Unit 03 Circular and Rotational Motion

STUDENT'S LEARNING OUTCOMES (SLO's)

After studying this unit, the students will be able to:

- Express angles in radians
- Define and calculate angular displacement, angular velocity and angular acceleration [This involves use of $S = r\theta$, $v = r\omega$, $\omega = 2\pi r/T$, $a = r\omega^2$, and $a = v^2/r$ to solve problems]
- Use equations of angular motion to solve problems involving rotational motions.
- Discuss qualitatively motion in a curved path due to a perpendicular force.
- Define and calculate centripetal force [Use $F_c = mr\omega^2$, $F_c = mv^2/r$]
- Analyze situations involving circular motion in terms of centripetal force [e.g. situations in which centripetal acceleration is caused by a tension force, a frictional force, a gravitational force, or a normal force.]
- Define and calculate average orbital speed for a satellite. [from the equation $v = 2\pi r / T$ where r is the average radius of the orbit and T is the orbit period; apply this equation to solve numerical problems]
- Explain why the objects in orbiting satellites appear to be weightless.
- Describe how artificial gravity is created to counter weightlessness.
- Define and calculate moments of inertia of a body and angular momentum.
- Derive and apply the relation between torque, moment of inertia and angular acceleration. Illustrate the applications of conservation of angular momentum in real life. [such as by flywheels to store rotational energy, by gyroscopes in navigation systems, by ice skaters to adjust their angular velocity]
- Describe how a centrifuge is used to separate materials using centripetal force

3.1 ANGULAR MEASUREMENTS

Q 1. What is the unit of angular displacement? Define it.

Ans. The SI unit of angular displacement is radian.

Radian

Definition

If the length of the arc is equal to the radius of the circle, then the angle is called one radian.

The angle drawn at the centre of the circle by an arc AB as shown in the figure is equal to radius *r* of the circle, therefore, this angle is equal to one radian.

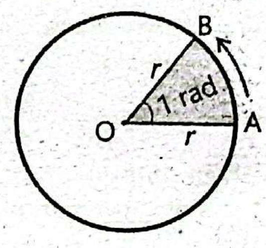


Fig. 1

Q 2. What is meant by angular displacement? Explain.

Ans. Angular Displacement

Definition

Angular displacement is the angel subtended at the centre of a circle by a particle moving along the circumference in a given time

Explanation

Consider the motion of a single particle P of mass *m* in a circular path of radius *r*. Suppose this motion is taking place by attaching the particle P at the end of a massless rigid rod of length *r* whose other end is pivoted at the centre O of the circular path, as shown in Fig. 2 (a). As the particle is moving on the circular path, the rod OP rotates in the plane of the circle. The axis of rotation passes through the pivot O and is normal to the plane of rotation. Consider a system of axes as shown in Fig. 2 (b). The z-axis is taken along the axis of rotation with the pivot O as origin of coordinates. Axes x and y are taken in the plane of rotation. While OP is rotating, suppose at any instant t, its position is OP_1, making angle θ with x-axis. At a later time $t + \Delta t$, let its position be OP_2 making angle $\theta + \Delta\theta$ with x-axis (Fig. 2 c).

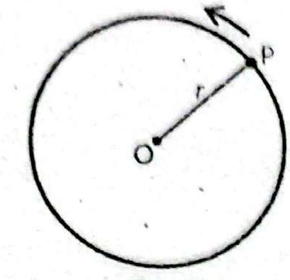


Fig. 2 (a)

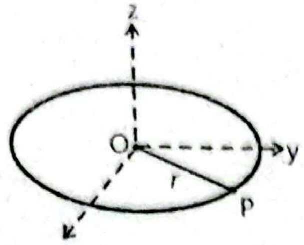


Fig. 2 (b)

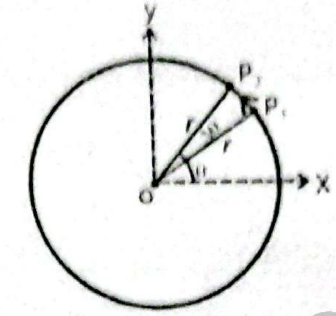


Fig. 2 (c)

Angle $\Delta\theta$ defines the angular displacement of OP during the time interval Δt. For very small values of $\Delta\theta$, the angular displacement is a vector quantity.

Direction of Angular Displacement

The angular displacement $\Delta\theta$ is assigned a positive sign when the sense of rotation of OP is anticlockwise.

The direction associated with $\Delta\theta$ is along the axis of rotation and is given by right hand rule; Fig 2 (d).

Right-hand Rule

Grasp the axis of rotation in right hand with fingers curling in the direction of rotation; the thumb points in the direction of angular displacement.

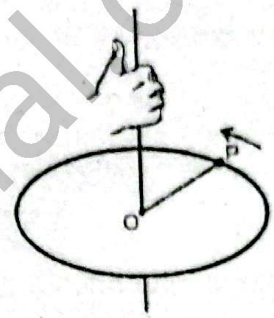


Fig. 2 (d)

3. Prove that; S = rθ. Also show that; 1 rad = 57.3°.

Ans. Units of Angular Displacement

Consider an arc of length S of a circle of radius r; Fig. 3 which subtends an angle θ at the centre of the circle. Its value in radians (rad) is given as:

$$\theta = \frac{S}{r}$$

or $\quad S = r\theta$ (where θ is in radian) ...... (i)

If OP is rotating, the point P covers a distance $S = 2\pi r$ in one revolution of P. In radian, it would be:

$$\frac{S}{r} = \frac{2\pi r}{r} = 2\pi$$

So $\quad$ 1 revolution $= 2\pi$ rad $= 360°$

$$1\text{rad} = \frac{360°}{2\pi} = 57.3°$$

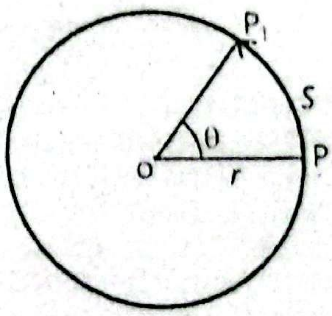


Fig. 3

4. Define the following:

(i) Angular velocity **(ii) Average angular velocity** **(iii) Instantaneous angular velocity**

Ans. (i) Angular Velocity

Definition

The rate at which the angular displacement changes with time is known as angular velocity.

Unit: Angular velocity is measured in radians per second, which is the SI unit. Sometimes, it is also measured in revolution per minutes.

(ii) Average Angular Velocity

Definition

It is defined as the ratio of the angular displacement to the time taken by the particle to undergo this displacement.

In Fig. 2(c), if $\Delta\theta$ is the angular displacement during the time interval Δt, the average angular velocity ω_{av} during this interval is given as:

$$\omega_{av} = \frac{\Delta\theta}{\Delta t}$$

(iii) Instantaneous Angular Velocity

Definition

It is defined as the limit of the average angular velocity as the time interval approaches to zero.

Thus $$\omega_{inst} = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}$$

Q 5. Define the following:

(i) Angular Acceleration (ii) Average angular acceleration (iii) Instantaneous angular acceleration

Ans. (i) Angular Acceleration

The time rate of change of angular velocity is known as **angular acceleration**.

Unit: The SI unit of angular acceleration is radian per square second (rad s^{-2}).

(ii) Average Angular Acceleration

Definition

The average angular acceleration is defined as the rate of change of angular velocity.

If ω_i and ω_f are the values of instantaneous angular velocity of a rotating body at instant t_i and t_f, the average angular acceleration during the time interval $t_f - t_i$ is given by

$$\alpha_{av} = \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{\Delta\omega}{\Delta t}$$

(iii) Instantaneous Angular Acceleration

Definition

The instantaneous angular acceleration is the time derivative of angular velocity.

