiparallel vectors $\vec{A}$ and $\vec{B}$ is:
(A) AB Cosθ　　(B) AB
(C) 0 ✓　　(D) –AB

2.4 EQUATIONS OF MOTIONS

Q 6. Derive first equation of motion by graphical method.

Ans. Derivation of First Equation of Motion

Suppose a body is moving with uniform acceleration along a straight line with an initial velocity v_i. Suppose its velocity changes from initial value v_i to a final value v_f in time interval t. Then the acceleration produced in the body during this time interval is given as:

$$a = \frac{v_f - v_i}{t}$$

Rearranging, we can write

$$v_f - v_i = at$$

This is the first equation of motion. It correlates the final velocity attained by a body with initial velocity and the time interval t, when moving with constant acceleration a.

Derivation of First Equation of Motion by Graphically

First equation of motion can be derived using velocity-time graph for an object moving with initial velocity v_i final velocity v_f and constant acceleration a.

Let the velocity of a body at point A be v_i which changes to v_f at point B in time interval t; Fig. 6. A perpendicular BD is drawn from point B to x-axis and another perpendicular BE from B on y-axis, such that

OA = v_i = Initial velocity of the body

OE = DB = v_f = Final velocity of the body

From the graph, it can observed that;

DB = DC + CB

DB = OA + CB　　(As OA = DC)

Therefore　v_f = CB + v_i

The value of CB in the above equation can be determined by taking the slope of line AB, which is equal to acceleration a.

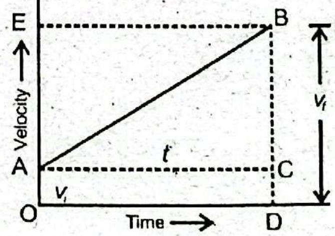

Fig. 6: Velocity-Time Graph

$$a = \frac{CB}{AC}$$

As　$AC = t$

So　$$a = \frac{CB}{t}$$

or　$CB = at$

Combining Eqs. (2.12) and (2.13), we have

$$v_f = v_i + at$$

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This is the first equation of motion.

7. Derive second equation of motion of graphical method.

Ans. Second Equation of Motion

Suppose a body is moving with uniform acceleration a along a straight line with an initial velocity v_i, which become v_f, after time interval t. Let it covers a distance S in a particular direction during time t, then using the definition of velocity as rate of change of displacement, we can write

Velocity = Displacement / Time

or Displacement = Velocity × Time

If velocity of the body is not constant, we can use average velocity instead of velocity.

Thus Displacement = Average velocity × Time

$$\text{Displacement} = \frac{(\text{Initial velocity} + \text{Final velocity})}{2} \times \text{Time}$$

$$S = \frac{(v_i + v_f)}{2} \times t$$

Using first equation of motion,

$$S = \frac{(v_i + v_i + at)}{2} \times t$$

$$S = \frac{(2v_i + at)}{2} \times t$$

$$2S = 2\,v_i t + at^2$$

$$S = v_i t + \frac{1}{2} at^2$$

This is the second equation of motion.

Derivation of Second Equation of Motion Graphically

Second equation of motion can be derived using velocity-time graph for a body moving with initial velocity v_i which attains a final value v_f in time interval t. While moving with constant acceleration a, it covers a displacement S in time t.

It can be seen from the graph that distance travelled by the body is, $S = v \times t$.

Also S = Area of the figure OABD

S = (Area of the rectangle OACD) + (Area of the triangle ABC)

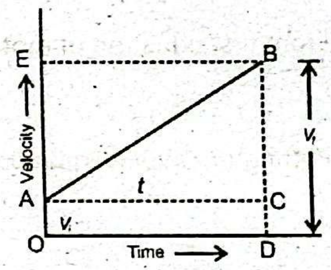

Fig. 7: Velocity-Time Graph

$$S = (OA \times OD) + \frac{1}{2}(AC \times BC)$$

As OA = v_i and OD = AC = t. So, the above equation becomes:

$$S = v_i \times t + \frac{1}{2}(t \times BC)$$

Here BC = at (From graphical representation of first equation of motion). By putting this value in the above equation, we have

$$S = v_i t + \frac{1}{2}(t \times at)$$

$$S = v_i t + \frac{1}{2} a t^2$$

This is the second equation of motion.

8. Derive third equation of motion by graphical method.

Ans. Derivation of Third Equation of Motion

Consider a body moving along a straight line with an initial velocity v_i which attains a final value v_f in time t. Let the displacement of the body be S during this time interval. Then, we can write:

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$$\text{Displacement} = \left(\frac{\text{Initial velocity} + \text{Final velocity}}{2}\right) \times \text{Time}$$

$$S = \frac{(v_i + v_f)}{2} \times t$$

$$2S = (v_i + v_f) \times t$$

Using the first equation of motion:

$$v_f = v_i + at$$

or $$t = \frac{v_f - v_i}{a}$$

Putting the value of t in Eq. (2.15)

$$2S = (v_i + v_f)\left(\frac{v_f - v_i}{a}\right)$$

$$2aS = v_f^2 - v_i^2$$

This is the third equation of motion.

Derivation of Third Equation of Motion Graphically:

In the speed-time graph Fig. 8, the total distance S travelled by a body is given by the area OABD under the graph, such that

$$S = \frac{1}{2}(\text{Sum of parallel sides}) \times \text{Height}$$

$$S = \frac{1}{2}(OA + BD) \times OD$$

Since $OA = v_i$, $BD = v_f$ and $OD = t$

The above equation becomes:

$$S = \frac{1}{2}(v_i + v_f) \times t$$

From first equation of motion

$$t = \frac{v_f - v_i}{a}$$

Putting t in above equation

$$S = \frac{1}{2}(v_i + v_f)\frac{(v_f - v_i)}{a}$$

or $$S = \frac{1}{2}(v_f + v_i)\frac{(v_f - v_i)}{a}$$

$$2aS = v_f^2 - v_i^2$$

This is the third equation of motion.

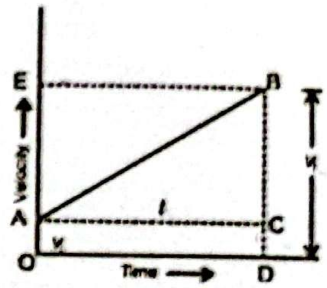

Fig. 8: Velocity-Time Graph

Q 9. Define acceleration due to gravity and state the use of equations of motion.

Ans. Acceleration due to Gravity

Definition

Acceleration due to gravity is the rate at which an object accelerates when it falls freely towards the Earth under the influence of gravity alone.

In the absence of **air resistance,** all objects in free fall at the surface of the Earth, move towards the Earth with a uniform acceleration. This acceleration is known as acceleration due to gravity, denoted by **g** and its average value at the Earth's surface is $9.8\ m\ s^{-2}$ in the downward direction.

Use of Equations of Motion:

The three equations of motion are fundamental in physics for describing the motion of objects under uniform acceleration. These equations relate displacement, velocity, acceleration, and time, and they are used in solving problems involving linear motion.

These equations of motion can only be applied to those objects, which are moving in a straight line with constant acceleration.

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Example 2.2: A car travelling at 10 ms^{-1} accelerates uniformally at 2 m s^{-2}. Calculate its velocity after 5 s.

Solution:

Given that;

$v_i = 10\ ms^{-1}$

$a = 2\ ms^{-2}$

$t = 5\ s$

To Find:

$v_f = ?$

Calculations:

Using first equation of motion, we can write:

$$v_f = v_i + at$$

Putting the values

$$v_f = 10\ ms^{-1} + 2\ ms^{-2} \times 5s$$

$$v_f = 10\ ms^{-1} + 10\ ms^{-1}$$

$$v_f = 20\ ms^{-1}$$ **Ans.**

Example 2.3: A car travels with initial velocity of 15 ms^{-1}. It accelerates at a rate of 2 ms^{-2} for 4 seconds. Find the displacement of the car.

Solution:

Given that;

$v_i = 15\ ms^{-1}$

$a = 2\ ms^{-2}$

$t = 4\ s$

To Find:

Displacement $S = ?$

Calculations:

By using 2nd equation of motion

$$S = v_i t + \frac{1}{2}at^2$$

Putting the values

$$S = (15\ ms^{-1} \times 4\ s) + \frac{1}{2}(2\ ms^{-2})(4s)^2$$

$$S = 76\ m$$ **Ans.**

Example 2.4: In a short distance race, a contestant in a car starts from rest and reaches the velocity of 300 km h^{-1}, after covering a distance of 0.45 km at a constant acceleration. Find this constant acceleration.

Solution:

Given that;

Initial velocity = $v_i = 0$

Final velocity = $v_f = 300\ km\ h^{-1}$

$$v_f = \frac{300 \times 1000}{60 \times 60}\ ms^{-1}$$

$$= 83.33\ ms^{-1}$$

Distance covered = S = 0.45 km = 0.45 × 1000 m = 450 m

To Find:

Acceleration = $a = ?$

Calculations:

Using third equation of motion, we have

$$v_f^2 - v_i^2 = 2aS$$

Putting the values

$$(83.33\ ms^{-1})^2 - (0)^2 = 2 \times a \times 450\ m$$

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$$a = \frac{6943.88\ m^2s^{-2}}{900\ m}$$

$a = 7.72\ m\ s^{-2}$ **Ans.**

2.5 MOTION UNDER GRAVITY

10. Write down the three equations of motion for freely falling bodies.

Ans. Motion Under Gravity

When an object moves under the influence of the gravity force, its motion is known as a motion under gravity. The gravity of the Earth, pulls the object toward it with an acceleration of 9.8 ms^{-2}, which is known as acceleration due to gravity.

The equations of motion for a freely falling body, on putting $a = g$, become:

$$v_f = v_i + gt \quad \text{......... (i)}$$

$$S = h = v_i t + \frac{1}{2} g t^2 \quad \text{......... (ii)}$$

$$v_f^2 - v_i^2 = 2gh \quad \text{......... (iii)}$$

Example 2.5: An iron ball of mass 1 kg is dropped from a tower. The ball reaches the ground in 3.34 s. Find: (a) the velocity of the ball on striking the ground, (b) the height of the tower.

Solution: Since the ball is falling under the action of gravity, we shall putt $a = g$ in equations of motion.

Given that:

Mass of the ball	$m = 1$ kg
Time taken to reach ground	$t = 3.34$ s
Initial velocity	$v_i = 0$
Acceleration	$a = g = 9.8\ ms^{-2}$

To Find:

(a)	Find velocity	$v_f = ?$
(b)	Height of tower	$h = ?$

Calculations:

(a) Using first equation of motion:

$$v_f = v_i + gt$$

$$v_f = 0 + (9.8\ ms^{-2})\ (3.34\ s)$$

$v_f = 32.7\ ms^{-1}$ **Ans.**

(b) Using third equation of motion:

$$v_f^2 - v_i^2 = 2gh$$

$$(32.7\ ms^{-1}) - (0)^2 = 2 \times 9.8\ ms^{-2} \times h$$

$$h = \frac{1069.29\ m^2\ s^{-2}}{19.6\ m\ s^{-2}}$$

$h = 54.56$ m **Ans.**

MCQs

1. **A bullet shot straight up returns to its starting point in 10s, its initial speed was:**
   (A) $9.8ms^{-1}$ (B) $24ms^{-1}$
   (C) $49ms^{1}$ ✓ (D) $98ms^{1}$
2. **A cricket ball is hit so that it travels straight up in air and it requires 3 s to reach the maximum height. Its initial velocity is:**
   (A) $10ms^{-1}$ (B) $15ms^{-1}$
   (C) $29.4ms^{-1}$ ✓ (D) $12.2ms^{1}$
3. **The equations of motion are not useful for objects moving with:**
   (A) Uniform velocity (B) Uniform acceleration
   (C) Variable acceleration ✓
   (D) Variable velocity
4. **A ball is allowed to fall freely from certain height. It covers a distance in 1st second equals to:**
   (A) 2g (B) g
   (C) g/2 ✓ (D) None of these

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5. **The distance covered by a freely falling body in two seconds is:**

(A) 9.8m (B) 19.6m ✓
(C) 44.4m (D) 49m

6. **The velocity of a free falling body just before hitting the ground is 9.8ms^{-1}, the height through which it has fallen will be:**

(A) 98m (B) 19.6m
(C) 4.9m ✓ (D) 9.8m

7. **A body having uniform acceleration of 10ms^{-2} has the velocity of 100ms^{-1}. In what time its velocity will be double?**

(A) 8 s (B) 10 s ✓
(C) 12 s (D) 14 s

8. **The value of 'g' at the centre of the Earth is:**

(A) Infinite (B) 2g
(C) 3g (D) Zero ✓

9. **Acceleration of bodies of different masses allowed to fall freely is (air friction is negligible):**

(A) Same in magnitude and direction ✓
(B) Same in magnitude only
(C) Same in direction only
(D) Different for different bodies

10. **If a body is moving with constant velocity of 10ms^{-1} towards west, then its acceleration is:**

(A) 1ms^{-2} (B) 10ms^{-2}
(C) 30ms^{-2} (D) Zero ✓

11. **A car starts from rest and covers a distance of 100m in one second with uniform acceleration. Its acceleration is:**

(A) 50ms^{-2} (B) 100ms^{-2}
(C) 200ms^{-2} ✓ (D) 300ms^{-2}

12. **When a body is thrown straight up, its velocity becomes zero at the highest point and its acceleration will be:**

(A) Zero (B) +9.8ms^{-2}
(C) -9.8ms^{2} ✓ (D) Undetermined

13. **Velocity of an object dropped from a building at any instant 't' is given by**

(A) $\frac{1}{2}gt^2$ (B) $v_it + \frac{1}{2}gt^2$
(C) at (D) gt ✓

14. **The distance covered by a freely falling body in first 2 seconds, when its initial velocity was zero will be:**

(A) 9.8m (B) 39.2m
(C) 19.6m ✓ (D) 4.9m

15. **Distance travelled by free falling object in first second is:**

(A) 4.9m ✓ (B) 9.8m
(C) 16.6m (D) 10m

16. **Which of the given variable is present in all the equations of motion?**

(A) Acceleration ✓ (B) distance
(C) Time (D) Torque

17. **What does the equation; $v_f = v_i + at$ represent?**

(A) Final position in terms of velocity
(B) Final velocity in terms of acceleration and time ✓
(C) Initial velocity in term of time
(D) Distance covered in time t

18. **The equation; $S = v_it + \frac{1}{2}at^2$ is used to calculate:**

(A) Final velocity (B) Initial velocity
(C) Displacement ✓ (D) Acceleration

19. **An object is projected vertically upward. At what point its acceleration is zero?**

(A) At the starting point (B) At the starting point
(C) Never ✓ (D) When it comes down

20. **When an object is in free fall, what force is acting on it (Neglecting air resistance)?**

(A) Frictional force (B) Magnetic force
(C) Gravitational force ✓ (D) Electric force

21. **An object thrown vertically upward returns to the same point. Its final velocity at the point of return is:**

(A) Zero
(B) Equal Initial velocity but in opposite direction ✓
(C) Greater than initial velocity
(D) Less than initial velocity.

2.6 PROJECTILE MOTION

11. What is projectile motion? Write the expressions for horizontal and vertical distances.

Ans. Projectile motion

Definition

Projectile motion is two-dimensional motion under constant acceleration due to gravity.

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Explanation

Consider the motion of a ball, when it is thrown horizontally from a certain height. It is observed that the ball travels forward as well as falls downward, until it strikes something such as ground.

Suppose that the ball leaves the hand of the thrower at point A and that its velocity at that instant is completely horizontal. Let this velocity be v_x.