The instantaneous angular acceleration is the limit of the ratio $\frac{\Delta\omega}{\Delta t}$ as Δt approaches zero. Therefore, instantaneous angular acceleration is given by

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t}$$

Q 6. Derive a relation between angular and tangential or linear velocity.

or Prove that; v = rω

Ans. Relation between Angular and Linear Velocity

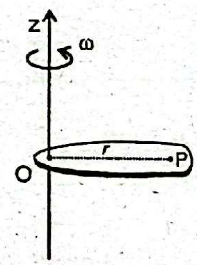


Fig. 4 (a)

Consider a rigid body rotating about z-axis with an angular velocity ω; Fig. 4 (a). Imagine a point P in the rigid body at a perpendicular distance r from the axis of rotation. OP represents the reference line of the rigid body. As the body rotates, the point P moves along a circle of radius r with a linear velocity **v** whereas the line OP rotates with angular velocity ω; Fig. 4(b). We have to find the relation between ω and **v**. As the axis of rotation is fixed, so the direction of ω always remains the same. For the linear velocity of the point P, let us consider only its magnitude only.

$$P_1P_2 = \Delta S$$

Suppose during its motion, the point P moves through a distance $P_1P_2 = \Delta S$ in a time interval Δt during which reference line OP covers an angular displacement $\Delta\theta$ radian. So, ΔS and $\Delta\theta$ are related by equation; $S = r\theta$ as:

$$\Delta S = r\Delta\theta$$

Dividing both sides by Δt

$$\frac{\Delta S}{\Delta t} = r\frac{\Delta\theta}{\Delta t}$$

In the limit, when linear $\Delta t \to 0$ the ratio $\Delta S/\Delta t$ represents v, the magnitude of the linear velocity with which point P is moving on the circumference of the circle. Similarly $\Delta\theta/\Delta t$ represents the angular velocity ω of the reference line OP. So, the above equation becomes:

$$v = r\omega$$

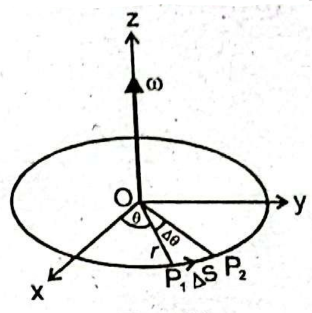


Fig. 4 (b)

7. Derive a relation between angular and linear acceleration. # or Prove that; $a_t = r\alpha$.

Ans. Relation between Linear and Angular Acceleration

From Fig. 4(b), it can be seen that the point P is moving along the arc P_1P_2. When $\Delta t \to 0$, the length of arc P_1P_2 becomes very small and its direction represents the direction of tangent to the circle at point P_1. Thus, the velocity with which point P is moving on the circumference of the circle has a magnitude v and its direction is always along the tangent to the circle at that point. That is why, the linear velocity of the point P is also known as **tangential velocity**.

Similarly, the equation $v = r\omega$ shows that if the reference line OP is rotating with an angular acceleration a, the point P will also have a linear or tangential acceleration a_t. Using the equation; $v = r\omega$, it can be shown that the two accelerations are related by

$$a_t = r\alpha$$

8. Write the equations of angular motion analogous to linear motion.

Ans. Equations of Angular Motion

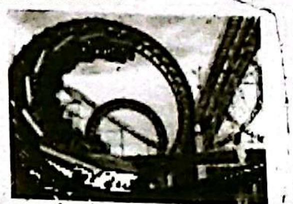
The equations;

$$\omega_{av} = \frac{\Delta\theta}{\Delta t}, \quad \omega = \lim_{\Delta t \to 0}\frac{\Delta\theta}{\Delta t}, \quad \alpha_{av} = \frac{\Delta\omega}{\Delta t}$$

and $\alpha = \lim_{\Delta t \to 0}\frac{\Delta\omega}{\Delta t}$ of angular motion are exactly analogous to those in linear motion if θ, ω and α be replaced by S, v and a, respectively. Given below are the angular equations together with their linear counterparts.

Linear Equations	Angular Equations
$v_f = v_i + at$	$\omega_f = \omega_i + \alpha t$
$2aS = v_f^2 + v_i^2$	$2\alpha\theta = \omega_f^2 - \omega_i^2$
$S = v_i t + \frac{1}{2}at^2$	$\theta = \omega_i t + \frac{1}{2}\alpha t^2$

Point to Ponder!



You may feel scared at the top of roller coaster ride in the amusement parks but you never fall down even when you are upside down. Why?

Ans. You do not fall out of a roller coaster even when you are upside down because of inertia, centripetal force, and safety restrains.

Example 3.1: An electric fan rotating at 3.0 rev s^{-1} is switched off. It comes to rest in 18.0 s. Assuming deceleration to be uniform, find its value. How many revolutions did it turn before coming to rest?

Solution:

Given that;

$\omega_i = 3.0 \text{ rev s}^{-1}$, $\omega_f = 0$, $t = 18.0 \text{ s}$

To Find:

$\alpha = ?$ and $\theta = ?$

Calculations:

Using the formula:

$$\alpha = \frac{\omega_f - \omega_i}{t}$$

$$\alpha = \frac{(0 - 3.0)\text{ rev s}^{-1}}{18.0\text{ s}} = -0.167\text{ rev s}^{-2}$$ **Ans.**

Putting the values

Now using the formula:

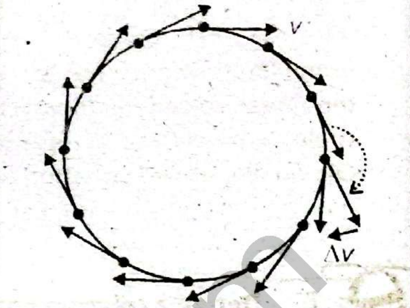
$$\theta = \omega_i t + \frac{1}{2}\alpha t^2$$

Putting the values

$$\theta = 3.0\text{ rev s}^{-1} \times 18.0\text{ s} + \frac{1}{2}(-0.167\text{ rev s}^{-2}) \times (18.0\text{ s})^2$$

θ = 27 rev **Ans.**

Do you know?



Direction of motion changes continuously in circular motion.

mQ's ✓

1. **One radian is equal to:**
(A) 77.3° (B) 76.3°
(C) 77.3° (D) 57 3° ✓

2. **30° is equal to how many radians?**
(A) $\frac{\pi}{8}$ (B) $\frac{\pi}{6}$ ✓
(C) $\frac{\pi}{5}$ (D) $\frac{\pi}{2}$

3. **One radian equals to:**
(A) 2π *rev* (B) $\frac{\pi}{4}$ *rev*
(C) $\frac{\pi}{2}$ *rev* (D) $\frac{1}{2\pi}$ *rev* ✓

4. **The angle subtended at the centre by circumference of a circle is:**
(A) π *radian* (B) π *radian*
(C) 2π *radian* ✓ (D) $\frac{\pi}{2}$ *radian*

5. **100 radians equals to:**
(A) 57.3° (B) 573°
(C) 5730° ✓ (D) 5.73°

6. **The dimensions of angular velocity are:**
(A) $[LT^{-2}]$ (B) $[LT^{1}]$
(C) $[L^{-1}T]$ (D) $[T^{-1}]$ ✓

7. **The direction of angular velocity is found by**
(A) Left hand rule (B) Right hand rule ✓
(C) Head to tail rule (D) None

8. **The dimension of angular acceleration is:**
(A) $[T^{-1}]$ (B) $[LT^{2}]$
(C) $[T^{-2}]$ ✓ (D) $[T^{-3}]$

9. **An angle through which a body moves is:**
(A) Angular velocity (B) Angular acceleration
(C) Angular displacement ✓ (D) Angular momentum

10. **1 revolution equals to:**
(A) 57° (B) 90°
(C) 180° (D) 360° ✓

11. **The SI unit of angular displacement is:**
(A) meter (B) degree
(C) revolution (D) radian ✓

12. **π-radian is equal to:**
(A) 0° (B) 90°
(C) 57.3° (D) 180° ✓

13. **60° is equal to:**
(A) $\frac{\pi}{8}$ (B) $\frac{\pi}{6}$
(C) $\frac{\pi}{5}$ (D) $\frac{\pi}{3}$ ✓

14. **Which quantity of the following is dimensionless?**
(A) Angular velocity (B) Angular acceleration
(C) Centripetal force (D) Angular displacement ✓

15. **A wheel of radius 2 m turns through an angle of 57.3°. It covers a circular distance of:**
(A) 2 *m* ✓ (B) 4 *m*
(C) 57.3 *m* (D) 113.6 *m*

16. **A wheel of diameter 1 m makes 60 revolution per minute. The linear speed of a point on its rim in ms^{-1} is:**
(A) π ✓ (B) 2π
(C) $\frac{\pi}{2}$ (D) 3π

17. **2 radians are equal to:**
(A) 114.6° ✓ (B) 57.3°
(C) 75.3° (D) 37.5°

18. **The angular velocity of the minute hand of a clock is:**
(A) $2\pi\ rad\ s^{-1}$ (B) $\pi\ rad\ s^{-1}$
(C) $\frac{\pi}{60}\ rad\ s^{-1}$ (D) $\frac{\pi}{1800}\ rad\ s^{-1}$ ✓

19. **A body starting from rest attains angular acceleration of 5 rad s^{-2} in 2 seconds. Its angular velocity will be:**

(A) 10 $rad\ s^{-1}$ ✓ (B) 7 $rad\ s^{-1}$
(C) 3 $rad\ s^{-1}$ (D) 2 $rad\ s^{-1}$

20. **The time rate of change of angular displacement is called:**

(A) Linear velocity (B) Linear speed
(C) Angular momentum (D) Angular velocity ✓

21. **2 revolutions are equal to:**

(A) π rad (B) $\frac{3\pi}{2}$ rad
(C) 2π rad (D) 4π rad ✓

22. **2° is equal to:**

(A) 0.035 *rad* ✓ (B) 0.30 *rad*
(C) 0.35 *rad* (D) 0.0035 *rad*

23. **1 rev per minute is equal to:**

(A) $\frac{\pi}{6}$ rad / s (B) $\frac{\pi}{30}$ rad / s ✓
(C) $\frac{\pi}{15}$ rad / s (D) 2π rad / s

24. **If a car moves with a uniform speed of 2ms^{-1} in a circle of radius 0.4 m, its angular speed is:**

(A) 4 rad s^{-1} (B) 5 rad s^{-1} ✓
(C) 2 rad s^{-1} (D) 0.4 rad s^{-1}

25. **A body is revolving with an angular velocity 10 rad s^{-1} in a circle of radius 20 cm. Its linear velocity is:**

(A) 2 ms^{-1} ✓ (B) 200 ms^{-1}
(C) 20 ms^{-1} (D) 10 ms^{-1}

26. **The angular displacement per second is called angular:**

(A) Acceleration (B) Rotation
(C) Velocity ✓ (D) Speed

27. **The SI unit of angular acceleration is:**

(A) rad s^{-2} ✓ (B) rev s^{-2}
(C) degree s^{-2} (D) m s^{-2}

28. **90° is equal to how many radians?**

(A) $\frac{\pi}{8}$ (B) $\frac{\pi}{6}$
(C) $\frac{\pi}{5}$ (D) $\frac{\pi}{2}$ ✓

29. **The angle 2π rad is equal to:**

(A) 0° (B) 180°
(C) 90° (D) 360° ✓

30. **The angular displacement for daily rotation of the Earth is:**

(A) 0 rad (B) π rad
(C) 2π rad ✓ (D) 4π rad

31. **All points on rigid body rotating about a fixed axis do not have the same:**

(A) Speed ✓ (B) Angular Speed
(C) Angular Acceleration (D) Angular displacement

32. **Which of the following is correct?**

(A) $\omega = vr$ (B) $v = r / \omega$
(C) $v = r\omega$ ✓ (D) $\omega = r/v$

33. **A wheel of radius 50 cm having the angular speed of 5 rad s^{-1} will have linear speed in m:**

(A) 1.5 (B) 2.5 ✓
(C) 3.5 (D) 4.5

34. **When a body moves in a circle, the angle between its linear velocity and angular velocity is:**

(A) 180° (B) 90° ✓
(C) 0° (D) 45°

35. **The direction of angular velocity is determined by**

(A) Left hand rule (B) Head to tail rule
(C) Right hand rule ✓ (D) General rule

36. **A body rotating with angular velocity of 2 radian s^{-1} and linear velocity is also 2ms^{-1}, then radius is:**

(A) 1*m* ✓ (B) 0.5*m*
(C) 4*m* (D) 2*m*

37. **If a body is moving in the counter clockwise direction, then the direction of angular velocity will be:**

(A) Towards the center
(B) Away from the center
(C) Along the linear velocity
(D) Perpendicular to both radius and linear velocity ✓

38. **The velocity of moon around the Earth is:**

(A) 10^2 ms^{-1} (B) 10^3 ms^{-1} ✓
(C) 10^4 ms^{-1} (D) 10^5 ms^{-1}

39. **If a body is moving counter clockwise, then angular displacement is:**

(A) Minimum (B) Zero
(C) Negative (D) Positive ✓

3.2 CENTRIPETAL FORCE

Q9. Define centripetal force. Prove that; $F_c = \frac{mv^2}{r}$ or $F_c = mr\omega^2$.

Ans. Centripetal Force

Definition

The force needed to bend the straight path of the particle into a circular path is called the centripetal force.

Expression for Centripetal force

When a constant force acts perpendicular to the velocity of a body moving in a circular path it will change the direction but magnitude of velocity (speed) will remain the same. Such force makes the body move in a circle by producing a radial (or centripetal) acceleration and is called centripetal force (Centre seeking) force. Consider a ball tied at the end of a string is whirling in a horizontal surface; Fig. 5. The ball will not continue in a circular path if the string is snapped. Careful observation shows at once that if the string snaps, when the ball is at the point *A* the ball will follow the straight-line path AB which is tangent AB at point A.

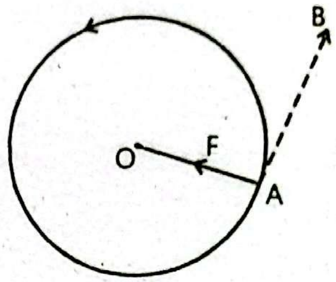


Fig. 5

Thus, a force is needed to change the direction of velocity or motion of a body continuously at each point in circular motion moving with uniform speed. The force that does not alter speed but only direction at each point is a perpendicular force which acts along the radius of the circular path. This force always pulls the object towards the centre of the circular path. Its direction is perpendicular to the tangential velocity at each point.

For a body of mass *m* moving with velocity *v* in a circular path of radius *r*, centripetal force F_c is given by

$$F_c = ma_c = \frac{mv^2}{r}$$

where $a = v^2/r$ is the centripetal acceleration and its direction is towards the centre of the circle. As $v = r\omega$, so the above equation becomes:

$$F_c = mr\omega^2$$

Example 3.2: If a CD spins at 210 rpm, what is the radial acceleration of a point on the outer rim of the CD? The CD is 12 cm in diameter.

Solution: We convert 210 rpm into a frequency in revolutions per second (Hz).

Thus $$f = 210\frac{\text{rev}}{\text{min}} \times \frac{1\ \text{min}}{60\ \text{s}} = 3.5\frac{\text{rev}}{\text{s}} = 3.5\ \text{Hz}$$

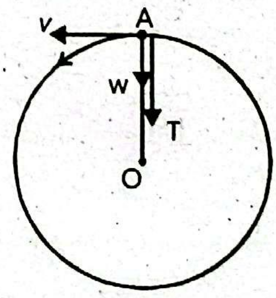
For each revolution, the CD rotates through an angle of 2π radians. The angular velocity is:

$$\omega = 2\pi f = 2\pi\ \text{rad} \times 3.5\ \text{s}^{-1} = 7.0\ \pi\ \text{rad s}^{-1}$$

The radial acceleration is:

$$a = \omega^2 r = (7.0\ \pi\ \text{rad s}^{-1})^2 \times 0.06\ \text{m} = 29\ \text{m s}^{-2}$$ **Ans.**

Example 3.3: A ball tied to the end of a string, is swung in a vertical circle of radius r under the action of gravity as shown in Fig. What will be the tension in the string when the ball is at the point A of the path and its speed is v at this point?