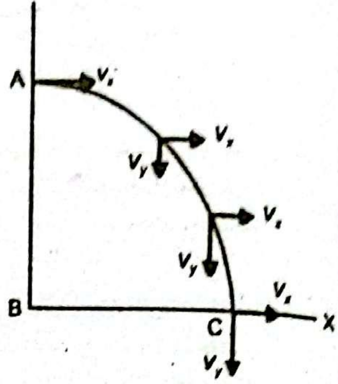

Fig. 9(a)

According to Newton's first law of motion, there will be no acceleration in horizontal direction, unless a horizontally directed force acts on the ball. Ignoring the air friction, only force acting on the ball during the flight is the force of gravity. There is no horizontal force acting on it. So, its horizontal velocity will remain unchanged and will be v_x, until the ball hits the ground. The ball moves with constant horizontal velocity component. Hence, horizontal distance x is given by

$$x = v_x \times t$$

For the vertical motion of the ball, it will accelerate downward under the force of gravity and, hence, $a = g$. This vertical motion is the same as for a freely falling body. Since initial vertical velocity is zero, hence, vertical distance y, is given by

$$y = \frac{1}{2}gt^2$$

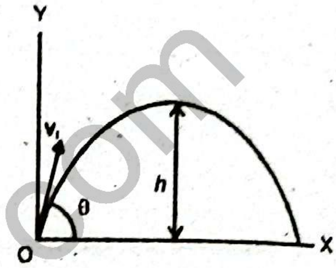

Fig. 9(b)

Examples of Projectile Motion

A football kicked off by a player; a ball thrown by a cricketer and a missile fired from a launching pad, all projected at some angles with the horizontal are examples of projectile motion.

12. Find the expressions for the magnitude of velocity at any instant and the angle which the resultant velocity makes with the horizontal for projectile motion.

Ans. Suppose that a projectile is fired along an angle θ with the horizontal by velocity $\mathbf{v_i}$; as Fig. 9(b). Let components of velocity $\mathbf{v_i}$ along the horizontal and vertical directions be $v_i \cos\theta$ and $v_i \sin\theta$, respectively. The horizontal acceleration is $a_x = 0$, because we have neglected air resistance and no other force is acting along this direction, whereas the vertical acceleration is $a_y = g$. Hence, the horizontal component v_{ix} remains constant and at any time t, we have

$$v_{fx} = v_{ix} = v_i \cos\theta$$

Now we consider the vertical motion. The initial vertical component of the velocity is $v_i \sin\theta$ in the upward direction.

The vertical component v_{fy} at any instant t can be determined by considering the upward motion of projectile as free fall motion ($a_y = -g$). Using 1st equation of motion:

$$v_{fy} = v_i \sin\theta - gt$$

The magnitude of velocity at any instant is:

$$v_{fy} = \sqrt{v_{fx}^2 + v_{fy}^2}$$

The angle ϕ which this resultant velocity makes with the horizontal can be found from:

$$\tan\phi = \frac{v_{fy}}{v_{fx}}$$

13. Derive the expression for:

(i) Time of flight **(ii) Height attained by the projectile**

(iii) Range of projectile

Ans. Height of the Projectile

Definition

The height of a projectile is the vertical position of the projectile above its point of launch at any given moment during its motion.

In order to determine the maximum height the projectile attains, we use the equation of motion:

$$2aS = v_f^2 - v_i^2$$

As body moves upward, $a = -g$, the initial vertical velocity $v_{iy} = v_i \sin\theta = v_i$ as $v_{fy} = 0 = v_f$, because the body comes to rest after reaching the highest point. Since

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$$S = \text{height} = h$$

$$-2gh = 0 - v_i^2 \sin^2 \theta$$

or
$$h = \frac{v_i^2 \sin^2 \theta}{2g}$$

The height of projectile will be reduced in presence of air resistance. In the presence of air resistance, the upward velocity of the projectile will decrease and hence, its height will also decrease during time t.

Time of Flight

Definition

The time taken by body to cover the distance from the place of its projection to the place where it hits the ground is called the time of flight.

This can be obtained by taking $S = h = 0$, because the body goes up and comes back to the same level, thus, covering no vertical distance. If the body is projecting with velocity v_i making angle θ with the horizontal, then its vertical component will be $v_i \sin \theta$. Hence, the equation of motion is:

$$S = v_i t + \frac{1}{2} g t^2$$

$$0 = v_i \sin \theta \, t - \frac{1}{2} g \, t^2$$

$$t = \frac{2 \, v_i \sin \theta}{g}$$

where t is the time of flight of the projectile when it is projected from the ground.

Range of the Projectile

Definition

Maximum distance which a projectile covers in the horizontal direction is called the range of the projectile.

To determine the range R of the projectile, we multiply the horizontal component of the velocity of projection with total time taken by the body to hit the ground after leaving the point of projection. Thus,

$$R = v_{ix} \times t$$

or
$$R = \frac{v_i \cos \theta \times 2 \, v_i \sin \theta}{g}$$

or
$$R = \frac{v_i^2}{g} 2 \sin \theta \cos \theta$$

As $2 \sin \theta \cos \theta = \sin 2\theta$, thus, the range of the projectile depends upon the velocity of projection and the angle of projection.

Therefore
$$R = \frac{v_i^2}{g} \sin 2\theta$$

For maximum range R, the factor $\sin 2\theta = 1$, so

$$2\theta = \sin^{-1}(1) \quad \text{or} \quad 2\theta = 90^\circ \quad \text{or} \quad \theta = 45^\circ$$

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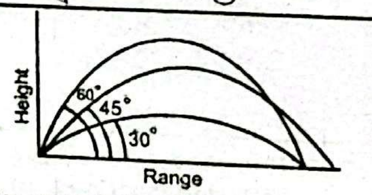

For an angle less than 45°, the height reached by the projectile and the range both will be less. When the angle of projectile is larger than 45°, the height attained will be more but the range is again less.

Example 2.6: A ball is thrown with a speed of 30 m s^{-1} in a direction 30° above the horizon. Determine the height to which it rises, the time of flight and the horizontal range.

Solution: Initially;

$$v_{ix} = v_i \cos \theta = 30 \text{ ms}^{-1} \times \cos 30^\circ = 25.98 \text{ m s}^{-1}$$

$$v_{iy} = v_i \sin \theta = 30 \text{ m s}^{-1} \times \text{sni } 30^\circ = 15 \text{ m s}^{-1}$$

Calculations:

As the time of flight is:

$$t = \frac{2v_i}{g} \sin \theta$$

$$= \frac{2 \times 30 \text{ m s}^{-1}}{9.8 \text{ m s}^{-2}} \times (0.5)$$

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So $t = \dfrac{2 \times 15\text{ m s}^{-1}}{9.8\text{ m s}^{-2}} = 3.1\text{ s}$ **Ans.**

Height $h = \dfrac{v_i^2 \sin^2\theta}{2g}$

So $h = \dfrac{(30\text{ ms}^{-1})^2 (0.5)^2}{19.6\text{ m s}^{-2}}$

$h = 11.5\text{ m}$ **Ans.**

Range $R = \dfrac{v_i^2}{g}\sin 2\theta = \dfrac{v_i^2}{g}\sin 60^\circ$

So $R = \dfrac{(30\text{ m s}^{-1})^2 \times 0.866}{9.8\text{ m s}^{-2}}$

$R = 79.53\text{ m}$ **Ans.**

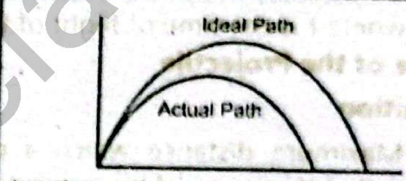
Q 14. What is the effect of air resistance on the range of a projectile?

Ans. Effect of Air Resistance on Range of a Projectile

Air resistance will slow down projectile forward motion, reducing its velocity v_i. The reduction in v_i will result in a decrease in the range of projectile.

Furthermore, air resistance is not constant throughout the flight of the object. As the object slows down, the air resistance experienced by it also decreases. This means that the object retards more slowly and accelerates more slowly as it falls down. This results in a trajectory that is not perfectly parabolic but is skewed, with steeper descent than ascent.

For your Information



In the presence of air friction the trajectory of a high speed projectile falls short of a parabolic path.

MCQs

1. **If the body of mass 2*kg* moving with 15 ms^{-1} collides with stationary body of same mass, then after elastic collision the 2nd body will move with the velocity of:**
(A) 15ms^{-1} ✓ (B) 30ms^{-1}
(C) Zero (D) None of these

2. **Rocket equation is given as:**
(A) $a = \dfrac{m}{mv}$ (B) $a = \dfrac{mv}{m}$
(C) $a = \dfrac{mv}{M}$ ✓ (D) $a = \dfrac{m}{MV}$

3. **As rocket moves upward during its motion, its acceleration goes on:**
(A) increasing ✓ (B) decreasing
(C) Remains the same
(D) It moves with uniform velocity

4. **Rocket objects that are burnt go at a speed of over (consuming fuel at rate of 1000 kgs^{-1}).**
(A) 4000 ms^{-1} ✓ (B) 400 ms^{-1}
(C) 40000 cms^{-1} (D) 400 cms^{-1}

5. **If a shell explodes in midair, its fragments fly off in different directions. The total momentum of fragments:**
(A) Decrease (B) Increase
(C) Becomes zero (D) Remains the same ✓

6. **A typical rocket consumes fuel at rate of (ejecting gas at speed of 4000 ms^{-1})**
(A) 10000 kgs^{-1} ✓ (B) 1000kgs^{-1}
(C) 100kgs^{-1} (D) 100000kgs^{-1}

7. **A 1500 kg has its velocity reduced from 20 ms^{-1} in 3.0 s. How large was the retarding force?**
(A) 500 N (B) 2500 N
(C) 1500 N (D) 10000 N ✓

8. **When a massive body of mass m_1 collides with lighter stationary body of mass m_2, the velocity of massive body after collision will be:**
(A) $v_1' = 2v_1$ (B) $v_2' = v_1$
(C) $v_1' = v_1$ ✓ (B) $v_2' = 2v_2$

9. **Which hurt you maximum when the time of collision is:**
(A) $\dfrac{1}{10}$ s (B) $\dfrac{1}{100}$ s
(C) $\dfrac{1}{1000}$ s ✓ (D) 1 s

10. **A ball is dropped from a height of 4.2 metres. To what height, it will rise if there is no loss after rebounding?**
(A) 4.2 *m* ✓ (B) 8.4 *m*
(C) 12.6 *m* (D) 2.4 *m*

11. **Two masses m_1 and m_2 will interchange their velocities after collision if:**
(A) $m_1 >> m_2$ (B) $m_1 = m_2$ ✓
(C) $m_2 >> m_1$ (D) m_2 is at rest

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12. **Motion of projectiles is:**
(A) One dimensional (B) two dimensional ✓
(C) three dimensional (D) Four dimensional

13. **The horizontal range is maximum when it is projected at an angle of:**
(A) 0° (B) 30°
(C) 45° ✓ (D) 60°

14. **The range of projectile is same for:**
(A) $0^\circ, 45^\circ$ (B) $35^\circ, 55^\circ$ ✓
(C) $15^\circ, 60^\circ$ (D) $30^\circ, 75^\circ$

15. **The horizontal component of velocity of a projectile.**
(A) Increases (B) Decreases
(C) Remains the same ✓ (D) Decreases then increases

16. **The velocity of projectile is ma. 'mum:**
(A) At the highest point (B) At t. e point of launch ✓
(C) At half of the height (D) After sticking the ground

17. **The horizontal range or projectile at 30° with horizontal is the same as that at an angle of:**
(A) 45° (B) 60° ✓
(C) 90° (D) 120°

18. **A projectile is thrown upward with the velocity v_i making an angle θ with the horizontal, the maximum horizontal range is:**
(A) $\frac{v_i^2}{2}$ ✓ (B) $\frac{v_i^2}{2g}$
(C) $\frac{v_i^2}{g}$ (D) $\frac{v_i^2}{g}\sin 2\theta$

19. **The time of flight of a projectile when it is projected from the ground is:**
(A) $\frac{v_i}{g}\cos\theta$ (B) $\frac{v_i^2}{g}\sin\theta$
(C) $\frac{2v_i^2}{g}\sin\theta$ ✓ (D) $\frac{v_i^2}{g}\sin\theta$

20. **In the projectile motion, the vertical component of velocity:**
(A) Remains constant (B) Varies point to point ✓
(C) Becomes zero (D) Increases with time

21. **The angle of projection for which its maximum height and horizontal range are equal:**
(A) 46° (B) 56°
(C) 66° (D) 76° ✓

22. **A bomber drops its bomb when it is vertically above the target, it misses the target due to:**
(A) Horizontal component of velocity ✓
(B) Vertical component of velocity
(C) Pull of the Earth
(D) Acceleration due to gravity

23. **A ball is thrown up with $20ms^{-1}$ at an angle of 60°, with x-axis, the velocity of a ball at the top position is:**
(A) $0ms^{-1}$ (B) $10ms^{-1}$ ✓
(C) $20ms^{-1}$ (D) $16ms^{-1}$

24. **The shape of trajectory of short range projectile is:**
(A) Circular (B) Parabola ✓
(C) Ellipse (D) hyperbole

25. **The path followed by a projectile is known as its:**
(A) Range (B) Trajectory ✓
(C) Cycle (D) Height

26. **For a rocket, the change in momentum per second of the ejecting gases is equal:**
(A) Acceleration of the rocket
(B) Momentum of rocket
(C) Velocity ~~of the rocket~~
(D) Thrust acting on rocket ✓

27. **The acceleration along x-axis direction in case of projectile is:**
(A) Zero ✓ (B) Equal to Gravity
(C) Maximum (D) Constant

28. **Which shows correct relation between H and T of projectile?**
(A) $H = \frac{gt^2}{8}$ ✓ (B) $H = \frac{8T^2}{g}$
(C) $H = \frac{8g}{T^2}$ (D) $H = \frac{8g}{gt^2}$

29. **If the initial velocity of a projectile becomes doubled, the time of flight will become:**
(A) Double ✓ (B) Remains the same
(C) 3 times (D) 4 times

30. **Time of flight of a projectile is:**
(A) $v_i \sin\theta$ (B) $\frac{v_i \sin\theta}{2g}$ ✓
(C) $\frac{v_i \sin\theta}{g}$ (D) $\frac{2 v_i \sin\theta}{g}$

31. **If maximum height of the projectile is equal to the range, then angle of projection of projectile will be:**
(A) 30° (B) 60°
(C) 45° (D) 76° ✓

32. **The maximum height attained by a projectile is:**
(A) $\frac{v_i^2 \sin^2\theta}{2g}$ ✓ (B) $\frac{v_i^2 \sin^2\theta}{g}$
(C) $\frac{v_i^2 \cos^2\theta}{2g}$ (B) $\frac{v_i^2 \cos^2\theta}{g}$

33. **The horizontal range of a projectile is:**
(A) $\frac{2v_i \sin\theta}{g}$ (B) $\frac{v_i^2 \sin^2\theta}{2g}$
(C) $\frac{v_i^2 \sin^2\theta}{g}$ ✓ (D) $\frac{v_i \sin^2\theta}{2g}$

34. **If the angle of projection is greater that 45°, then:**
(A) Height attained is more but range is less ✓
(B) Height attained is less but range is more
(C) Range and height attained is less
(D) Both height attained and range are more

35. **A ball is thrown with an initial speed of $30\ ms^{-1}$ in direction 30° above the Horizontal. Its vertical component of velocity is:**
(A) $25.98\ ms^{-1}$ (B) $30\ ms^{-1}$
(C) $1\ ms^{-1}$ (D) $15\ ms^{-1}$ ✓

36. **The horizontal component of acceleration of projectile is equal to:**
(A) $0\ ms^{-2}$ ✓ (B) $4.9\ ms^{-2}$
(C) $9.8\ ms^{-2}$ (D) $-9.8\ ms^{-2}$

37. **The range of projectile becomes half of the maximum range at angle of projection.**
(A) 15° ✓ (B) 25°
(C) 45° (D) 72°

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38. **The time to reach the maximum height by projectile is:**

(A) $\frac{v_i^2 \sin^2 \theta}{g}$ (B) $\frac{v_i^2 \sin \theta}{g}$ ✓

(C) $\frac{v_i^2 \sin \theta}{g^2}$ (B) $\frac{v_i^2 \sin \theta^2}{g^2}$

39. **Horizontal range is equal to 4 times of its maximum height only if angle of projection is:**

(A) 90° (B) 45° ✓

(C) 60° (D) 30°

2.7 MOMENTUM

15. Define momentum and impulse. Show that rate of change of momentum of a body is equal to the applied force.

Ans. Momentum

Definition

The quantity of motion of a moving body is called momentum.

or The product of mass and velocity of an object.