Solution: For the ball to travel in a circle, the force acting on the ball must provide the required centripetal force. In this case, at point A, two forces act on the ball, the pull of the string and the weight w of the ball. These forces act along the radius at A, and so their vector sum must furnish the required centripetal force. We, therefore, have

$$T + w = \frac{mv^2}{r}$$

As $w = mg$,

Therefore,

$$\therefore \quad T = \frac{mv^2}{r} - mg = m\left(\frac{v^2}{r} - g\right)$$

If $\frac{v^2}{r} = g$, then T will be zero and the centripetal force is just equal to the weight.

10. Enlist some examples of centripetal force.

Ans. Examples of Centripetal Force:

In every circular or orbital motion, centripetal force is needed which is provided by some agency.

1. When a ball is whirled in a horizontal circle with the help of a string, then tension in the string provides necessary centripetal force.
2. For an object placed on a turntable, the friction is the centripetal force.
3. The gravitational force is the cause of the Earth orbiting around the Sun, Moon and artificial satellites revolving around the Earth.
4. A normal or perpendicular magnetic force compels a charge particle moving along a straight path into a circular path.
5. When a vehicle takes turn on a road, it also needs centripetal force which is provided by the friction between the tyres and the road.
6. In case of riders on a roller coaster looping the loop, centripetal force is provided by the normal force from the seat and gravity (depending on position in loop).

Tidbits

Hence, vehicle will not be able to take turn and may skid or may even be toppled. To overcome this difficulty, the highway road is banked on turns. That is, the outer edge of the track is kept slightly higher than that of the inner edge.

Q11. What is centrifuge? State some everyday applications of centripetal force.

Ans. Centrifugal Force

Definition

The force which pulls the objects away from the centre of a circular path is known as centrifugal force.

The centrifuge; Fig. 6(a) functions on this basic principle.

Fig. 6 (a)

Centrifuge

It is one of the most useful laboratory device. It helps to separate out denser and lighter particles from a mixture. The mixture is rotated at high speed for a specific time. In a laboratory setup, sample tubes are used where the denser particles will settle at the bottom and lighter particles will rise to the top of the sample tubes; Fig. 6 (b).

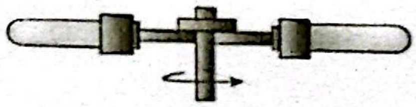


Fig. 6 (b)

Applications of Centripetal Force

We know that an object moves in a circle because of centripetal force. If the magnitude of applied force falls short of required centripetal force then the object will move away from the centre of the circle.

Washing Machine Dryer

The **dryer** of the washing machine also functions on the principle of centrifuge. The dryer consists of a long cylinder with hundreds of small holes on its wall. Wet clothes are piled up in this cylinder, which is then rotated rapidly about its axis. Water moves outward to the walls of the cylinder and thus, drained out through the holes. In this way, clothes become dry quickly.

Fig. 7

Cream separator

It is another practical device which is used to separate cream form the milk. In this machine, milk is whirled rapidly. Since milk is a mixture of light and heavy particles, when it is rotated, the light particles gather near the axis of rotation whereas the heavy particles will go outwards and hence, cream can easily be separated from milk.

Centripetal force has several practical application in everyday life, technology and engineering, such as; Vehicles turning on curves, satellites in orbit, amusement park riders, planetary motion, athletics and supports, rotating space stations, etc.

Fig. 8

For your information

Satellites are objects that orbit in nearly circular path around the Earth. They are put into orbit by rockets and are held in orbits by the gravitational pull of the Earth. The low flying Earth satellites have acceleration 9.8 m s^{-2} towards the centre of the Earth. If there is no gravitational pull, they would fly off in a straight line along tangent to the orbit. When the satellite is moving in a circle, it has an acceleration $\frac{v^2}{r}$

MCQ's ✓

1. **A body of mass 8 *kg* moves along a circle of radius 4*m* with a constant speed of 8 ms^{-1}, the centripetal force on the body is:**
(A) 48 *N* (B) 8 *N*
(C) 18 *N* (D) 128 *N* ✓

2. **Centripetal force performs:**
(A) Maximum work (B) Minimum work
(C) Negative work (D) No work ✓

3. **Centripetal acceleration is also called:**
(A) Tangential acceleration (B) Radial acceleration ✓
(C) Angular acceleration (D) Rotational acceleration

4. **In angular motion, the centripetal force is:**
(A) mr^2 (B) $mr^2\omega$
(C) $mr\omega^2$ ✓ (D) $mr^2\omega^2$

5. **The centripetal forces is always directed:**
(A) away from the centre along the radius
(B) along the direction of motion
(C) opposite to the motion of the body
(D) towards the centre along the radius ✓

6. **If linear velocity and radius are both made to half of a body moving around a circle, the centripetal force becomes:**
(A) F_c (B) $\frac{F_c}{2}$ ✓
(C) $\frac{F_c}{4}$ (D) $2F_c$

7. **A body is rotated in a vertical circle by a string. The tension in the string is minimum at the:**
(A) Top ✓ (B) Bottom
(C) Mid-position between top and bottom
(D) Remains same

8. **The expression for centripetal force is given by**
(A) $\frac{mv^2}{r^2}$ (B) $\frac{m^2v^2}{r}$
(C) $\frac{m^2v^2}{r^2}$ (D) $mr\omega^2$ ✓

9. **A 20 *N* centripetal force revolving a body along a circular path of radius 1*m*, the work done by the centripetal force is:**
(A) 20 J (B) 40 J
(C) 10 J (D) Zero J ✓

10. **Centripetal force is directed along:**
(A) tangent to circle (B) radius ✓
(C) Axis of rotation (D) x-axis

11. **Magnitude of centripetal force on a mass 'm' moving with angular speed ω in a circle of radius 'r' is:**
(A) mr^2 (B) $\frac{m\omega^2}{r}$
(C) $mr\omega^2$ ✓ (D) $mr^2\omega^2$

12. **If the velocity is doubled, then the force required to move it in a circle becomes:**
(A) One half (B) Four times ✓
(C) Two times (D) Remains same

13. **Magnitude of centripetal acceleration is:**
(A) $r\omega^2$ (B) $r^2\omega$
(C) $\frac{\omega^2}{r}$ ✓ (D) $r^2\omega^2$

14. **For uniform circular motion, tangential acceleration equals:**
(A) Centripetal acceleration (B) Centrifugal acceleration
(C) Angular acceleration (D) Zero ✓

3.3 ARTIFICIAL SATELLITES

Q 12. What are artificial satellites? Calculate the minimum time period necessary to put a satellite into the orbit near the surface of the Earth.

Ans. Artificial Satellites

Artificial satellites are the objects that orbit around the Earth due to gravity.

Explanation

In a circular orbit around the Earth, the centripetal acceleration is supplied by gravity, thus

$$g = \frac{v^2}{R}$$

where *v* is the orbital velocity and *R* is the radius of the Earth (6400 km). Putting the values in the equation;

$$v = \sqrt{gR}$$

$$= \sqrt{9.8\ \text{m s}^{-2} \times 6.4 \times 10^6\ \text{m}}$$

$$v = 7.9 \times 10^3\ \text{m s}^{-1} = 7.9\ \text{km s}^{-1}$$

This is the minimum velocity necessary to put a satellite into the orbit, called the **critical velocity**. The period T is given by

$$T = \frac{2\pi R}{v} = 2 \times 3.14 \times \frac{6400 \text{ km}}{7.9 \text{ km s}^{-1}}$$

$$T = 5060 \text{ s} = 84 \text{ min approx.}$$

If, however, a satellite in a circular orbit is at a distance h much greater than R above the Earth's surface, then taking into consideration the experimental fact that the gravitational acceleration decreases inversely as the square of the distance from the centre of the Earth; Fig. 9.

The higher the satellite, the slower will be the required speed and longer it will take to complete one revolution around the Earth.

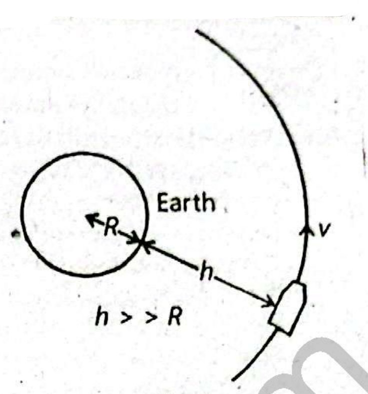


Fig. 9

13. Define orbital velocity. Derive an expression for it.

Ans. Orbital Velocity

Definition

It is the tangential velocity required to put a satellite into the orbit around the Earth.