Mathematically,

$$\text{Momentum} = p = \text{Mass} \times \text{Velocity} = mv$$

It is a vector quantity. The SI unit of momentum is kg ms^{-1} or Ns.

Momentum and Newton's Second law of Motion

Consider a body of mass m moving with an initial velocity v_i. Suppose an external force F acts upon it for time t after which velocity becomes v_f. The acceleration a produced by this force is given by

$$a = \frac{v_f - v_i}{t} \quad \text{......... (i)}$$

By Newton's second law, the acceleration is given as:

$$a = \frac{F}{m} \quad \text{......... (ii)}$$

Equating Eqs. (i) and (ii), we have

$$\frac{F}{m} = \frac{v_f - v_i}{t}$$

$$F \times t = mv_f - mv_i \quad \text{......... (iii)}$$

where mv_i is the initial momentum and mv_f is the final momentum of the body.

Equation (iii) shows that change in momentum is equal to the product of force and the time for which force is applied.

From Eq. (iii)

$$F = \frac{m v_f - m v_i}{t}$$

Newton's Second Law in Terms of Momentum

Newton's second law of motion can also be stated in terms of momentum as:

Time rate of change of momentum of a body is equal to the applied force.

Impulse

Definition

Impulse can be defined as:

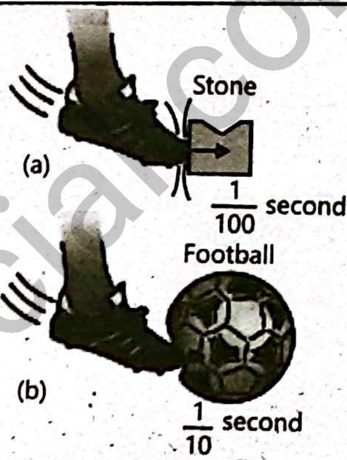
The change of momentum of an object when the object is acted upon by a force fro a very short interval of time.

Impulse is the change in momentum of an object. If the initial momentum of an object in p_i and the final momentum is p_f, then the object has received the impulse J. Such that;

$$\text{Impulse} = p_f - p_i = \Delta p$$

Momentum is a vector quantity, so impulse is also a vector quantity.

Point to Ponder!



Which hurt you in the above situations (a) or (b) and think why?

Ans. In situation (a)—stone, when we cick, we are hurt more because of its hardness, density and surface texture.

Point to Ponder!

Can a moving object experience impulse?

Ans. Yes, a moving object can experience impulse. Even if an object is already moving, if a force is applied to it, causing its velocity to change, it experiences an impulse.

Examples

(i) A moving car that hits, the brakes experience a negative impulse.

(ii) A Soccer ball being kicked again while it is already rolling experiences an additional impulse.

So yes, impulse is not limited to stationary objects; any change in momentum – including changes in speed or direction – involves impulses.

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Newton's second law of motion states that the rate of change of momentum of an object is equal to the resultant force F acting on the object. Thus

$$F = \frac{p_f - p_i}{\Delta t}$$

$$\text{Impulse} = F \times \Delta t = mv_f - mv_i$$

Example 2.7: A 1500 kg car has its velocity reduced from 20 m s^{-1} to 15 m s^{-1} in 3.0 s. How large was the average retarding force?

Solution:

Given that;

$v_f = 15\ ms^{-1}$

$v_i = 20\ ms^{-1}$

$m = 1500$ kg

To Find:

$F = ?$

Calculations:

Using the equation

$$F \times t = mv_f - mv_i$$

Putting the values

$$F \times 3.0\ s = 1500\ kg \times 15\ m\ s^{-1} - 1500\ kg \times 20\ m\ s^{-1}$$

or $F = -2500\ kg\ m\ s^{-2}$

$= -2500\ N = -2.5\ kN$ **Ans.**

The negative sign indicates that the force is retarding one.

16. State and prove law of conservation of momentum for an isolated system.

Ans. Law of Conservation of Momentum

Statement

The total linear momentum of an isolated system remains constant.

Explanation

Consider an isolated system on which no external agency exerts any force. For example, the molecules of a gas enclosed in a glass vessel at constant temperature constitute an isolated system. The molecules can collide with one another because of their random motion, but being enclosed by glass vessel, no external agency can exert a force on them.

Consider an isolated system of two smooth hard interacting balls of masses m_1 and m_2, moving along the same straight line, in the same direction, with velocities $\mathbf{v}_1$ and $\mathbf{v}_2$ respectively. Both the balls collide and after collision, ball of mass m_1 moves with velocity v_1' and m_2 moves with velocity v_2' in the same direction.

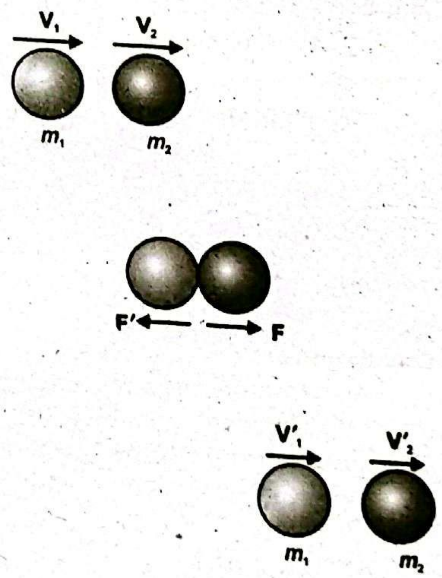

Fig. 10

To find the change in momentum of mass m_1, the equation;

$F \times t = mv_f - mv_i$ can be used as:

$$\mathbf{F} \times t = m_1\mathbf{v}_1' - m_1\mathbf{v}_1 \qquad \text{...... (i)}$$

Similarly, for the ball of mass m_2

$$\mathbf{F} \times t = m_2\mathbf{v}_2' - m_2\mathbf{v}_2 \qquad \text{...... (ii)}$$

Adding these two expressions:

$$(\mathbf{F} + \mathbf{F}')\, t = (m_1\mathbf{v}_1' - m_1\mathbf{v}_1) + (m_2\mathbf{v}_2' - m_2\mathbf{v}_2)$$

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Since the action force **F** is equal and opposite to the reaction force **F'**, thus **F'** = –**F**, or **F** + **F'** = 0. so the left hand side of the equation is zero. Hence,

$$0 = (m_1\mathbf{v}_1' - m_1\mathbf{v}_1) + (m_2\mathbf{v}_2' - m_2\mathbf{v}_2)$$

In other words, change in momentum of 1[st] ball + change in momentum of the 2[nd] ball is zero.

$$\text{or} \quad (m_1\mathbf{v}_1 + m_2\mathbf{v}_2) = (m_1\mathbf{v}_1' + m_2\mathbf{v}_2') \quad \text{...... (iii)}$$

Which means that total initial momentum of the system before collision is equal to the total final momentum of the system after collision. Consequently, the total change in momentum of the isolated two ball system is zero.

For such a group of objects, if one object within the group experiences a force, there must exist an equal but opposite reaction force on other object in the same group. As a result, the change in momentum of the group of objects as a whole is always zero. This is the law of conservation of momentum.

Did You Know?

When a moving car stops quickly, the passengers move forward towards the wind shield. Seatbelts change the forces of motion and prevent the passengers from moving. Thus, the chance of injury is greatly reduced.

Did You Know?

A motorcycle's safety helmet is padded so as to extend the time of any collision to prevent serious injury.

Isolated System

Definition

An isolated system is a physical system that does not exchange matter and energy with its surroundings.

It means;

- No mass enters or leaves the system.
- No energy is transferred to or from the system.

Example 2.8: Two spherical balls of 2.0 kg and 3.0 kg masses are moving towards each other will velocities of 6.0 m s^{-1} and 4 m s^{-1}, respectively. What must be the velocity of the smaller ball after collision, if the velocity of the bigger ball is 3.0 m s^{-1}?

Solution:

Given that;

$m_1 = 2$ kg , $m_2 = 3$ kg ,

$v_1 = 6$ ms^{-1} , $v_2 = -4$ ms^{-1}

$v_2' = -3$ ms^{-1}

To Find:

$v_1' = ?$

Calculations:

As both the balls are moving towards one another, so their velocities are of opposite sign. Let us suppose that the direction of motion of 2 kg ball is positive and that of the 3 kg is negative.

The momentum of the system before collision is:

$$m_1v_1 + m_2v_2 = 2\text{ kg} \times 6\text{ m s}^{-1} + 3\text{ kg} \times (-4\text{ m s}^{-1})$$

$$= 12\text{ kg m s}^{-1} - 12\text{ kg m s}^{-1} = 0$$

Momentum of the system after collision $= m_1v_1' + m_2v_2'$

$$= 2\text{ kg} \times v_1' + 3\text{ kg} \times (-3)\text{ m s}^{-1}$$

From the law of conservation of momentum

$$\begin{bmatrix}\text{Momentum of the system} \\ \text{before collision}\end{bmatrix} = \begin{bmatrix}\text{Momentum of the system} \\ \text{after collision}\end{bmatrix}$$

$$0 = 2\text{ kg} \times v_1' - 9\text{ kg m s}^{-1}$$

$$v_1' = 4.5\text{ m s}^{-1}$$ **Ans.**

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1. **The rate of change in momentum of a body is equal to:**
(A) Displacement (B) Velocity
(C) Acceleration (D) Applied force ✓

2. **Impulse can be defined as:**
(A) $1 = F \times d$ (B) $1 = F \times t$ ✓
(C) $1 = F \times v$ (D) $1 = F / P$

3. **If force of 10N acts on a body of mass 5kg for one second, then its rate of change of momentum will be:**
(A) $10\ kg\ ms^{-2}$ ✓ (B) $50\ kg\ ms^{-2}$
(C) $5\ kg\ ms^{-2}$ (D) $2\ kg\ ms^{-2}$

4. **Total change in momentum of a moving body is equal to its:**
(A) K.E. (B) Impulse ✓
(C) Force (D) Inertia

5. **An alternate unit to kg ms^{-1} is:**
(A) $J\ s$ (B) $N\ s$ ✓
(C) Nm (D) N

6. **The SI unit of impulses is:**
(A) $kgms^{-1}$ (B) Nm
(C) Ns ✓ (D) Nm^2

7. **At what speed the momentum and kinetic energy of a body having the same value?**
(A) $1ms^{-1}$ (B) $1ms^{-1}$ ✓
(C) $4ms^{-1}$ (D) $8ms^{-1}$

8. **Impulse has same unit as that of:**
(A) Force (B) Energy
(C) Mass (D) Linear momentum ✓

9. **kgms^{-1} is the unit of:**
(A) Force (B) Momentum ✓
(C) Power (D) Work done

10. **Rate of change of momentum of a freely falling object is equal to:**
(A) K.E. (B) Momentum
(C) Acceleration (D) Weight ✓

11. **Two object collide in an isolated system. The total momentum after the collision is:**
(A) Equal to total K.E.
(B) Zero
(C) Equal to total momentum before collision ✓
(D) Greater than total momentum before collision.

12. **A truck and car has the same momentum. Which of the following is true?**
(A) The car has more velocity than the truck. ✓
(B) The truck has more velocity than the car.
(C) Both have the same velocity.
(D) It is not possible to have same momentum.

2.8 ELASTIC AND INELASTIC COLLISIONS

Q 17. Define elastic and inelastic collisions. Also give examples.

Ans. Elastic Collision

Definition

A collision in which the system of bodies conserves both its total linear momentum and total kinetic energy is called elastic collision.

Examples

(i) When two similar trolleys are travelling toward each other with equal speed. They collide, bouncing off each other with no loss in speed. This collision is perfectly elastic because no energy has been lost.
(ii) When a ball at a billiard table hits another ball, it is an example of elastic collision.
(iii) When you throw a ball on the ground and it bounces back to you hand, there is not net change in the K.E., and hence, it is an elastic collision.
(iv) Collisions between gas molecules in an ideal gas are considered elastic.

Inelastic Collision

Definition

A collision in which the K.E. of the system is not conserved but momentum is conserved is called inelastic collision.

Examples

(i) When a cricketer or baseball player is catching a fast-moving ball. As the ball is caught, it comes to rest with the player's hands or glove, and K.E. is dissipated, showing an inelastic interaction.
(ii) When a ball is dropped from a certain height and it is unable to rise to its original height. This is done to inelastic collision.
(iii) The accident of two vehicles.
(iv) A football player tackling another and they both fall together. Here K.E. is not conserved.

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18. Explain elastic collision in one dimension. Show that relative velocities before and after collision are the same. Also discuss its different cases.

Ans. Elastic Collision in One Dimension

Consider two smooth, non-rotating balls of masses m_1 and m_2 moving initially with velocities $\mathbf{v_1}$ and $\mathbf{v_2}$ respectively, in the same direction. They collide and after collision, they move along the same straight line without rotation. Let their velocities after the collision be $\mathbf{v_1}'$ and $\mathbf{v_2}'$ respectively; Fig. (11).

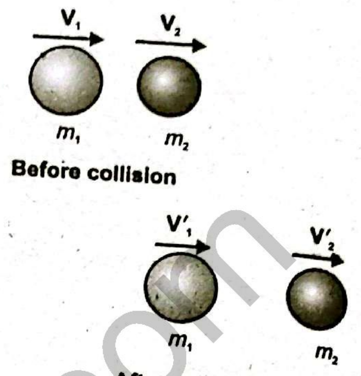

Fig. 11

We take the positive direction of the velocity and momentum to the right. By applying the law of conservation of momentum, we have

$$(m_1\mathbf{v_1} + m_2\mathbf{v_2}) = (m_1\mathbf{v_1}' + m_2\mathbf{v_2}')$$

$$m_1(\mathbf{v_1} - \mathbf{v_1}') = m_2(\mathbf{v_2}' - \mathbf{v_2}) \quad \text{...... (i)}$$

As the collision is elastic, so the *K.E.* is conserved. From the conservation of *K.E.*, we have

$$\frac{1}{2}m_1\mathbf{v_1}^2 + \frac{1}{2}m_2\mathbf{v_2}^2 = \frac{1}{2}m_1\mathbf{v_1}'^2 + \frac{1}{2}m_2\mathbf{v_2}'^2$$

or
$$m_1(\mathbf{v_1}^2 - \mathbf{v_1}'^2) = m_2(\mathbf{v_2}'^2 - \mathbf{v_2}^2)$$

or
$$m_1(\mathbf{v_1} + \mathbf{v_1}')(\mathbf{v_1} - \mathbf{v_1}') = m_2(\mathbf{v_2}' + \mathbf{v_2})(\mathbf{v_2}' - \mathbf{v_2}) \quad \text{...... (ii)}$$

Dividing Eq. (i) by (ii)

$$(\mathbf{v_1} + \mathbf{v_1}') = (\mathbf{v_2}' + \mathbf{v_2})$$

or
$$(\mathbf{v_1} - \mathbf{v_2}) = (\mathbf{v_2}' - \mathbf{v_1}') = -(\mathbf{v_1}' - \mathbf{v_2}')$$

We note that, before collision $(v_1 - v_2)$ is the velocity of first ball relative to the second ball. Similarly $(v_1' - v_2')$ is the velocity of the first ball relative to the second ball after collision. It means that relative velocities before and after the collision has the same magnitude but are reversed after the collision. In other words, the magnitude of relative velocity of approach is equal to the magnitude of relative velocity of separation.

In equations (i) and (ii) m_1, m_2, $\mathbf{v_1}$ and $\mathbf{v_2}$ are known quantities. We solve these equations to find the values of $\mathbf{v_1}'$ and $\mathbf{v_2}'$ which are unknown. The results are:

$$\mathbf{v_1}' = \frac{m_1 - m_2}{m_1 + m_2}\mathbf{v_1}' + \frac{2m_2}{m_1 + m_2}\mathbf{v_2} \quad \text{......... (iii)}$$

$$\mathbf{v_2}' = \frac{2m_1}{m_1 + m_2}\mathbf{v_1}' + \frac{m_2 - m_1}{m_1 + m_2}\mathbf{v_2} \quad \text{......... (iv)}$$

There are some cases of special interest, which are discussed below:

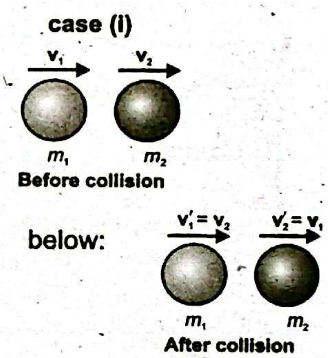

Fig. 12

(i) When m_1 and m_2:

From Eq. (iii) and (iv), we find that;

$$\mathbf{v_1}' = \mathbf{v_2}$$

and
$$\mathbf{v_2}' = \mathbf{v_1}$$

(ii) When m_1 and m_2 and $\mathbf{v_2} = 0$:

In this case, the mass m_2 be at rest, and $\mathbf{v_2} = 0$, then Eqs. (iii) and (iv) give

$$\mathbf{v_1}' = 0 \quad ; \quad \mathbf{v_2}' = \mathbf{v_1}$$

When $m_1 = m_2$ then ball of mass m1after collision will come to a stop and m_2 will take off with the velocity that m_1 originally had; Fig. (13). Thus, when a billiard ball m_1, moving on a table collides with exactly similar ball m_2 at rest, the ball m_1 stops while m_2 begins to move with the same velocity, with which m_1 was moving initially.

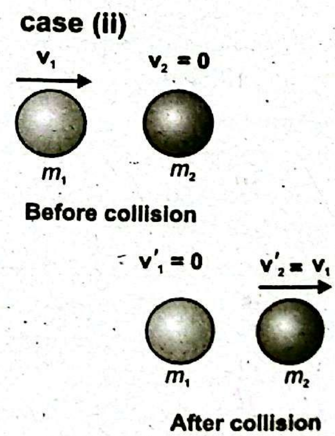

Fig. 13

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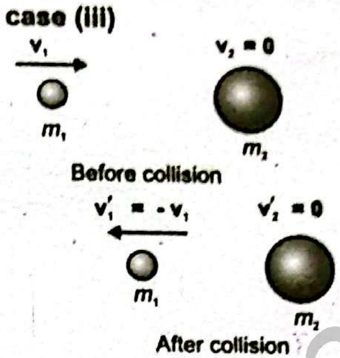
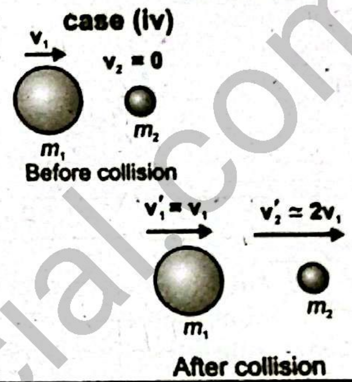
(iii) When a light body collides with a massive body at rest:
In this case initial velocity $\mathbf{v_2} = 0$ and $m_2 >> m_1$. Under these conditions m_1 can be neglected as compared to m_2. From Eq. (iii) and (iv), we have $\mathbf{v_1}' = -\mathbf{v_1}$ and $\mathbf{v_2}' = 0$.
The result is shown in Fig. (14). This means that m_1 will bounce back with the same velocity while m_2 will remain stationary. This fact is used of by the squash player.



(iv) When a massive body collides with light stationary body:
In this case, $m_1 >> m_2$ and $\mathbf{v_2} = 0$, so m_2 can be neglected in Eqs. (iii) and (iv). This gives $\mathbf{v_1}' \approx \mathbf{v_1}$ and $\mathbf{v_2}' \approx 2\mathbf{v_1}$. Thus, after the collision, there is practically no change in the velocity of massive body, but the lighter one bounces off in the forward direction with approximately twice the velocity of the incident body; Fig. (15).



2.9 INELASTIC COLLISION IN ONE DIMENSION

Q 19. Describe an inelastic collision in one dimension.

Ans. Inelastic Collision in one Dimensions
Consider two bodies having masses m_1 and m_2, moving with velocities $\mathbf{v_1}$ and $\mathbf{v_2}$, along the same line such that $\mathbf{v_1} > \mathbf{v_2}$. In such a case m_1 is regarded as projectile and m_2 as target. After time t both the bodies make inelastic collision and stick together. Let their combined mass become $m_1 + m_2$ which moves with final velocity $\mathbf{v_f}$ after collision.
Since the collision is perfectly inelastic, the total momentum of balls is conserved. Using law of conservation of momentum.

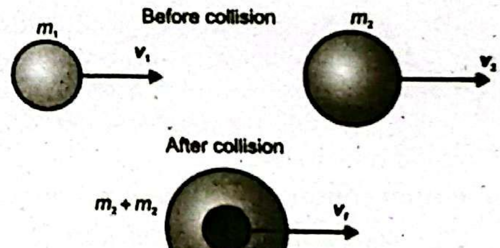


Fig. 16

Total momentum of system before collision = Total momentum of the system after collision

$$m_1\mathbf{v_1} + m_2\mathbf{v_2} = (m_1 + m_2)\,\mathbf{v_f}$$

$$\mathbf{v_f} = \frac{m_1}{m_1 + m_2}\mathbf{v_1} + \frac{m_2}{m_1 + m_2}\mathbf{v_2}$$

which gives the common velocity of the body after inelastic collision.
In a special case when the target m_2 is at rest, $v_2 = 0$, the above equation becomes:

$$\mathbf{v_f} = \frac{m_1}{m_1 + m_2}\mathbf{v_1}$$

It shows that velocity of m_1 is reduced by the mass ratio i.e., $\frac{m_1}{m_1 + m_2}$

Example 2.9: A 70 g ball collides with another ball of mass 140 g. The initial velocity of the first ball is 9 m s^{-1} to the right while the second ball is at rest. If the collision were perfectly elastic. What would be the velocity of the two balls after the collision?

Solution:

Given that;

$m_1 = 70$ g $v_1 = 9$ m s^{-1} $v_2 = 0$

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$m_2 = 140$ g

To Find:

$v_1' = ?$ $v_2' = ?$

Calculations:

We know that;

$$v_1' = \frac{m_1 - m_2}{m_1 + m_2} v_1$$

$$= \left(\frac{70\text{ g} - 140\text{ g}}{70\text{ g} + 140\text{ g}}\right) \times 9\text{ m s}^{-1} = -3\text{ m s}^{-1}$$

$$v_2' = \frac{2\,m_1}{m_1 + m_2} v_1$$

$$= \left(\frac{2 \times 70\text{ g}}{70\text{ g} + 140\text{ g}}\right) \times 9\text{ m s}^{-1}$$

$v_2' = 6\text{ m s}^{-1}$ **Ans.**

2.10 ELASTIC COLLISION IN TWO DIMENSIONS

20. Find the expression for conservation of momentum and energy for an elastic collision in two dimensions.

Ans. Elastic collision in Two Dimension

Consider the motion of two balls of mass m_1 and m_2 in a straight line with velocities $\mathbf{v_1}$ and $\mathbf{v_2}$ respectively undergoing an elastic collision with each other. Fig. (17).

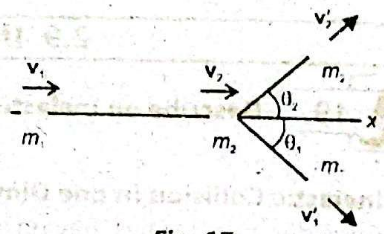

Fig. 17

Assume the bodies move off in different directions after collision with velocities $\mathbf{v_1}'$ and $\mathbf{v_2}'$ making angles θ_1 and θ_2 respectively with x-axis.

As, the collision is elastic, so we apply both the laws of conservation of momentum and law of conservation of kinetic energy. Momentum is a vector quantity, we resolve it into its rectangular components and apply the law of conservation of momentum along both axes.

Momentum conservation along x-axis is:

Momentum before collision = momentum after collision

$$m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = m_1\mathbf{v}_1' \cos\theta_1 + m_2\mathbf{v}_2' \cos\theta_2 \quad \text{......... (i)}$$

Momentum conservation along y-axis:

Momentum before collision = Momentum after collision

$$0 = m_1\mathbf{v}_1' \sin\theta_1 - m_2\mathbf{v}_2' \sin\theta_2 \quad \text{......... (ii)}$$

Conservation of Energy

Kinetic Energy before collision = Kinetic Energy after collision

$$\frac{1}{2}m_1\mathbf{v}_1^2 + \frac{1}{2}m_2\mathbf{v}_2^2 = \frac{1}{2}m_1\mathbf{v}_1'^2 + \frac{1}{2}m_2\mathbf{v}_2'^2 \quad \text{......... (iii)}$$

2.11 INELASTIC COLLISION IN TWO DIMENSIONS

21. Find the expressions for momentum and energy in case of inelastic collision in two dimensions.

Ans. Inelastic Collision in Two Dimensions:

The macroscopic collisions are generally inelastic and do not conserve kinetic energy. The perfect inelastic collision is one in which the colliding objects stick together to make a single mass after collision.

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Explanation

Let us take two balls having masses m_1 and m_2 moving with velocities, $\mathbf{v}_1$ and $\mathbf{v}_2$, respectively, in a two-dimensional xy-plane. Assume that the first body is moving along the x-axis while the second body moves in a direction, making an angle θ with x-axis. Both the bodies collide at the origin Fig. (18).

After collision, bodies stick together, having combined mass $M = m_1 + m_2$, which moves with velocity v_f, making an angle ϕ with x-axis.

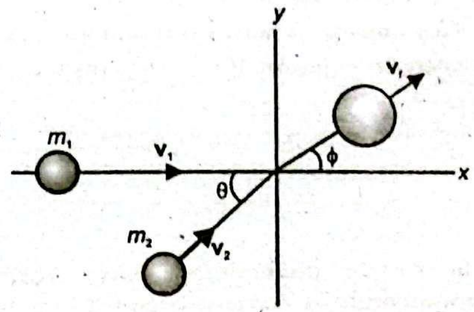

Fig. 18

Momentum in the x-direction:

$$m_1\mathbf{v}_1 + m_2\mathbf{v}_2 \cos\theta = M\,\mathbf{v}_f \cos\phi \quad \text{...... (i)}$$

Momentum in the y-direction:

$$0 + m_2\mathbf{v}_2 \sin\theta = M\,\mathbf{v}_f \sin\phi \quad \text{...... (ii)}$$

Equation (i) and (ii) can be used to find the final velocity.

Kinetic Energy: Since collision is inelastic, the kinetic energy of colliding system is not conserved. The loss of kinetic energy is computed as follows:

Initial Kinetic Energy: The total initial kinetic energy $K.E_i$ of the system before the collision is:

$$(K.E)_i = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \quad \text{...... (iii)}$$

Since *K.E.* is a scalar quantity, so velocities involving in the formula of *K.E.* does not require to break velocities into their components.

Final Kinetic Energy: The total final kinetic energy $K.E_f$ after the collision (when the objects stick together) is:

$$(K.E)_f = \frac{1}{2}Mv_f^2 \quad \text{......... (iv)}$$

where v_f is magnitude of the final velocity which can be calculated from Eq. (iv).

Energy Loss in the Collision: Since the collision is inelastic, there is a loss in kinetic energy, represented by $\Delta K.E.$

$$\Delta K.E. = (K.E.)_i - (K.E.)_f$$

This lost kinetic energy is transformed into other forms of energy, such as heat, sound, or in deformation.

Q 22. States some real-life examples of inelastic collision.

Ans. Some Examples of an Inelastic Collision

(i) When a karate chop breaks a pile of bricks, it's an example of an inelastic collision. In this type of collision, the objects involved don't bounce back after impact. Instead, some of the energy from the strike is absorbed by the bricks, converting into heat, sound, and the force needed to break them. This means the energy goes into breaking the bricks rather than causing the hand to rebound. If the Karate chop is not perfectly vertical and involves some horizontal motion, the momentum transfer and the resulting forces will have both horizontal and vertical components.

(ii) In a car crash, the collision is an inelastic nature. When the vehicles collide and absorb the impact energy, causing them to crumple and deform. This energy absorption slows down the cars, stopping them from bouncing back. Most of the kinetic energy is lost, turning into heat, sound, and damage to the vehicles.

(iii) In real-world collisions, a ball and bat show an inelastic behaviour. When the bat hits the ball, some of the kinetic energy is lost because the ball deforms, and energy is also converted into heat and sound. Even though the bat is rigid, it does not transfer energy perfectly and absorbs some energy itself. The ball compresses upon impact, which leads to further energy loss. Consequently, not all of the initial kinetic energy is conserved, making the collision overall an inelastic.

mQ's

1. In an elastic collision:

(a) K.E. is not conserved.
(b) Momentum is not conserved.
(c) Both K.E. and momentum are conserved. ✓
(d) Only mechanical energy is conserved.

2. Two bodies collide elastically in one dimension, which of the following quantities is conserved?

(a) Only momentum
(b) Only K.E.
(c) Neither momentum nor K.E.

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(d) Momentum and K.E. ✓

3. **Two objects of masses m_1 and m_2 undergo an elastic head-on-collision. If $m_1 = m_2$, then after the collision:**
(a) Both come to rest.
(b) They exchange their velocities. ✓
(c) They stick together.
(d) They move in opposite directions with the same velocity.

4. **In a two dimensional elastic collision, if total momentum is conserved and K.E. is not conserved, what can you conclude about the collision?**
(a) It is elastic. (b) It is inelastic. ✓
(c) It is impossible. (d) It is perfectly elastic.

5. **In an inelastic collision, which of the following quantities is always conserved?**
(a) Velocity (b) K.E.
(c) Mechanical energy (d) Momentum ✓

6. **Which of the following best describes energy transformation in an inelastic collision?**
(a) K.E. is fully conserved.
(b) Total energy is not conserved.
(c) Some K.E. is converted into other forms of energy (like heat and sound). ✓
(d) Mass is lost during collision.

7. **Two objects collide in elastically. What can be said about their total K.E. before and after the collision?**
(a) It becomes zero. ✓ (b) It increases.
(c) It decreases. (d) It stays the same.

8. **In two-dimensional inelastic collision, which of the following is always conserved?**
(a) Total K.E.
(b) Linear momentum in both x and y directions. ✓
(c) Angular momentum
(d) Speed of each object.

9. **In two-dimensional inelastic collision, K.E:**
(a) is conserved in all directions.
(b) is increased due to deformation.
(c) is partially lost and converted to other forms. ✓
(d) has no relation to the motion.

2.12 ROCKET PROPULSION

Q23. Write a note on rocket propulsion.

Ans. Rocket Propulsion

Rockets move by expelling burning gas through engines at their rear. The ignited fuel turns to a high pressure gas which is expelled with extremely high velocity from the rocket engines. Fig. (19) The rocket gains momentum equal to the momentum of the gas expelled from the engine but in opposite direction. The rocket engines continue to expel gases after the rocket has begun moving and hence rocket continues to gain more and more momentum. So, instead of travelling at steady speed the rocket gets faster and faster so long the engines are operating.