Expression for Orbital Velocity

A satellite going round the Earth in a circular path; Fig. 10. Let the mass of the satellite be m_s and v is its orbital speed. The mass of the Earth is M and r represents the radius of the orbit. A centripetal force $m_s v^2/r$ is required to hold the satellite in the orbit. This force is provided by the gravitational force of attraction between the Earth and the satellite. Equating the gravitational force to the required centripetal force, we have

$$\frac{Gm_sM}{r^2} = \frac{m_sv^2}{r}$$

or

$$v = \sqrt{\frac{GM}{r}}$$

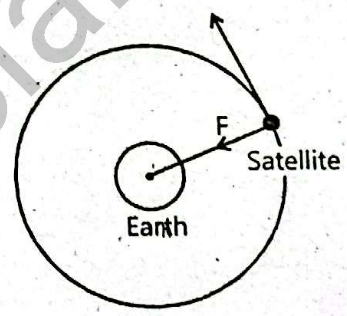


Fig. 10

This shows that the mass of the satellite is unimportant in describing the satellite's orbit. Thus, any satellite orbiting at distance r from the Earth's centre must have the orbital speed given by the above equation. Any speed less than this will bring the satellite tumbling back to the Earth.

Example 3.4: An Earth satellite is in circular orbit at distance of 384,000 km from the Earth's surface. What is its period of one revolution in days? Take mass of the Earth $M = 6.0 \times 10^{24}$ kg and its radius $R = 6400$ km.

Solution:

Given that;

$h = 384{,}000$ km , $R = 6400$ km , $M = 6.0 \times 10^{24}$ kg

To Find:

$T = ?$

Calculations:

As $r = R + h = (6400 + 384000)$ km $= 390400$ km

Using $v = \sqrt{\frac{GM}{r}}$

Putting the values

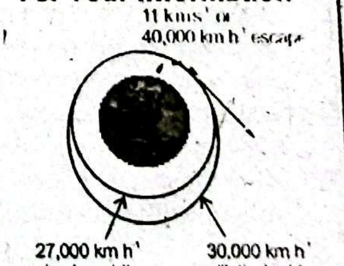
$$v = \sqrt{\frac{6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2} \times 6 \times 10^{24} \text{ kg}}{390400 \text{ km}}}$$

$$= 1.025 \text{ km s}^{-1}$$

Also $T = \frac{2\pi R}{v} = 2 \times 3.14 \times 390400 \text{ km} \times \frac{1}{1.025 \text{ km s}^{-1}} \times \frac{1 \text{ day}}{60 \times 60 \times 24 \text{ s}}$

$T = 27.7$ days **Ans.**

For Your Information



Satellites Orbits

14. What is meant by weightlessness in satellites? Show that the supporting force acting on a body inside the satellite is zero.

Ans. Weightlessness in Satellites

When a satellite is launched by a rocket in its desired orbit around the Earth, then it has been observed practically that everything inside the satellite experiences weightlessness because the satellite is accelerating towards the centre of the Earth as a freely falling body.

Consider a satellite of mass M revolving in its orbit of radius r around the Earth. A body of mass m inside the satellite suspended by a spring balance from the ceiling of the satellite is under the action of two forces. That is, its weight mg acting downward, while the supporting force, called normal force F_N or tension in the spring acting upward; Fig. 11. Their resultant force is equal to the centripetal force required by the mass m which is acting towards the centre of the Earth, and is expressed as:

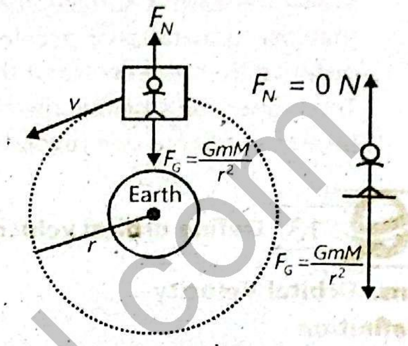


Fig. 11

$$F_c = mg - F_N$$

where $$F_c = \frac{mv^2}{r}$$

Hence $$\frac{mv^2}{r} = mg - F_N \qquad \text{......... (i)}$$

It may be noted that the centripetal force responsible for the revolution of the satellite of mass M around the Earth is provided by the gravitational force of attraction between the Earth and the satellite. Therefore,

$$F_g = F_c$$

or $$Mg = \frac{Mv^2}{r} \qquad \text{or} \qquad g = \frac{v^2}{r}$$

Hence, Eq. (i) becomes:

$$mg = mg - F_N \qquad \text{or} \qquad F_N = 0$$

This shows that the supporting force acting on a body inside the satellite is zero. Therefore, the bodies as well as the astronauts in a satellite find themselves in a state of apparent weightlessness.

> **Can you Tell?**
>
> When a bucket full of water is rapidly whirled in a vertical circular path, water does not fall out even if the bucket is inverted at the maximum height. Why is it so?
>
> **Ans.** When a bucket full of water is rapidly whirled in a vertical circular path, the water does not fall out even when the bucket is inverted at the top of a circle. This happens due to centripetal force and inertia.

15. Why an artificial gravity is created in a spacecraft? Derive an expression for frequency with which the spaceship rotates to provide artificial gravity.

Ans. Artificial Gravity

Definition

It refers to the simulation of the force of gravity in a space environment where it does not naturally exists, such as in orbit or deep space. It is typically created to allow humans to live and work more comfortably and safely in space, mimicking the effects of Earth's gravity.

In a gravity free space, there will be no force that will push anybody to any side of the spacecraft. If this spacecraft is to stay in the orbit over an extended period of time, the weightlessness may affect the performance of the astronauts present in that spacecraft. To overcome this difficulty, an artificial gravity can be created in the spacecraft. This could enable the crew of the space ships to function in an almost normal manner. For this situation to prevail, the spaceship is set into rotation around its own axis. The astronaut then is pressed towards the outer rim and exerts a force on the 'floor' of the spaceship in much the same way as on the Earth.

Expression for Frequency

Consider a spacecraft of the shape; Fig. 12. The outer radius of the spaceship is R and it rotates around its own central axis with angular speed w, then its angular acceleration a_c is

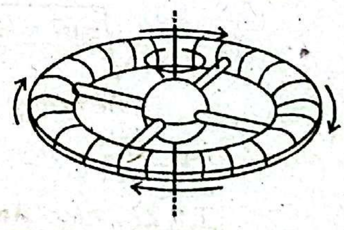


Fig. 12

$$a_c = R\,\omega^2$$

But $\omega = \frac{2\pi}{t}$ where t is the period of revolution of spaceship.

Hence $$a_c = R\frac{(2\,\pi)^2}{t^2} = R\frac{4\pi^2}{t^2}$$

As frequency f = 1/t, therefore,

$$a_c = R\,4\pi^2 f^2$$

or $$f^2 = \frac{a_c}{4\pi^2 R} \quad \text{or} \quad f = \frac{1}{2\pi}\sqrt{\frac{a_c}{R}}$$

As described above, the force of gravity provides the required centripetal acceleration, therefore,

$$a_c = g$$

So $$f = \frac{1}{2\pi}\sqrt{\frac{g}{R}}$$

When the spaceship rotates with this frequency, the artificial gravity like the Earth is provided to the inhabitants of the spaceship.

Tidbits

The moment you switch on your mobile phone, your location can be tracked immediately by global positioning system.

Do you know?

Your weight slightly changes when the velocity of the elevator changes at the start and end of a ride, not during the rest of the ride when that velocity is constant.

3.4 MOMENT OF INERTIA

16. Define moment of inertia. Prove that torque acting on a rotating rigid body is equal to the product of its moment of inertia and angular acceleration.

Ans. Moment of Inertia

Definition

Moment of inertia is the rotational analogue of mass in linear motion. It is defined as:

The quantity expressed by the body resisting angular acceleration which is the sum of product of the mass of every particle with its square of a distance from the axis of rotation.