A rocket carries its own fuel in the form of a liquid or solid hydrogen and oxygen. It can, therefore work at great heights where very little or no air is present. In order to provide enough upward thrust to overcome gravity, a typical rocket consumes about 10000 kg s^{-1} of fuel and ejects the burnt gases at speeds of over 4000 m s^{-1}. In fact, more than 80% of the launch mass of a rocket consists of fuel only. One way to overcome the problem of mass of fuel is to make the rocket from several rockets linked together.

When one rocket has done its job, it is discarded leaving others to carry the space craft further up at ever greater speed.

If m is the mass of the gases ejected per second with velocity **v** relative to the rocket, the change in momentum per second of the ejecting gases is m**v**. This equals the thrust produced by the engine on the body of the rocket. So, the acceleration '**a**' of the rocket is

Fig. 19: Fuel and oxygen mix in the combustion chamber. Hot gases exhaust the chamber at a very high velocity. The gain in momentum of the gases equals the gain in momentum of the rocket. The gas and rocket push against each other and move in opposite directions.

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$$a = \frac{mv}{M}$$

where M is the mass of the rocket. When the fuel in the rocket is burned and ejected, the mass M of rocket decreases and hence the acceleration increases.

MCQs

1. What causes a rocket to move forward in space where there is no atmosphere?
(a) Push against the vacuum.
(b) Expelling fuel backward. ✓
(c) Solar radiation pressure
(d) Gravitation pull of Earth

2. The momentum of gases expelled from a rocket engine is:
(a) Equal to rocket's momentum in the opposite direction. ✓
(b) Zero
(c) Greater than the rocket's momentum
(d) Unrelated to the rocket's motion.

3. The velocity of the rocket increases as it moves because:
(a) Gravity decreases (b) Air resistance increases
(c) Fuel becomes more reactive
(d) Mass of rocket decreases ✓

4. Rocket propulsion is based on which fundamental principle?
(a) Conservation of energy
(b) Conservation of momentum
(c) Newton's first law of motion
(d) Newton's 3rd law of motion

5. The velocity of a rocket increases when:
(a) It gains mass.
(b) The exhaust velocity is increased. ✓
(c) Gravity increases
(d) Air resistance increases.

ADDITIONAL SHORT ANSWER QUESTIONS

SQ

1. Can a scalar quantity be negative?
Ans. Yes, a scalar quantity like temperature and charge can be negative.

2. What is the resultant of two vectors acting in the same direction?
Ans. The sum of their magnitudes in the same direction.

3. What is the resultant of two vector acting in the opposite directions?
Ans. The difference of their magnitudes in the direction of the larger vector.

4. What is the result of two equal vectors at 90° to each other?
Ans. The resultant vector has a magnitude of $\sqrt{2}$ times one vector.

5. Can a vector have a negative magnitude?
Ans. No, the magnitude of a vector is always positive.

6. Can the components of a vector be negative?
Ans. Yes, depending on the direction of the vector relative to the axes.

7. What is the significance of resolving a vector into components?
Ans. It simplifies vector calculations, especially when adding or subtracting vectors.

8. What happens to the components if the vector lies along the x-axis?
Ans. The y-component is zero and the x-component equals the magnitude of the vector.

9. What is the scalar product of two perpendicular and two parallel vectors?
Ans. The scalar product of two perpendicular vectors is zero and two parallel vectors is one.

10. Can the scalar product be zero?
Ans. Yes, if the angle between the vectors is greater than 90°.

11. What does a positive scalar product indicate about the angle between two vectors?
Ans. The angle is less than 90°, meaning generally the vectors are pointing in the same direction.

12. What is the direction of the vector product?
Ans. It is given by the right hand rule, perpendicular to the plane containing the two vectors.

13. What is the significance of vector product?
Ans. It represents quantities like torque and angular momentum, which have direction perpendicular to the plane of action.

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14. What is the relation between impulse and momentum?

Ans. Impulse is equal to the change in momentum.

i.e., Impulse = $\Delta p = mv_f - mv_i = m\Delta v$

15. What is the SI unit of impulse?

Ans. The SI unit of impulse is same as momentum i.e., Ns or kg ms^{-1}.

16. How can a small force produce a large momentum?

Ans. By acting over a long period of time.

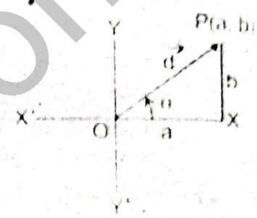
17. Why are airbags used in cars?

Ans. Airbags increase the time of impact, reducing the force and thus reducing injuries by lowering the impulse force.

18. If a force acts on a body for zero time, what is the impulse?

Ans. Zero, since impulse depends on time.

19. If a vector lies in xy-plane. Is it possible, one of its rectangular components is zero? Explain.



Ans. No, it is not Possible.

Reason:

When a vector lies in xy–plane, so it must have non-zero x and y components.

20. Can the magnitude of vector ever be zero? Explain.

Ans. Yes, the magnitude of the vector can be zero, if it is a null vector.

Mathematically $\vec{A} + (-\vec{A}) = \vec{0}$

21. Define the terms:

(i) Unit vector **(ii) Components of a vector.**

Ans. (a) Unit Vector: A vector of unit magnitude is called unit vector. Unit vector represents the direction of the vector.

$$\text{Unit Vector} = \frac{\text{Vector}}{\text{Magnitude of the vector}}$$

$$\boxed{\hat{A} = \frac{\vec{A}}{A}}$$

(b) Components of a Vector: The effective part of a vector along a particular direction is called the component of the vector. The parts of a vector the combined effect of which is the vector itself are called components of the vector.

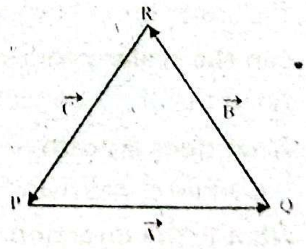
If A_x is component, of the vector along *x–axis* and A_y is component of the vector along y–axis, then magnitude of the vector is given by,

$$\boxed{A = \sqrt{A_x^2 + A_y^2}}$$

In three dimensions, we can write,

$$\boxed{A = \sqrt{A_x^2 + A_y^2 + A_z^2}}$$

22. The vector sum of three vectors gives a zero resultant. What can be the orientation of the vectors?



Ans. If the three vectors can be represented by the three sides of a closed triangle taken in order, then vector sum of the three vectors gives zero resultant.

According to head to tail rule,

Sum of the three vectors = $\vec{A} + \vec{B} + \vec{C} = \vec{O}$

23. Vector $\vec{A}$ lies in the xy-plane. For what orientation will both of its rectangular components be negative? For what orientation will its components have opposite signs?

Ans. (i) If vector $\vec{A}$ lies in the xy-plane both of its rectangular components will be negative in the III quadrant.

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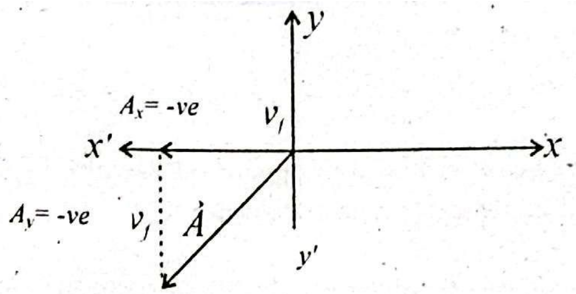
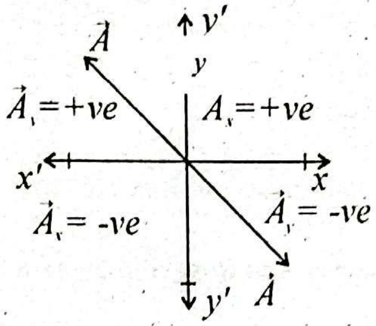


(ii) In the II quadrant A_x is negative A_y is positive. In the IV quadrant A_x is positive A_y is negative.



Therefore, in the II and the IV quadrants, components of $\vec{A}$ have opposite signs.

24. If one of the rectangular components of a vector is not zero, can its magnitude be zero? Explain.

Ans. No, if one of the rectangular components of a vector is not zero, then magnitude of the vector cannot be equal to zero.

Reason:

Suppose $A_x \neq 0$

$A_y = 0$

$A_z = 0$

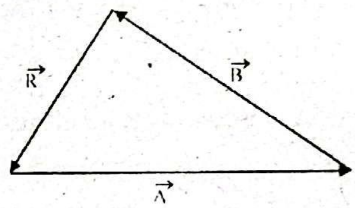
The magnitude of vector $\vec{A}$ is given by,

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2} = \sqrt{A_x^2 + 0 + 0} = A_x \neq 0$$

25. Can a vector have a component greater than the vector's magnitude?

Ans. Case I: If two components are rectangular then the magnitude of component will be less than the magnitude of a resultant.

Case II: But in case when components are not rectangular then the magnitude of a component can be greater than the magnitude of resultant as shown in figure.



26. Can the magnitude of a vector have a negative value?

Ans. No, magnitude of a vector cannot have a negative value because magnitude of a vector $\vec{A}$ is given by:

$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$ and A_x^2, A_y^2 & A_z^2 have positive values even if A_x or A_y or A_z is negative. Actually, negative sign indicates direction and not magnitude.

27. Under what circumstances would a vector have components that are equal in magnitude?

Ans. We know that magnitudes of rectangular components of vector $\vec{A}$ are given by,

$A_x = A \cos\theta$; $A_y = A \sin\theta$

When the two components are equal, then

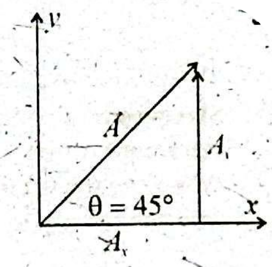
$A \cos\theta = A \sin\theta$

Divided by $\cos\theta$ on both sides

or $\dfrac{A \sin\theta}{\cos\theta} = \dfrac{A \cos\theta}{\cos\theta}$

or $\tan\theta = 1$

or $\theta = \tan^{-1}(1)$



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or $\theta = 45°, 135°, 225°, 315°$

This shows that components of a vector are equal in magnitude when the vector makes an angle of 45°, 135°, 225° or 315° with *x-axis*.

28. Is it possible to add a vector quantity to a scalar quantity? Explain.

Ans. No, it is not possible to add a vector quantity to a scalar quantity.

Reason:

Only the quantities of the same nature can be added with one another. Scalar quantities and vector quantities do not have the same nature. Scalars are added by simple mathematics.

Vectors are added by head to tail rule. Therefore, it is not possible to add a vector quantity to a scalar quantity.

29. Can you add zero to a null vector?

Ans. No, zero cannot be added to a null vector.

Reason:

Zero is a scalar quantity and null vector is a vector. They are different in nature. Therefore, zero cannot be added to a null vector.

30. Two vectors have unequal magnitudes. Can their sum be zero? Explain.

Ans. No, then sum cannot be zero.

Reason:

Sum of two vectors will be zero only if the two vectors are equal in magnitude but opposite in direction. Therefore, if two vectors have unequal magnitude, then their sum cannot be zero.

31. If all the components of the vectors $\vec{A}_1$ & $\vec{A}_2$ were reversed, how would this alter $\vec{A}_1 \times \vec{A}_2$?

Ans. If all the components of vectors $\vec{A}_1$ & $\vec{A}_2$ are reversed, there will be no change in the vector product $\vec{A}_1 \times \vec{A}_2$.

This is because $(-\vec{A}_1) \times (\vec{A}_2) = \vec{A}_1 \times \vec{A}_2$.

32. Name the three different conditions that could make $\vec{A}_1 \times \vec{A}_2 = 0$.

Ans. When

$$\vec{A}_1 \times \vec{A}_2 = A_1 A_2 \sin\theta\, \hat{n}$$

(i) When $\vec{A}_1$ is a null vector so $|\vec{A}_1| = 0$

So $$\vec{A}_1 \times \vec{A}_2 = (A_1)\, A_2 \sin\theta\, \hat{n} = 0$$

(ii) When $\vec{A}_2$ is a null vector so $|\vec{A}_2| = 0$

$$\vec{A}_1 \times \vec{A}_2 = A_1\,(0) \sin\theta\, \hat{n} = 0$$

(iii) When $\theta = 0°$ or $180°$

$$\vec{A}_1 \times \vec{A}_2 = A_1 A_2 \sin\theta = 0$$

33. Identify true or false statements and explain the reason.

(a) A body in equilibrium implies that it is neither moving nor rotating.

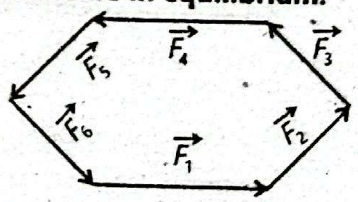
(b) If coplanar forces acting on a body form a closed polygon, then the body is said to be in equilibrium.

Ans. **(i) Statement:** A body in equilibrium implies that it is not moving or rotating.

A body moving with a uniform linear velocity or rotating with a uniform angular velocity is in equilibrium *i.e.*,

$$\Sigma \vec{F} = \vec{0} \quad \text{and} \quad \Sigma \vec{\tau} = \vec{0}$$

Therefore, the given statement is false.



(ii) Statement: If coplanar forces acting on a body form a closed polygon, the body is said to be in equilibrium. This statement is true.

When the forces acting on a body are in the form of a closed polygon, then the head of the last vector coincides with the tail of the first vector, therefore, that $\Sigma F = 0$, and the body is in equilibrium

$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 + \vec{F}_5 + \vec{F}_6 = 0$$

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$$\Sigma\vec{F} = 0$$

34. Explain the circumstances in which the velocity 'v' and acceleration 'a' are constants.

Ans. When the body moves with uniform velocity, the ***change in velocity is zero***. So, acceleration is zero. It remains zero as far as the body moves with constant velocity. Hence, acceleration remains constant (i.e., 0 ms^{-2}) throughout the motion of the body.

35. An object is thrown upward with initial velocity. How much height does it gain in turns of velocity?

Ans. $v_f = 0$ (At Max. height)

$a = -g$ (upward motion)

$t = ?$

$S = h = ?$

Let 'v_i' be the initial velocity.

1st Equation of Motion	3rd Equation of Motion
$v_f = v_i + at$ $0 = v_i - gt$ $gt = v_i$ $t = \frac{v_i}{g}$	$S = v_i t + \frac{1}{2}at^2$ $h = v_i\left(\frac{v_i}{g}\right) + \frac{1}{2}(-g)\left(\frac{v_i}{g}\right)^2$ $h = \frac{v_i}{g} - \frac{v_i}{2g} = \frac{v_i^2}{g}$

36. A rubber ball and lead ball of same size are moving with same velocity. Which ball have greater momentum and why?

Ans. Momentum of an object is given by

$$p = mv$$

Let the rubber ball has mass (m_1) and velocity v

Lead ball has mass (m_2) and velocity v

So, $p_1 = m_1 v$

$p_2 = m_2 v$

or $\frac{p_1}{p_2} = \frac{m_1 v}{m_2 v}$

$\frac{p_1}{p_2} = \frac{m_1}{m_2}$

Since $m_2 > m_1$

or $p_2 > p_1$

So, momentum of the lead ball is greater than the momentum of the rubber ball.

37. What is the difference between open and closed system?

Ans. Open System: An open system is the one which can exchange both energy and matter with its surroundings.

Closed System: A closed system is the one which can exchange only energy and no matter with its surroundings.

38. Why a motorcycle's safety helmet is padded?

Ans. A motorcycle's safety helmet is padded so as to extend the time of any collision to prevent serious injury.

39. How the hairs act like a crumple zone on your skull?

Ans. The hair on the skull act as a soft zone.

Explanation:

As $I = F \times \Delta t$, so $F \propto \frac{1}{\Delta t}$

For the naked skull a force of 5N is sufficient to fracture, because here the time of collision Δt will be smaller and the impulsive force 'F' will be more effective.

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For the covered skull with hair, the time of collision Δt is greater and the impulsive force 'F' will not be so effective. In this case, a force of 50N instead of 5N will be required.

40. Which will be more effective in knocking a bear down?
(i) A rubber bullet or (ii) A lead bullet of the same momentum, why?

Ans. As $I = \Delta P = F \times \Delta t$, so $F \propto \frac{1}{\Delta t}$

For knocking the bear down, impulse acts on the bear. In case of the rubber bullet, the time of collision Δt will be smaller (due to bouncing) and the impulsive force 'F' will be more effective.

But in case of the lead bullet, the time of collision Δt will be greater (due to penetration) and the impulsive force 'F' will be smaller.

So, the rubber bullet will be more effective to knock down the bear.

41. What is projectile motion?

Ans. Projectile motion is the curved path followed by an object thrown or projected into the air, under the influence of gravity only.

42. What are two components of projectile motion?

Ans. Horizontal motion (constant velocity) and vertical motion (accelerated motion due to gravity).

43. What is the horizontal acceleration of a projectile?

Ans. Zero, because there is no force acting horizontally (ignoring air resistance).

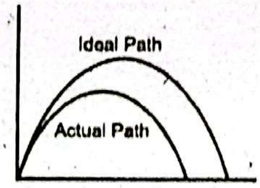
44. Why does a projectile fall back to the ground?

Ans. Because of the downward acceleration due to gravity.

45. In the absence of friction, how the vertical and horizontal components of velocity change?

Ans. Due to air friction, the vertical and horizontal velocities decrease and therefore, the height and range both decrease which make fall short of a parabolic trajectory, as shown in the figure.

For your Information



In the presence of air friction, the trajectory of a high speed projectile fall short of a parabolic path.

46. Derive the formula for the vertical distance covered by the projectile when it is thrown from a certain height h.

Ans. Let the ball (projectile) is thrown from the certain height (h). The ball will accelerate downward under the force of gravity.

So, $a_y = g$, $v_{iy} = 0$

and $y = h$

Using 2nd equation of motion:

$$S = v_i t + \frac{1}{2} a t^2$$

For vertical motion:

$$y = v_{iy} t + \frac{1}{2} a_y t^2$$

or $$h = (0)\, t + \frac{1}{2} g t^2$$

$$h = \frac{1}{2} g t^2$$

47. Which quantity remains same at all points on the trajectory of a projectile; either velocity or acceleration? Explain.

Ans. Because there is no horizontal directed force acting on the object on the trajectory of a projectile.

So, $F_x = 0$

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or $\quad m\,a_x = 0$

or $\quad a_x = 0$, As $m \neq 0$

So, on the trajectory of a projectile

v_x = constant	v_y = variable
$a_x = 0$	$a_y = \pm g \qquad (-g \uparrow \quad \downarrow +g)$

Hence, horizontal component of velocity remains same.

48. The horizontal range of projectile is four times of its maximum height. What is angle of projection?

Ans. According to given condition

$$R = 4H$$

or $$\frac{v_i^2 \sin^2\theta}{g} = 4^2 \frac{v_i^2 \sin^2\theta}{{}_1 2g}$$

or $$\left(\frac{v_i^2}{g}\right)(\sin^2\theta) = \left(\frac{v_i^2}{g}\right)(2\sin^2\theta)$$

or $$2\sin\theta\cos\theta = 2\sin^2\theta \qquad (\because \sin 2\theta = 2\sin\theta\cos\theta)$$

or $$1 \times \cos\theta = \sin\theta$$

or $$\frac{\sin\theta}{\cos\theta} = 1$$

or $$\tan\theta = 1$$

or $$\theta = \tan^{-1}(1)$$

$$\boxed{\theta = 45^\circ}$$

49. If the angle of projection of a projectile is zero. What is its maximum height?

Ans. According to given condition:

$$\theta = 0^\circ \qquad \text{(The angle of projection)}$$

Max. Height of the projectile:

$$H = \frac{v_i^2 \sin^2\theta}{2g}$$

$$H = \frac{v_i^2 (\sin 0^\circ)^2}{2g}$$

$$H = \frac{v_i^2 (0)^2}{2g} \qquad (\because \sin 0^\circ = 0)$$

$$H = \frac{0}{2g}$$

$$H = 0$$

50. For what value of the angle of projection the range of projectile is half of its maximum possible value?

Ans. The range of projectile is:

$$R = \frac{v_i^2}{g}\sin 2\theta$$

For Max. Range:

$$\therefore \sin 2\theta = 1 \text{ (Max. Value)}$$

$$2\theta = \sin^{-1}(1)$$

$$2\theta = 90^\circ$$

$$\theta = 45^\circ$$

As $$R = \frac{v_i^2}{g}\sin 2\theta$$

So $$R_{max} = \frac{v_i^2}{g}$$

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Hence $R = R_{max} \sin 2\theta$

According to given condition

$$\frac{R_{max}}{2} = R$$

or $$\frac{R_{max}}{2} = R_{max} \sin 2\theta$$

or $$\sin 2\theta = \frac{1}{2}$$

or $$2\theta = \sin^{-1}\left(\frac{1}{2}\right)$$

or $$2\theta = 30^\circ$$

or $$\theta = 15^\circ$$

51. Give one example of an elastic collision.

Ans. Collision between law steel balls or gas molecules.

52. Give one example of an inelastic collision.

Ans. A car crash where the cars stick together after colliding.

53. Is momentum conserved in both elastic and inelastic collisions?

Ans. Yes, momentum is always conserved in both elastic and inelastic collisions.

54. Is K.E. conserved in an inelastic collision?

Ans. No, some K.E. is converted into other forms of heat, sound or deformation.

55. What happens to the total energy in an inelastic collision?

Ans. Total energy is conserved, but K.E. is not.

56. What is perfectly inelastic collision?

Ans. It is a type of inelastic collision where the colliding bodies stick together after impact.

57. What is conserved in all types of collisions?

Ans. Linear momentum.

58. Can K.E. increase after a collision?

Ans. Only in explosive collisions (in special case), otherwise in elastic it remains the same, and in inelastic, it decreases.

59. Write two significances of velocity-time graph?

Ans. It significance is as follows:

(i) To determine speed of body at any instant of time.

(ii) To determine acceleration of a body.

(iii) to determine total distance travalled by a body in a given time interval.

60. When two objects move in opposite direction one will have positive momentum and other negative. Why?

Ans. As we known;

$$P = m \times v$$

(i) Momentum is a vector quantity and has the same direction as velocity. The object moving in direction of velocity will have positive momentum while object moving in oppositive direction of velocity will have negative momentum.

(ii) Also, for linear momentum to be conserived after collision, both objects must rebound with same velocities but in opposite directions.

61. How would you elaborate the importance of head rest of car seat?

Ans. If a car crashes into back of another car, the heard rest of the car seat can save us from neck injury. It helps to accelerate the head forward with same rate as rest of our body.

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62. Why kinetic energy is not conserved in inelastic collision?

Ans. In an inelastic collision, kinetic energy is not conserved because during collision same of the kinetic energy is transferred to other forms of energy like thermal energy, sound energy, material deformation etc.

63. Find the change in momentum for an object subjected to a given force for a given time and state law of motion in terms of momentum.

Ans. According to Newton's second law of motion,

$$\text{Force} = \text{Mass} \times \text{Acceleration}$$

or $$F = ma = \frac{m(v_f - v_i)}{t}$$

$$F = \frac{mv_f - mv_i}{t} = \frac{p_f - p_i}{t} = \frac{\Delta p}{t}$$

Hence in terms of momentum, Newton's second law of motion may be defined as follows: according to this law, time rate of change of momentum of a body is equal to the force applied on the body.

64. Define impulse and show that how it is related to linear momentum?

Ans. Impulse: When a large force acts for a very short interval of time, the product of force and time is called impulse.

$$\text{Impulse} = \text{Force}\times\text{Time} = F\times t$$

We know that

or $$F = ma = \frac{m(v_f - v_i)}{t}$$

or $$F = \frac{mv_f - mv_i}{t}$$

or $$F \times t = mv_f - mv_i = \text{change in momentum of the body}$$

65. State the law of conservation of linear momentum, pointing out the importance of isolated system. Explain, why under certain conditions, the law is useful even though the system is not completely isolated?

Ans. Law of Conservation of Momentum:

Total linear momentum of an isolated system always remains constant.

Importance of Isolated system:

Law of conservation of momentum holds only for isolated systems otherwise it is not valid.

Application for not completely isolated systems:

When the effect force (like frictional and gravitational forces) is negligibly small as compared to the forces between the interacting objects, then this law become applicable.

66. Explain the difference between elastic and inelastic collisions. Explain, how would a bouncing ball behave in each case? Give plausible reasons for the fact that kinetic energy is not conserved in most cases.

Ans. Elastic Collision: It is that collision in which both linear momentum and kinetic energy are conserved.

Inelastic Collision: It is that collision in which linear momentum is conserved but kinetic energy is not conserved.

Explanation: Suppose a ball is dropped on the floor. If the ball rebounds to its original height, then the collision is elastic. But if the ball does not rebound to its original height, the collision is inelastic. In this case, some kinetic energy is lost due to friction, because some kinetic energy is converted into sound energy and heat energy during collision.

Non-conservation of kinetic energy: In most cases, kinetic energy is not conserved. A part of kinetic energy is lost due to friction. Some kinetic energy is converted into sound energy and heat energy during collision.

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67. Explain, what is meant by projectile motion? Derive expressions for:
(a) The time of flight **(b) The range of projectile**
Show that the range of projectile is maximum when projectile is thrown at an angle of 45° with the horizontal.

Ans. Projectile Motion: When a body having a constant horizontal velocity moves under the action of gravity, it follows a curved path. Such a motion of the body is called projectile motion. Thus projectile motion is the two-dimensional motion under constant acceleration due to gravity. Path followed by the projectile is called trajectory.

Time of Flight (T): The time taken by the projectile from the point of projection to the point where it hits the ground is called time of flight. Thus vertical distance is zero *i.e.*,

$$S = y = 0 \text{ also } a = -g, t = T$$

We know that

$$S = v_i t + \frac{1}{2}at^2$$

For vertical motion,

$$y = v_{iy}t - \frac{1}{2}gt^2$$

or $$y = v_i \sin\theta\, t - \frac{1}{2}gt^2$$

or $$0 = v_i \sin\theta T - \frac{1}{2}gT^2$$

or $$v_i \sin\theta T = \frac{1}{2}gT^2$$

or $$T = \frac{2v_i \sin\theta}{g}$$

Horizontal range (R): Maximum distance covered by the projectile in the horizontal direction is called horizontal range of projectile.

For horizontal motion,

$$S = R \quad , \quad a = 0$$

$$v_i = v_{ix} = v_i \cos\theta \quad , \quad t = T = \frac{2v_i \sin\theta}{g}$$

We know that

$$S = v_i t + \frac{1}{2}at^2$$

or $$R = v_i \cos\theta \times \frac{2\, v_i \sin\theta}{g} + \frac{1}{2} \times 0 \times t^2$$

$$R = \frac{v_i^2 \sin\theta \cos\theta}{g}$$

$$R = \frac{v_i^2 \sin\theta}{g}$$

Maximum horizontal range (R_{max}): We know that horizontal range of projectile is given by

$$R = \frac{v_i^2 \sin 2\theta}{g}$$

Horizontal range R is maximum when sin 2θ is maximum

Since $\sin 2\theta = 1$

$$2\theta = 90°$$

$$\theta = 45°$$

Maximum range is given by,

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$$R_{max} = \frac{v_i^2 \times 1}{g} = \frac{v_i^2}{g}$$

68. What is rocket propulsion?

Ans. It is the motion of a rocket caused by the explosion of gases at high speed in the opposite direction.

69. What is the source of thrust in a rocket?

Ans. The high-speed exhaust gases expelled from the rocket engine.

70. Why can a rocket work in space where there is no air?

Ans. Because rocket propulsion does not require air; it depends on the reaction force from expelling gas.

71. What is thrust?

Ans. It is the force that propels a rocket forwards generated by the explosion of gas.

72. What is the role of fuel in rocket propulsion?

Ans. The fuel burns to produce hot gases, which are expelled to generate thrust.

73. What happens to the mass of a rocket as it moves upward?

Ans. It decreases because fuel is burnt and gases are expelled.

74. Why does a rocket accelerate as it rises?

Ans. Because its mass decreases while thrust remains, increasing acceleration according to Newton's second law.

75. What is the source of thrust in a rocket?

Ans. The thrust comes from the high-speed exhaust gases produced by burning fuel.

76. What is meant by the term "recoil velocity" in rockets?

Ans. It is the velocity with which the rocket moves in the opposite direction of the expelled gases.

77. What is the momentum conservation principle in rocket propulsion?

Ans. The total momentum before and after gas ejection is conserved; the rocket gains momentum equal and opposite to that of the gases.

78. What is the effect on the speed of a fighter plane chasing another when it opens the fire?

Ans. When the fighter plane opens fire, its momentum will be in back direction due to reaction force in backward direction and therefore, its speed will decrease. When the pursued plane opens fire in back direction, its momentum will act on the plane in forward direction due to reaction force in the forward direction and therefore, its speed will increase.

79. How does the rocket propulsion take place?

Ans. Motion of rocket is based on the law of conservation of ***momentum*** and Newton's ***third*** law of motion.

Propulsion:

- Rocket moves up by ejecting burning gasses from its rear part.
- When fuel is burned, it turns to high pressure gases.
- These gases come out from the engine with very high velocity.
- The rocket gains momentum equal to the momentum of expelled gases but in opposite direction.
- Rocket continues to gain more and more momentum as long as engine of the rocket continues to expel gases.

SOLVED EXERCISE

MULTIPLE CHOICE QUESTIONS

Tick (✓) the correct answer.

2.1 The angle at which dot product becomes equal to cross product:
(a) 65° (b) 45° ✓ (c) 76° (d) 30°

2.2 The projectile gains its maximum height at an angle of:
(a) 0° (b) 45° (c) 60° (d) 90° ✓

2.3 The scalar product of two vectors is maximum if they are:
(a) perpendicular (b) parallel ✓ (c) at 30° (d) at 45°

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2.4 The range of projectile is same for two angles which are mutually:
(a) perpendicular (b) supplementary (c) complementary✓ (d) 270°

2.5 The acceleration at the top of a trajectory of projectile is:
(a) maximum (b) minimum (c) zero (d) g✓

2.6 SI unit of impulse is:
(a) $kg\ m\ s^{-2}$ (b) N m (c) N s✓ (d) $N\ m^2$

2.7 The rate of change of momentum is:
(a) force✓ (b) impulse (c) acceleration (d) power

2.8 As rocket moves upward during its journey, then its acceleration goes on:
(a) increasing✓ (b) decreasing
(c) remains same (d) it moves with uniform velocity

2.9 Elastic collision involves:
(a) loss of energy (b) gain of energy
(c) no gain, no loss of energy✓ (d) no relation between energy and elastic collision

SHORT ANSWER QUESTIONS

2.1 State right hand rule for two vectors with reference to vector product.

Ans. The right hand rule for two vectors w.r.t.the vector (cross product) is stated as follows:

"If you point the fingers of your right hand in the direction of the first vector ($\vec{A}$) and then curl them toward the direction of the second vector ($\vec{B}$), your thumb will point in the direction of the cross product ($\vec{A} \times \vec{B}$)."

2.2 Define impulse and show how it is related to momentum.

Ans. Impulse is defined as the change in momentum of an object when a force is applied over a period of time. Mathematically, impulse is equal to the product of the force F applied to an object and the time duration Δt for which the force is applied:

$$\text{Impulse} = F \times \Delta t$$

Momentum is the quantity of motion an object has, and it is the product of the object's mass m and its velocity v.

Thus $p = mv$

Relation between Impulse and Momentum:

The relationship between impulse and momentum comes from Newton's second law, which states that the change in momentum of an object is equal to the impulse applied to it. This can be written as:

$$\Delta P = J$$

where:

- ΔP is the change in momentum (final momentum P_f minus initial momentum P_i),
- J is the impulse.