Explanation

Consider a mass m attached to the end of a massless rod. Assume that the bearing at the pivot point O is frictionless. Let the system be in a horizontal plane. A force F is acting on the mass perpendicular to the rod and hence, this will accelerate the mass according to:

$$F = ma$$

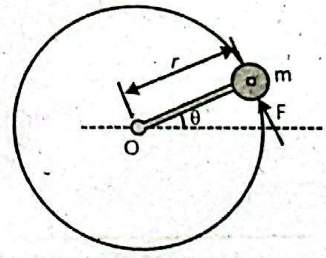


Fig. 13

In doing so, the force will cause the mass to rotate about O. Since tangential acceleration a_T is related to angular acceleration α by the equation,

$$a_T = r\alpha$$

So $$F = mr\alpha$$

As turning effect is produced by torque τ, it would, therefore, be better to write the equation for rotation in terms of torque. This can be done by multiplying both sides of the above equation by r. Thus,

$$rF = \tau = \text{torque} = mr^2\alpha$$

which is rotational analogue of the Newton's second law of motion, $F = ma$.

Here F is replaced by τ, a by α and m by mr^2. The quantity mr^2 is known as the moment of inertia and is represented by I. The moment of inertia plays the same role in angular motion as the mass in linear motion. It may be noted that moment of inertia depends not only on mass m but also on r^2.

17. Find an expression of moment of inertia for a rigid body of 'n' small pieces.

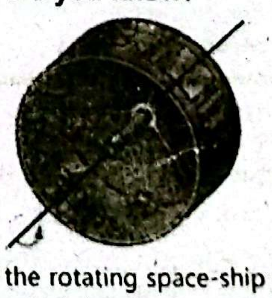
Ans. Most rigid bodies have different mass concentration at different distances from the axis of rotation, which means the mass distribution is not uniform. As shown in Fig. 14, the rigid body is made up of n small pieces of masses m_1, m_1, ... at distances r_1, r_2, ... from the axis of rotation O. Let the body be rotating with the angular acceleration a, so the magnitude of the torque acting on m_1 is

$$\tau_1 = m_1 r_1^2 \alpha_1$$

Similarly, the torque on m_2 is

$$\tau_2 = m_2 r_2^2 \alpha_2 \text{ and so on.}$$

Do you know?



The surface of the rotating space-ship pushes on an object with which it is in contact and thereby provides the centripetal force needed to keep the object moving on a circular path.

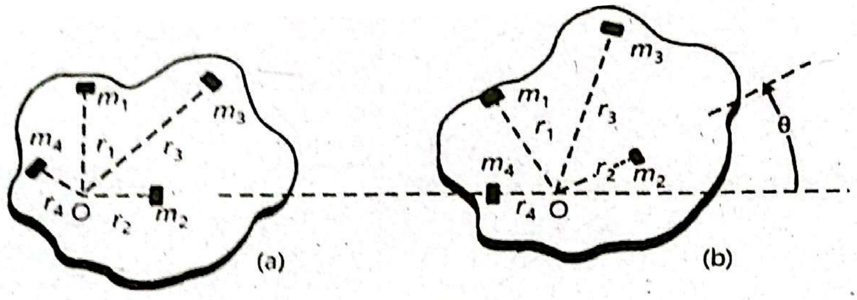


Fig. 14

Each small piece of mass within a large, rigid body undergoes the same angular acceleration about the pivot point.

Since the body is rigid, so all the masses are rotating with the same angular acceleration α.

Total torque τ_{total} is then given by

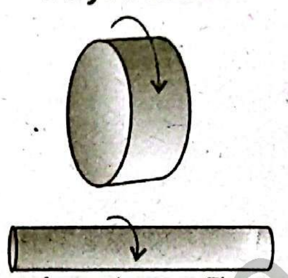
$$\tau_{total} = (m_1r_1^2 + m_2r_2^2 + \ldots + m_n r_n^2)\,\alpha$$

$$= (\sum_{i=1}^{n} m_i r_i^2)\,\alpha$$

Where I is the moment of inertia of the body and is expressed as:

$$I = \sum_{i=1}^{n} m_i r_i^2$$

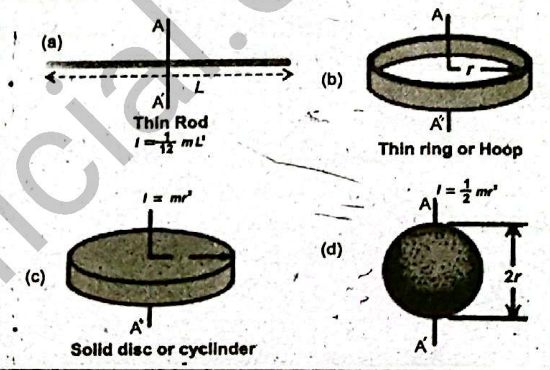
Do you know?



Two cylinders of equal mass. The one with the larger diameter has the greater rotational inertia.

For Your Information

Moments of inertia of various bodies about axis AA′.



mCQs

1. **Moment of inertia of solid sphere is:**
   (A) mr^2 (B) $\frac{1}{2}mr^2$
   (C) $\frac{2}{5}mr^2$ ✓ (D) $\frac{1}{12}mr^2$
2. **The force and torque are analogous to:**
   (A) Velocity (B) Mass and weight
   (C) Moment of inertia (D) Each other ✓
3. **Moment of inertia of thin rod is given by**
   (A) $\left(\frac{1}{12}\right)mL^2$ ✓ (B) $\left(\frac{2}{5}\right)mR^2$
   (C) $12\,mL^2$ (D) $\left(\frac{5}{7}\right)2mR^2$
4. **If the angular velocity of the particle rotating in a circle is doubled, then its moment of inertia:**
   (A) Remains same (B) Becomes half ✓
   (C) Becomes doubled (D) Becomes 4 times
5. **Moment of inertia is measured in:**
   (A) $kg\,m^{-2}$ (B) kgm^2 ✓
   (C) $N\,s$ (D) $rad\,s^{-1}$
6. **Torque acting on a body is given by**
   (A) $\tau = I\alpha$ ✓ (B) $\tau = I\alpha^2$
   (C) $\tau = I^2\alpha^2$ (D) $\tau = I^2\alpha$
7. **Moment of inertia of a particle is equal to:**
   (A) m^2r (B) mr
   (C) m^2r^2 (D) mr^2 ✓
8. **In angular motion, the centripetal force is:**
   (A) m^2r (B) $mr^2\omega$
   (C) $mr\omega^2$ ✓ (D) $mr^2\omega^2$
9. **The angular acceleration is produced by**
   (A) Momentum (B) Torque ✓
   (C) Pressure (D) Power
10. **Choose the quantity which plays the same role in angular motion as that of mass in linear motion.**
    (A) Angular acceleration (B) Torque
    (C) Moment of inertia ✓ (D) Angular momentum
11. **The ratio of moment of inertia of disc and hoop is:**
    (A) $\frac{1}{2}$ ✓ (B) $\frac{1}{4}$
    (C) $\frac{3}{4}$ (D) $\frac{1}{3}$
12. **The ratio of moment of inertia of a disc and sphere of the same radius is:**
    (A) $\frac{2}{5}$ (B) $\frac{5}{4}$ ✓
    (C) $\frac{1}{2}$ (D) $\frac{5}{2}$
13. **Rotational inertia of two equal masses of cylinders, but one has larger diameter will be:**
    (A) Lesser (B) Larger ✓
    (C) Same (D) None of these

14. **A body rotates with a constant angular velocity of 100 rad s^{-1} about a vertical axis, the required torque to sustain this motion will be:**
(A) Zero *Nm* ✓ (B) 100 *Nm*
(C) 200 *Nm* (D) 300 *Nm*

15. **Moment of inertia of 100 kg sphere having radius 50 cm will be:**
(A) 10 $kg\ m^2$ ✓ (B) 5 $kg\ m^2$
(C) 500 $kg\ m^2$ (D) 2.5 $kg\ m^2$

16. **In rotational motion analogous of force is:**
(A) Torque ✓ (B) Rotational inertia
(C) Mass (D) Momentum

17. **The moment of inertia of solid disc or cylinder is:**
(A) mr^2 (B) $\frac{1}{2}mr^2$ ✓
(C) $\frac{1}{4}mr^2$ (D) $\frac{1}{2}m^2r$

18. **If moment of inertia of a body becomes double, then angular momentum becomes:**
(A) One half ✓ (B) Double
(C) Three times (D) Four times

3.5 ANGULAR MOMENTUM

Q 18. What is meant by angular momentum? Show that orbital angular momentum; L = Iω.