So, the change in momentum of an object is equal to the impulse applied to it. This principle is known as the Impulse-momentum Theorem.

In summary:

Thus $F\Delta t = m(v_f - v_i)$

2.3 Differentiate between an elastic and an inelastic collision.

Ans. Elastic and inelastic collisions are two different types of collisions based on the conservation of kinetic energy and other factors:

1. Elastic Collision

(i) Kinetic Energy: In an elastic collision, kinetic energy is conserved before and after the collision. The total kinetic energy of the system remains the same.

(ii) Momentum: Momentum is conserved, just like in any other type of collision.

(iii) Deformation: There is no permanent deformation of the objects, and they do not lose energy to sound, heat, or other forms of energy.

Example: Idealized collisions like two billiard balls colliding, or atoms in an ideal gas.

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2. Inelastic Collision

(i) Kinetic Energy: In an inelastic collision, kinetic energy is not conserved. Some of the kinetic energy is converted into other forms of energy, such as heat, sound, or internal energy.

(ii) Momentum: Momentum is still conserved, as it always is in isolated systems.

(iii) Deformation: Objects can undergo deformation, and they may stick together or lose some energy due to internal processes.

Example: A car crash or a ball hitting the ground and bouncing back with less speed.

2.4 Show that rate of change in momentum is equal to force applied. Also state Newton's second law of motion in terms of momentum.

Ans. To show that the rate of change in momentum is equal to the force applied, we can start from the basic definition of momentum.

i.e., $P = m\vec{v}$

Now, the rate of change of momentum with respect to time is:

$$\frac{\Delta P}{\Delta t} = \Delta\,(mv)$$

Assuming the mass m is constant, this becomes:

$$\frac{\Delta P}{\Delta t} = m\Delta v$$

The term Δv is the acceleration a. So, we have:

$$\frac{\Delta P}{\Delta t} = ma$$

By Newton's second law, force is the product of mass and acceleration. So,

$$F = ma$$

Thus $$\frac{\Delta P}{\Delta t} = F$$

This shows that the rate of change of momentum is equal to the force applied.

Newton's Second Law in Terms of Momentum

Newton's second law of motion, when expressed in terms of momentum, states that;

$$F = \frac{\Delta P}{\Delta t}$$

This means that the force acting on an object is equal to the rate of change of its momentum with respect to time.

2.5 State law of conservation of linear momentum. Also state condition under which it holds.

Ans. The law of conservation of linear momentum states that the total linear momentum of an isolated system remains constant if no external forces act on it.

Mathematically, it can be expressed as:

$$p_{initial} = p_{final}$$

where $p = mv$ is the linear momentum, m is the mass, and v is the velocity of the objects.

Condition under which it holds:

- The system must be isolated, meaning no external forces or torque are acting on the system.
- The forces must be internal to the system, such as the forces between objects in the system, which cancel out in terms of the net effect on the system's total momentum.

In cases where external forces are present (*e.g.*, friction, gravity), the law does not apply in its strict form unless those external forces are balanced.

2.6 Show that range of projectile is maximum at an angle of 45°.

Ans. To show that the range of a projectile is maximum at an angle of 45°, let us first derive the formula for the range of a projectile.

When a projectile is launched at an angle θ with an initial velocity v_i, the horizontal and vertical components of the initial velocity are:

- Horizontal component: v_{ix}

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- Vertical component: v_{fy}

The time of flight T can be calculated by considering the vertical motion. The total time of flight is the time it takes for the projectile to rise and fall back to the same level, and is given by

$$T = \frac{2\, v_i \sin\theta}{g}.$$

The horizontal range R, or the distance the projectile travels horizontally, is given by the horizontal velocity multiplied by the time of flight.

i.e., $$R = v_{ix} \times T = v_i \cos\theta \frac{(2\, v_{ix} \sin\theta)}{8}$$

Simplifying this expression:

$$R = \frac{v_i^2 \sin 2\theta}{8}$$

Now, to maximize the range R, we need to maximize 2θ because the other terms (v_i and g) are constants. The sine function achieves its maximum value of 1 when:

$$\sin 2\theta = 1 \text{ which occurs when } 2\theta = 90^\circ$$

Thus $$\theta = 45^\circ$$

Conclusion: The range R of a projectile is maximum when the angle of launch θ is 45°.

2.7 Find the time of flight of a projectile to reach the maximum height.

Ans. The formula for time of flight is:

$$t = \frac{v_i \sin\theta}{g}$$

At maximum height, the vertical component of the velocity becomes zero. The formula is derived by setting the final vertical velocity $v_y = 0$ in the equation given below:

$$v_y = v_i \sin\theta - gt$$

$$0 = v_i \sin\theta - gt$$

or $$gt = v_i \sin\theta$$

or $$t = \frac{v_i \sin\theta}{g}$$

2.8 The maximum horizontal range of a projectile is 800 m. Find the value of height attained by the projectile at 0 = 60°.

Ans. Given that;

Maximum horizontal range R_{max} = 800 m

Angle of projection for maximum height θ = 60°

The formula for maximum range of a projectile is:

$$R_{max} = \frac{v_i^2}{g}$$

or $$v_i^2 = R_{max} \times g$$

$$= 800\text{ m} \times 9.8\text{ ms}^{-2}$$

or $$v_i^2 = 7840\text{ m}^2\text{s}^{-2}$$

So $$v_i^2 \approx 88.55\text{ ms}^{-1}$$

The maximum height H is given by

$$H = \frac{v_i^2 \sin^2\theta}{2g}$$

Putting the values

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$$H = \frac{(88.55\ ms^{-1})^2 \times \sin^2(60^\circ)}{2 \times 9.8\ ms^{-2}}$$

$$H = 300\ m$$

CONSTRUCTED RESPONSE QUESTIONS

2.1 Why does a hunter aiming a bird in a tree miss the target exactly at the bird?

Ans. This happens because of the mechanics involved in aiming at a moving target and the effects of gravity. If a hunter aims directly at the bird, gravity pulls the bullet (or arrow, if using a bow) downward as it travels through the air. In this case, the bullet or arrow will fall to the ground before reaching the bird.

However, the reason the hunter misses exactly at the bird has to do with the concept of lead in shooting, which is where the hunter aims ahead of the target (considering its movement and the projectile's trajectory). But if the bird is stationary, gravity is still at play, and the projectile will be pulled down, meaning the shot will miss at the point where the bird was located.

In essence, the hunter's aim at the bird does not take into account the gravitational drop of the projectile, causing a miss that appears perfectly aligned with the bird at that moment.

2.2 A person falling on a heap of sand does not hurt more as compared to a person falling on a concrete floor. Why?

Ans. When a person falls onto a heap of sand, the sand acts as a cushion that absorbs the impact and slows down the person's descent. This gradual deceleration reduces the force exerted on the body. In contrast, when falling onto a concrete floor, there is very little flexibility. The concrete does not absorb the impact, leading to a rapid deceleration, which results in a much greater force being applied to the body, causing more injury.

The key factors are:

(i) **Impact Force:** The sand spreads out the force over a longer time, reducing the peak force on the body.

(ii) **Surface:** The sand gives way and molds around the body, reducing the sudden stop. The concrete, being rigid, doesn't deform, causing the body to stop abruptly.

(iii) **Energy Absorption:** The sand absorbs some of the energy of the fall, whereas concrete reflects almost all of it back to the body.

This difference in how the materials interact with the body explains why falling on sand generally causes less injury compared to falling on concrete.

2.3 State the conditions under which birds fly in air.

Ans. Birds can fly in air under the following conditions:

(i) **Sufficient air density**
The air must be dense enough to provide lift when birds flap their wings. This is why most birds cannot fly at extremely high altitudes where the air is too thin.

(ii) **Wing structure and strength**
Birds need strong, light weight wings with the right shape (air foil) to generate lift and thrust.

(iii) **Muscle power**
Birds must have strong flight muscles, especially the pectoral muscles, to flap their wings with enough force.

(iv) **Favourable weather conditions**
Calm or moderate wind conditions are ideal. Strong winds, storms, or heavy rain can prevent or hinder flight.

(v) **Adequate space**
Birds need enough space to take off, glide and land safety.

(vi) **Health and energy**
Birds must be healthy and have enough energy (from food) to sustain flight.

2.4 Describe the circumstances for which velocity and acceleration of a vehicle are:

(i) v is zero but a is not zero

(ii) a is zero but v is not zero

(iii) perpendicular to one another

Ans. Here is a breakdown of the situations described:

(i) **Velocity is zero but acceleration is not zero:** This occurs when the vehicle is at rest at a particular moment but is experiencing a change in velocity. A typical example is when a car is at a stop sign, but the driver suddenly

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starts to accelerate, meaning the velocity is zero at that instant but the car is accelerating from rest. This could also happen when an object reaches the maximum height in projectile motion, where the velocity momentarily becomes zero but acceleration due to gravity continues to act downward.

(ii) Acceleration is zero but velocity is not zero: This occurs when the vehicle moves at a constant speed in a straight line. For example, when a car moves at a constant velocity on a flat, straight road, the acceleration is zero because there is no change in the speed or direction of the vehicle. This is a scenario of uniform motion, where the velocity is constant.

(iii) Perpendicular to one another: This situation occurs when the velocity and acceleration vectors are at a 90-degree angle to each other. An example of this would be an object moving in uniform circular motion. For instance, a car moving in a circle at constant speed: its velocity is tangential to the circle (pointing along the direction of motion), while the acceleration (centripetal acceleration) points toward the center of the circle, perpendicular to the velocity.

2.5 Describe briefly effects of air resistance on the range and maximum height of a projectile.

Ans. Air resistance reduces both the **range** and the maximum height of a projectile. It opposes the motion of the projectile, causing it to slow down more quickly than it would in a vacuum. As a result, the projectile does not travel as far horizontally (shorter range) and does not rise as high vertically (lower maximum height).

COMPREHENSIVE QUESTIONS

2.1 Define and explain scalar product. Write down its important characteristics.

Ans. *See Q.* 4.

2.2 Define and explain vector product of two vectors. Discuss important characteristics of vector product.

Ans. *See Q.* 5.

2.3 Derive three equations of motion by graphical method.

Ans. *See Q.* 6, *Q.* 7 and *Q.* 8.

2.4 What is projectile motion? Explain.

Ans. *See Q.* 11 and *Q.* 12.

2.5 Derive the following expressions for projectile motion:

(i) time of flight

(ii) height attained

(iii) range for projectile.

Ans. *See Q.* 13.

2.6 Explain elastic collision in one dimension. Show that magnitude of relative velocities before and after collision are equal.

Ans. *See Q.* 18.

2.7 Explain elastic collision in two dimensions.

Ans. *See Q.* 20.

2.8 Explain an inelastic collision in one and two dimensions.

Ans. *See Q.* 19 and *Q.* 21

NUMERICAL PROBLEMS

2.1 The magnitude of cross and scalar products of two vectors are $4\sqrt{3}$ and 4, respectively. Find the angle between the vectors.

Solution:

Given that:

Magnitude of cross product of two vectors = $4\sqrt{3}$

Magnitude of dot product of two vectors = 4

Calculations:

of cross product:

$[\vec{A} \times \vec{B}] = AB \sin\theta = 4\sqrt{3}$ ...... (i)

Magnitude of dot product:

$[\vec{A} \cdot \vec{B}] = AB \cos\theta = 4$ ...... (ii)

Dividing Eq. (i) by 2 (ii)

$$\frac{AB \sin\theta}{AB \cos\theta} = \frac{4\sqrt{3}}{4}$$

or $\tan\theta = \sqrt{3}$

or $\theta = \tan^{-1}(\sqrt{3})$

$\theta = 60^\circ$

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2.2 A helicopter is ascending vertically at the rate of 19.6 m s^{-1}. When it is at a height of 156.8 m above the ground, a stone is dropped. How long does the stone take to reach the ground?

Solution:

Given Data:

Initial velocity = v_i = 19.6 ms^{-1}

Acceleration = a = −g = −9.8 ms^{-2}

Distance = S = −156.8 m

Required:

Time = t = ?

Calculations:

1st Method: When stone is dropped from point P, it will not come down directly but it will rise to maximum height up to point Q and then will fall freely.

$$h' = 156.8 \text{ m}$$

First, we calculate time taken from P to Q.

At point P, v_i = 19.6 ms−1

At point Q, $v_f = 0$

Using equation of motion,

$$v_f = v_i - g\,t$$

$$0 = 19.6 - 9.8\,t1$$

$$9.8\,t_1 = 19.6$$

$$t_1 = \frac{19.6}{9.8}$$

$$t_1 = 2s$$

Height from P to Q = h_1

$$2gh_1 = v_f^2 - v_i^2$$

$$2(-9.8)\,h_1 = (0)^2 - (19.6)^2$$

$$-19.6\,h_1 = -19.6 \times 19.6$$

$$h_1 = 19.6m$$

Total height = $h = h' + h_1$

$$h = 156.8 + 19.6$$

$$h = 176.4$$

Time from Q to R

v_i = 0 (at Q) for downward motion

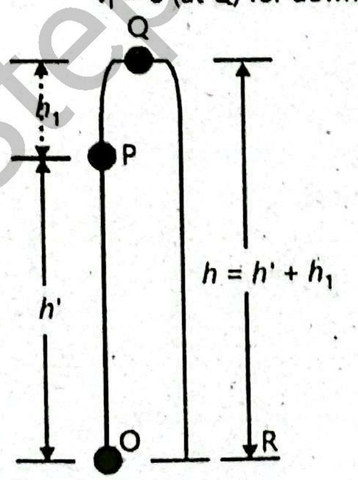

Fig: Problem 3.1

v_i = 0 (at Q) for downward motion

$$h = v_i t_2 + \frac{1}{2} g t_2^2$$

$$176.4 = (0)\,t_2 + \frac{1}{2}(9.8)\,t_2^2$$

$$176.4 = (4.9)\,t_2^2$$

$$t_2^2 = \frac{176.4}{4.9}$$

$$t_2^2 = 36$$

Taking square root on both sides

$$t_2 = 6s$$

Total time = $t = t_1 + t_2$

$$T = 2 + 6$$

Required time = 8s

2nd Method:

Initial velocity = v_i = 19.6 ms^{-1}

Acceleration = a = −g = −9.8 ms^{-2}

Distance = S = − 156.8 m

Time = t = ?

Using:

$$S = v_i t + \frac{1}{2} a t^2$$

$$S = v_i t + \frac{1}{2} g t^2$$

or $$-156.8 \text{ m} = 19.6 \times t - \frac{1}{2} \times 9.8 \text{ ms}^{-2} \times t^2$$

or $$-32 = 4t - t^2$$

or $$t^2 - 4t - 32 = 0$$

or $$t^2 - 8t + 4t - 32 = 0$$

or $$t(t - 8) + 4(t - 8) = 0$$

or $$(t - 8)(t + 4) = 0$$

or $$t = 8, -4s$$

Time is always positive. Therefore, we neglect the negative value of t.

Thus **Required time is t = 8 s**

2.3 If |A + B| = |A − B|, then prove that A and B are perpendicular to each other.

Solution:

We are given that;

$$|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$$

We have to prove that $\vec{A}$ and $\vec{B}$ are perpendicular i.e.,

$$\vec{A}.\vec{B} = 0$$

Squaring both sides to eliminate the magnitudes:

$$|\vec{A} + \vec{B}|^2 = |\vec{A} - \vec{B}|^2$$

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Use the identity $|\vec{X}|^2 = \vec{X} \cdot \vec{X}$

$$(\vec{A} + \vec{B}) \cdot (\vec{A} + \vec{B}) = (\vec{A} - \vec{B}) \cdot (\vec{A} - \vec{B})$$

or $\vec{A} \cdot \vec{A} + 2\vec{A} \cdot \vec{B} + \vec{B} \cdot \vec{B} = \vec{A} \cdot \vec{A} - 2\vec{A} \cdot \vec{B} + \vec{B} \cdot \vec{B}$

or $2(\vec{A} \cdot \vec{B}) = -2(\vec{A} \cdot \vec{B})$

Adding $2(\vec{A} \cdot \vec{B})$ to both sides

$$2(\vec{A} \cdot \vec{B}) + 2(\vec{A} \cdot \vec{B}) = -2(\vec{A} \cdot \vec{B}) + 2(\vec{A} \cdot \vec{B})$$

or $4(\vec{A} \cdot \vec{B}) = 0$

or $\vec{A} \cdot \vec{B} = 0$

Conclusion:

Since the dot product is zero, so, $\vec{A} \perp$ar $\vec{B}$ i.e., angle between $\vec{A}$ and $\vec{B}$ is 90°.

2.4 A body of mass *M* at rest explodes into 3 pieces, two of which of mass *M / 4* each are thrown off in perpendicular directions with velocities of 3 m s⁻¹ and 4 m s⁻¹, respectively. Find the velocity of 3rd piece with which it will be flown away.

Solution:

Given that:

- A body of mass M at rest.
- It explodes into three pieces.
- Two of the pieces have mass m/4 and are ejected.
- One moves with velocity 3 ms^{-1} in some direction.
- The other moves with velocity 4 ms^{-1} in a direction perpendicular to the first.
- The third piece must have mass $M - 2\left(\frac{M}{4}\right) = \frac{M}{2}$

To find: The velocity of third piece

Conclusions:

Since initially the body was at rest, the total momentum before explosion is zero. So, the vector sum of the momenta of all three pieces must also be zero.

Let

- Piece 1 (mass = M/4) moves in the x-direction with velocity of 3 ms^{-1}.
- Piece 2 (mass = M/4) moves in the y-direction with velocity of 4 ms^{-1}.
- Piece 3 (mass = M/2) must move such that total momentum = 0.

Computing Momentum Components

x-component:

- Piece 1: $p_{x_1} = \frac{M}{4} \cdot 3\ ms^{-1} = \frac{3\ M}{4}\ ms^{-1}$
- Piece 2: $p_{x_2} = 0$
- Let piece 3 has velocity:

  $v = (v_x, v_y)$, so
- $p_{x_3} = \frac{M}{2} \times v_x$

Applying conservation in x-direction

$$\frac{3M}{4} + 0 + \frac{M}{2} v_x = 0$$

or $\frac{M}{2} v_x = -\frac{3M}{4}$

or $v_x = \frac{-3}{2} = 1.5\ ms^{-1}$

Applying conservation in y-direction:

$$0 + M + \frac{M}{2} v_y = 0$$

or $\frac{M}{2} v_y = -M$

or $v_y = -2\ ms^{-1}$

For finding the magnitude of velocity of third piece:

$$\sqrt{v_x^2 - v_y^2} = \sqrt{(-1.5\ ms^{-1})^2 + (-2\ ms^{-1})^2}$$

$$= \sqrt{2.25\ m^2s^{-2} + 4\ m^2s^{-2}}$$

$$= \sqrt{6.25\ m^2s^{-2}}$$

$$\sqrt{v_x^2 + v_y^2} = 2.5\ ms^{-1}$$

Thus, the third piece moves with the velocity of 2.5 ms^{-1}, opposite to the resultant velocity vector of two pieces.

2.5 A cricket ball is hit upward with velocity of 20 m s⁻¹ at an angle of 45° with the ground. Find its:
(a) time of flight (b) maximum height
(c) how far away it hits the ground

Solution:

Given data:

Angle $\theta = 45^\circ$

Velocity $v = 20\ ms^{-1}$

Acceleration due to gravity $g = 9.8\ ms^{-2}$.

To Find:

(a) Time of flight t = ?

(b) Maximum height h = ?

(c) Horizontal range R_H = ?

Calculations:

(a) For calculating time of flight, we use the Formula:

$$t = \frac{2\ v_i \sin\theta}{g}$$

Putting the values

$$t = \frac{2 \times 20\ ms^{-1} \times \sin 45^\circ}{9.8\ ms^{-2}}$$

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$$= \frac{2 \times 20\ ms^{-1} \times 0.707}{9.8\ ms^{-2}} \quad (\because \sin 45^o = 0.707)$$

$$\boxed{t = 2.9\ s}$$

(b) For the calculation of maximum height, we use the Formula:

$$H_{max} = \frac{v_i^2 \sin^2 \theta}{2g}$$

Putting the values

$$H_{max} = \frac{(20\ ms^{-1})^2 \times (\sin 45^o)^2}{2 \times 9.8\ ms^{-2}}$$

$$= \frac{400\ m^2s^{-2} \times (0.707)^2}{19.6\ ms^{-2}}$$

$$\boxed{H_{max} = 10.2\ m}$$

(c) To find horizontal range, we use the Formula:

$$R_H = \frac{v_i^2 \sin 2\theta}{g}$$

Putting the values

$$R_H = \frac{(20\ ms^{-1})^2 (\sin 90^o)}{9.8\ ms^{-2}}$$

$$= \frac{400\ m^2s^{-2} \times 1}{9.8\ ms^{-2}} \quad (\because \sin 90^o = 1)$$

$$\boxed{R_H = 41\ m}$$

2.6 A 20 g ball hits the wall of a squash court with a constant force of 50 N. If the time of impact of force is 0.50 s, find the impulse.

Solution:

Given data:

Mass of ball $m = 20\ g = 0.002\ kg$

Constant force $F = 50\ N$

Time of impact $t = 0.50\ N$

To Find:

Impulse = ?

Calculations:

The impulse exerted on an object is given by the Formula:

Impulse = Force × Time

Putting the values

Impulse = 50 N × 50 s = 25 N s

2.7 A ball is kicked by a footballer. The average force on the ball is 240 N, and the impact lasts for a time interval of 0.25 s.

(a) Calculate change in momentum

(b) State the direction of change in momentum

Solution:

Given data:

Average force $F = 240\ N$

Impact time $t = 0.25\ s$

To Find:

(a) Change in momentum $\Delta p = F \times t$

(b) Direction of change of momentum = ?

Calculations:

(a) Change in momentum = Impulse = $\Delta p = F \times t$

Putting the values

$$\Delta p = 240\ N \times 0.25\ s$$

$$\Delta p = 60\ kg\ ms^{-1}$$

(b) The direction of change in momentum is the same as the direction of the applied force, which is in the direction, the ball is kicked.

2.8 An aeroplane is moving horizontally at a speed of 200 m s^{-1} at a height of 8 km to drop a bomb on a target. Find horizontal distance from the target at which the bomb should be released.

Solution:

Given data:

Height = h = 8 km = 8000 m

Horizontal speed of aeroplane $v_x = 200\ ms^{-1}$

Acceleration due to gravity $g = 9.8\ ms^{-1}$

To find:

Horizontal distance = ?

Calculations:

Time taken to fall:

Using the equation for free fall

$$h = \frac{1}{2}gt^2$$

or $$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 8000\ m}{9.8\ ms^{-2}}}$$

$$= \sqrt{1632.65\ s^2} \approx 40.4\ s$$

Horizontal distance

$$S = v \times t$$

$$S = 200\ ms^{-1} \times 40.4\ s = 8080\ m = 80.8\ km$$

Thus, the bomb should be released 8080 m or 8.08 km before reaching the target.

2.9 Why does range *R* of a projectile remain the same when angle of projection is changed from 0 to 0′ = 90° − 0. Also show that for complementary angles of projection, the ratio R / R′ is equal to 1.

Solution:

The formula for the range R of a projectile is:

$$R = \frac{v_i^2 \sin 2\theta}{g}$$

Now, let us see what happens if we replace θ with $\theta' = 90^o - \theta$

$$R' = \frac{v_i^2 \sin(2\theta')}{g} = \frac{v_i^2 \sin[2(90^o - \theta)]}{g}$$

$$R' = \frac{v_i^2 \sin(180^o - 2\theta)}{g}$$

Since $\sin(180^o - x) = \sin x$, therefore,

$$R' = \frac{v_i^2 \sin(2\theta)}{g} = R$$

Conclusion:
The range is the same for complementary angles θ and $90^o - \theta$.
To show that for complementary angles of projection the ratio $\frac{R}{R'} = 1$.
From the above derivation:

$$R = \frac{v_i^2 \sin(2\theta)}{g} \text{ and } R' = \frac{v_i^2 \sin(2\theta')}{g}$$

Since $\sin(2\theta') = \sin(180^o - 2\theta) = \sin 2\theta$, then

$$\frac{R}{R'} = \frac{v_i^2 \sin(2\theta)/g}{v_i^2 \sin(2\theta)/g}$$

Thus, the range remains the same because $\sin(2\theta) = \sin[2(90 - \theta)]$ and the ratio $\frac{R}{R'} = 1$ for complementary angles.

2.10 A trolley of mass 1.0 kg moving with velocity 1.0 m s^{-1} collides with a similar trolley at rest:
(i) after collision, the 1st trolley comes to rest whereas the second starts moving with velocity of 1.0 m s^{-1} in the same direction. Show that it is an example of an elastic collision.
(ii) after the collision, they stick together and move away with a velocity of 0.5 m s^{-1}. Show that it is an example of an inelastic collision.

Solution:

Given data:
Mass of both trolleys m = 1.0 kg
Initial velocity of trolley 1 = v_1 = 1.0 ms^{-1}
Initial velocity of trolley 2 = v_2 = 0 ms^{-1}
Calculations: Let us analyze both situations using the principles of momentum and kinetic energy conservation.

(i) Elastic collision
After collision;
Final velocity of trolley 1 = v_1' = 0 ms^{-1}
Final velocity of trolley 2 = v_2' = 1.0 ms^{-1}

- **Conservation of momentum:**
According to law of conservation of momentum:
Initial momentum = Final momentum

$$mv_1 + mv_2 = mv_1' + mv_2'$$

$$1.0 \text{ kg} \times 1.0 \text{ ms}^{-1} + 1.0 \text{ kg} \times 0 \text{ ms}^{-1} = 1.0 \text{ kg} \times 0 \text{ ms}^{-1} + 1.0 \text{ kg} \times 1.0 \text{ ms}^{-1}$$

$$1 \text{ kg ms}^{-1} = 1 \text{ kg ms}^{-1}$$

So, momentum is conserved.

- **Conservation of K.E.:**
According to law of conservation of energy:

$$K.E_{Initial} = K.E_{Final}$$

$$K.E_{Initial} = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$$

$$\frac{1}{2} \times 1.0 \text{ kg} \times 1.0 \text{ ms}^{-1} + \frac{1}{2} \times 1.0 \text{ kg} \times 0 \text{ ms}^{-1} = 0.5 \text{ J}$$

$$K.E_{Final} = \frac{1}{2} \times 1.0 \text{ kg} \times 0 \text{ ms}^{-1} + \frac{1}{2} \times 1.0 \text{ kg} \times 1.0 \text{ ms}^{-1} = 0.5 \text{ J}$$

Thus $K.E_{Initial} = K.E_{Final}$
So, kinetic energy is conserved.
Conclusion: This is an elastic collision, as both momentum and kinetic energy are conserved.

(ii) Inelastic Collision
After collision, both trolleys stick together and move with velocity equal to 0.5 ms^{-1}.

- **Conservation of momentum**

$$P_{Initial} = mv_1 = 1.0 \text{ kg} \times 1.0 \text{ ms}^{-1} = 1 \text{ kg ms}^{-1}$$

As combined mass of both the trolleys equals 2 kg, therefore,

$$P_{Final} = mv = 2 \text{ kg} \times \text{ms}^{-1} = 1 \text{ kg ms}^{-1}$$

So, momentum is conserved.

- **Conservation of kinetic energy:**

$$K.E_{Initial} = \frac{1}{2}mv_1^2 = \frac{1}{2} \times 1.0 \text{ kg} \times 1.0 \text{ ms}^{-1} = 0.5 \text{ J}$$

$$K.E_{Final} = \frac{1}{2}mv^2 = \frac{1}{2} \times 2 \text{ kg} \times (0.5 \text{ ms}^{-1})^2 = 0.25 \text{ J}$$

As $K.E_{Initial}$ is not equal to $K.E_{Final}$, so K.E is not conserved.
Conclusion: This is an inelastic collision, as momentum is conserved but K.E is not.

2.11 A railway wagon of mass 4 × 10^4 kg moving with velocity of 3 m s^{-1} collides with another wagon of mass 2 × 10^4 kg which is at rest. They stick together and move off together. Find their combined velocity.

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Solution:

Given data:

Mass of first wagon = $m_1 = 4 \times 10^4$ kg

Velocity of first wagon = $v_1 = 3\ ms^{-1}$

Mass of second wagon = $m_2 = 2 \times 10^4$ kg

Velocity of second wagon = $v_2 = 0\ ms^{-1}$

To find:

Combined velocity of wagons v = ?

Calculations: After the collision, they stick together. So, it a perfectly inelastic collision.

Let the final combined velocity of both the wagons be v. According to law of conservation of momentum:

$$m_1v_1 + m_2v_2 = (m_1 + m_2)\ v$$

$$4 \times 10^4\ kg \times 3\ ms^{-1} + 2 \times 10^4\ kg \times 0\ ms^{-1} = (4 \times 10^4\ kg + 2 \times 10^4\ kg) \times v$$

$$1.2 \times 10^5 = 6 \times 10^4 \times v$$

$$v = \frac{1.2 \times 10^5}{6 \times 10^4}$$

$$v = 2\ ms^{-1}$$

Thus, final velocity after collision is $2\ ms^{-1}$.

2.12 A car with mass 575 kg moving at 15.0 m s^{-1} smashes into the rear end of a car with mass 1575 kg moving at 5 m s^{-1} in the same direction.

(a) What is the final velocity if the wrecked car lock together?

(b) How much kinetic energy is lost in the collision?

Solution:

Given data:

Mass of car 1 = m_1 = 575 kg

Velocity of car 1 before collision $v_1 = 15\ ms^{-1}$

Mass of car 2 = m_2 = 1575 kg

Velocity of car 2 before collision $v_2 = 5\ ms^{-1}$

To find:

(i) Final velocity of wrecked car v_f = ?

(ii) Loss of K.E. in the collision $K.E_{Loss}$ = ?

Calculations:

(a) We use law of conservation of momentum:

$$m_1v_1 + m_2v_2 = (m_1 + m_2)\ v_f$$

$$575\ kg \times 15\ ms^{-1} + 1575\ kg \times 5\ ms^{-1} = (575 + 1575)\ kg \times v_f$$

$$8625\ kg\ ms^{-1} + 7875\ kg\ ms^{-1} = 2150\ kg \times v_f$$

or

$$v_f = \frac{16500\ kg\ ms^{-1}}{2150\ kg\ ms^{-1}}$$

$$v_f = 7.67\ ms^{-1}$$

(b) Kinetic energy lost:

$$K.E_{Initial} = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

$$= \frac{1}{2} \times 575\ kg \times (15\ ms^{-1})^2 + \frac{1}{2}1575\ kg \times (5\ ms^{-1})^2$$

$$= (0.5)\ (575)\ (225)\ kg^2\ ms^{-2}$$

$$= (0.5)\ (1575)\ (25)\ kg\ m^2s^{-2}$$

$$= (64687.5 + 19687.5)\ kg\ m^2s^{-2} = 84375\ J$$

$$K.E_{Final} = \frac{1}{2}(m_1 + m_2)\ v_f^2 = \frac{1}{2}(2150\ kg)\ (7.67\ ms^{-1})^2$$

$$= (0.5)\ (2150)\ (58.85)\ kg\ m^2\ s^{-2}$$

$$K.E_{Final} = 63264.9\ J$$

$$K.E_{Lost} = K.E_{Initial} + K.E_{Final} = 84375\ J - 63264.9\ J$$

$$\approx 21110.1\ J$$

$$K.E_{Lost} = 2.11 \times 10^4\ J$$

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