Ans. Angular Momentum

Definition

It is defined as the property of any rotating object given by moment of inertia times angular velocity.

Angular momentum is a measure of the quantity of rotation of an object around a particle axis.

Angular momentum has important role in the study of rotational motion. It depends on the object's moment of inertia and its angular velocity.

A particle is said to possess an angular momentum about a reference axis if it so moves that its angular position changes relative to that reference axis.

The angular momentum **L** of a particle of mass *m* moving with velocity **v** and momentum **p**; Fig. 15 relative to the origin O is defined as:

$$\mathbf{L} = \mathbf{r} \times \mathbf{p}$$

where **r** is the position vector of the particle at that instant relative to the origin O. Angular momentum is a vector quantity. Its magnitude is:

$$L = rp \sin\theta = m\,rv \sin\theta$$

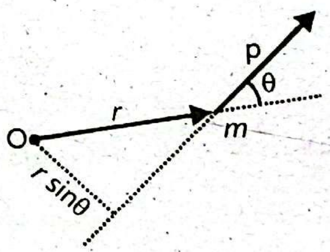


Fig. 15

where θ is the angle between **r** and **p**. The direction of **L** is perpendicular to the plane formed by **r** and **p** and its sense is given by the right hand rule of vector product.

If the particle is moving in a circle of radius *r* with uniform angular velocity ω, then angle between *r* and tangential velocity is 90°. Hence,

$$L = mrv \sin 90° = mrv$$

But $v = r\omega$

Hence $L = mr^2\omega$ or $L = I\omega$

Now consider a symmetric rigid body rotating about a fixed axis through the centre of mass, Fig 16. Each particle of the rigid body rotates about the same axis in a circle with an angular velocity m. The magnitude of the angular momentum of the particle of mass m_i is $m_i\, v_i\, r_i$ about the origin O. The direction of $\mathbf{L_i}$ is the same as that of m. Since $v_i = r_i\omega$, the angular momentum of the ith particle is $m_i\, r_i^2\omega$. Summing this over all particles gives the total angular momentum of the rigid body.

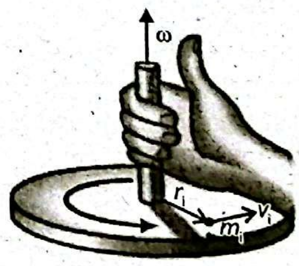


Fig. 16

$$L = \left(\sum_{i=1}^{n} m_i r_i^2\right)\omega = I\,\omega$$

where *I* is the moment of inertia of the rigid body about the axis of rotation.

Unit: The SI unit of angular momentum is kg m^2s^{-1} or J s.

Example 3.5: The mass of Earth is 6.00×10^{24} kg. The distance r from Earth to the Sun is 1.50×10^{11} m. As seen As seen from the direction of the North Star, the Earth revolves counter-clockwise around the Sun. Determine the orbital angular momentum of the Earth about the Sun, assuming that it traverses a circular orbit about the Sun once a year (3.16×10^7 s).

Solution:

Given that;

$m = 6.0 \times 10^{24}$ kg , $r = 1.50 \times 10^{11}$ m ,
$t = 3.16 \times 10^7$ s

To Find:

$L = ?$

Calculations:

To find the Earth's orbital angular momentum, we must first know its orbital speed from the given data. When the Earth moves around a circle of radius r, it travels a distance of $2\pi r$ in one year. Its orbital speed v_o is thus, $v_o = \frac{2\pi r}{t}$

Orbital angular momentum of the Earth = $L_o = mv_o r$

$$L = \frac{2\pi r^2 m}{t}$$

$$= \frac{2\pi (1.50 \times 10^{11}\ \text{m})^2 \times (6.00 \times 10^{24}\ \text{kg})}{3.16 \times 10^7\ \text{s}}$$

$L = 2.67 \times 10^{40}$ kg m^2 s^{-1} **Ans.**

The sign is positive because the revolution is counter clockwise.

For Your Information

It has been noticed that when ice on the polar caps of Earth melts and water flows away in the form of river; the moment of inertia of water and hence that of Earth about its axis of rotation increases due to conservation of angular momentum. Hence, the angular velocity of Earth will decrease, therefore, the duration of day increases slightly.

Point to ponder!

Why does the coasting rotating system slow down as water drips into the beaker?

Ans. The coasting rotating system slows down when water drips into the beaker due to conservation of angular momentum and the increase the moment of inertia.

3.6 LAW OF CONSERVATION OF ANGULAR MOMENTUM

19. State and explain the law of conservation of angular momentum.

Ans. Law of Conservation of Angular Momentum

Statement

It states that if no external torque acts on a system, the total angular momentum of the system remains constant.

Mathematically;

$$L_{total} = L_1 + L_2 + ... = \text{constant}$$

If a body of moment of inertia I_1spinning with angular speedω_1 alters its moment of inertia to I_2, then its angular speed ω_2 also changes so that its angular momentum remains constant.

Hence $I_1\omega_1 = I_2\omega_2$

The angular momentum is a vector quantity with direction along the axis of rotation. Hence, the direction of angular momentum along the axis of rotation also remains fixed. This is illustrated by the fact given below:

The axis of rotation of an object will not change its orientation unless an external torque causes it to do so.

20. State some examples of law of conservation of angular momentum.

Ans. Examples of conservation of angular momentum

A man diving from a diving board

A diver jumping from a springboard has to take a few somersaults in air before touching the water surface. After leaving the springboard, he curls his body by rolling arms and legs in. Due to this, his moment of inertia decreases, and he spins in midair with a large angular velocity. When he is about to touch the water surface, he stretches out his arms and legs. He enters the water at a gentle speed and gets a smooth dive. This is an example of the law of conservatic of angular momentum.

Fig.17: A man diving from a

The spinning ice skater

An ice skater can increase his angular velocity by folding arms and bringing the stretched leg close to the other leg. By doing so, he decreases his moment of inertia. As a result, angular speed increases. When he stretches his hands and a leg outward, the moment of inertia increases and hence angular velocity decreases.

diving board.

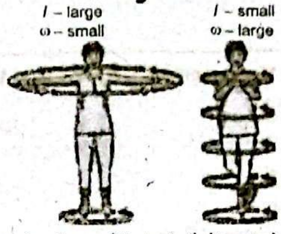


Fig. 18: An ice skater using angular momentum

A person holding some weight in his hands standing on a turntable

A person is standing on a turntable with heavy mass (dumb-bell) in his hands stretched out on both sides. As he draws his hands inward, his angular speed at once increases. This is because the moment of inertia decreases on drawing the hands inwards, which increases the angular speed.

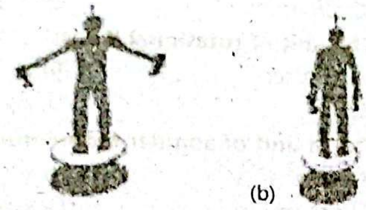


Fig. 19: Man with masses in his hands on a turntable. Conservation of angular momentum requires that as the man pulls his arms in, the angular velocity increases.

Do you know?

If you try to sit on a bike at rest, it falls. But if the bike is moving, the angular momentum of the spinning wheel resists any tendency to change and helps to keep the bike upright and stable.

Point to ponder!

Planets move around the Sun in elliptical orbits with Sun situated at one of its foci, thus, distance of a planet from the Sun is not constant when it is nearer the Sun. Its orbital velocity increases automatically. Why?

Ans. The reason a planet's orbital velocity increases when it is nearer the Sun is due to the law of conservation of angular momentum and the gravitational force acting between the planet and the Sun.

Flywheel

Flywheel is a mechanical device which consists of a heavy wheel with an axle. It is used to store rotational energy, smooth out output fluctuations and provides stability in a wide range of applications such as bicycles and other vehicles, industrial machinery, gyroscopes, ships and spacecrafts.

When a fly wheel spins, its angular momentum resists changes to its orientations, maintaining stability. This is useful in systems that need precise control over their orientation without external interference.

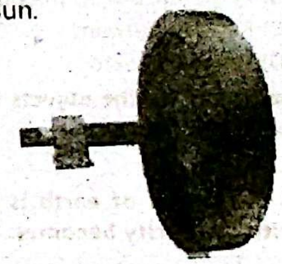


Fig. 20: A fly wheel

The Gyroscope

A gyroscope is a device which is used to maintain its orientation relative to the Earth's axis or resists changes in its orientation. It consists of a mounted flywheel pivoted in supporting rings. It works on the basis of law of conservation of angular momentum due to its large moment of inertia. When the gyroscope spins at a large angular speed, it gains large angular momentum. It is then difficult to change the orientation of the gyroscope's rotational axis due to its large moment of inertia. A change in orientation requires a change in its angular momentum. To change the direction of a large angular momentum, a corresponding large torque is required. Even if gyroscope is tilted, it still keeps levitated without falling. Hence, it is a reason why a gyroscope can be used to maintain orientation. The main applications of gyroscope are in the guiding system of aeroplanes, submarines and space vehicles in order to maintain a specific direction in space to keep steady course.

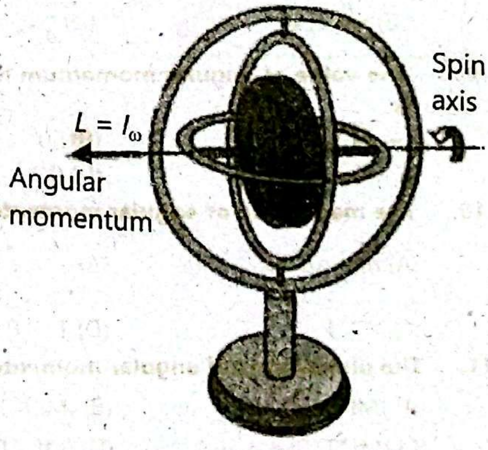


Fig. 21: The gryscope

1. **The time period of artificial satellite is given by**
(A) $\frac{2\pi r}{v}$ ✓ (B) $2\pi r v$
(C) $\frac{2\pi v}{r}$ (D) $\frac{\pi r}{v}$

2. **The relation of angular momentum are given by**
(A) $v_{esc} = \frac{1}{2v_o}$ (B) $v_{esc} = \sqrt{2}v_o$ ✓
(C) $v_{esc} = v_o$ (D) $v_{esc} = 2v_o$

3. **The SI units of angular momentum are given by**
(A) $J\,s^{-2}$ (B) $J\,s^{-1}$
(C) $J\,s$ ✓ (D) None of these

4. **The angular momentum L is defined by the equation:**
(A) $L = mv$ (B) $\vec{L} = \vec{P} \times \vec{r}$
(C) $\vec{L} = \vec{r} \times \vec{F}$ (D) $\vec{L} = \vec{r} \times \vec{P}$ ✓

5. **If a gymnast sitting on a stool with his arms stretched lowers his arms:**
(A) ω decreases (B) ω increases
(C) I decreases (D) Both (B) and (C) ✓

6. **The direction of angular momentum of body moving in a circle is:**
(A) Along the tangent
(B) Perpendicular to the plane of circle ✓
(C) Radially outward
(D) Radially inward

7. **Satellites are the objects that orbit around the:**
(A) Moon (B) Sun
(C) Earth ✓ (D) Star

8. **If the radius of earth is doubled, then the value of critical velocity becomes:**
(A) $\frac{1}{\sqrt{2}}v_o$ (B) $\frac{1}{2}v_o$
(C) $\sqrt{2}\,v_o$ ✓ (D) $\frac{1}{4}v_o$

9. **The value of angular momentum is maximum when θ is:**
(A) 90° ✓ (B) 60°
(C) 45° (D) 0°

10. **The magnitude of angular momentum is:**
(A) $mrv\sin\theta$ ✓ (B) $\vec{L} \times \vec{r}$
(C) $\vec{r} \times \vec{F}$ (D) $\vec{F} \times \vec{P}$

11. **The dimensions of angular momentum are:**
(A) $[MLT^{-2}]$ (B) $[MLT^{-1}]$
(C) $[ML^2T^{-1}]$ ✓ (D) $[ML^{-2}T^{-2}]$

12. **Orbital velocity near surface of earth is given by**
(A) $\sqrt{2gR}$ (B) $\sqrt{gR}$ ✓
(C) $\sqrt{\frac{2g}{R}}$ (D) $\sqrt{\frac{g}{R}}$

13. **Close orbiting satellites orbit the Earth at a height of about:**
(A) 400 km ✓ (B) 4000 km
(C) 400 m (D) 400 cm

14. **The diver spins faster when moment of inertia becomes:**
(A) Smaller ✓ (B) Greater
(C) Constant (D) Equal

15. **The time period of a low flying satellite is:**
(A) 1 year (B) 84 minutes ✓
(C) 28 hours (D) 1 day

16. **The unit of rotational K.E is:**
(A) *rad/sec* (B) *Js*
(C) *J* ✓ (D) $kg\,m^2$

17. **The SI unit of angular momentum is:**
(A) $kgm^2\,s^{-1}$ ✓ (B) $kgm^2\,s^{-2}$
(C) $kg^2\,ms^{-1}$ (D) $kgms^{-1}$

18. **A stone of mass 16 *kg* is attached to a string 144 *m* long and is whirled in a horizontal circle. The maximum tension the string can withstand is 16*N*, the maximum velocity of revolution that can be given to the stone without breaking it, will be:**
(A) 12 ms^{-1} ✓ (B) 20 ms^{-1}
(C) 16 ms^{-1} (D) 14 ms^{-1}

19. **Angular momentum of a rigid body is given by**
(A) $I\omega$ ✓ (B) $I^2\omega$
(C) $I\omega^2$ (D) $I^2\omega^2$

20. **In rotational motion, the torque is equal to rate of change of:**
(A) Angular velocity (B) Linear momentum
(C) Angular Momentum ✓ (D) Angular acceleration

21. **Angular momentum has the same units as:**
(A) Impulse × distance ✓ (B) Power × time
(C) Linear Momentum × time
(D) Work × frequency

22. **If the radius of earth is increased to four times of the present, critical velocity v_o becomes:**
(A) $\frac{v_o}{\sqrt{2}}$ (B) $\sqrt{2}\,v_o$
(C) $2\,v_o$ ✓ (D) $\frac{1}{2}v_o$

23. **The linear velocity of a disc when it reaches the bottom of an inclined plane of height '*h*' is:**
(A) $\sqrt{gh}$ (B) $\sqrt{\frac{4}{3}gh}$ ✓
(C) $\sqrt{\frac{2}{3}gh}$ (D) $\sqrt{\frac{1}{3}gh}$

24. **The speed of hoop at the bottom of an inclined plane is:**
(A) $\sqrt{gh}$ ✓ (B) $\sqrt{2gh}$
(C) $\sqrt{\frac{4}{3}gh}$ (D) $\sqrt{4gh}$

25. **Rotational *K.E* of a disc is given by**

(A) $\frac{1}{2}mv^2$ (B) $\frac{1}{4}mv^2$ ✓

(C) $\sqrt{gh}$ (D) $\sqrt{\frac{4}{3}gh}$

26. **The *K.E* of any hoop of radius '*r*' is given by**

(A) $\frac{1}{2}mr^2\omega^2$ ✓ (B) $\frac{1}{2}r^2\omega^2$

(C) $\frac{1}{2}r^2\omega^3$ (D) None

27. **The rotational K.E of solid sphere is:**

(A) $\frac{2}{5}mr^2\omega^2$ ✓ (B) $\frac{1}{2}mr^2\omega^2$

(C) $\frac{2}{3}mr^2\omega^2$ (D) $\frac{1}{5}I\omega^2$

28. **The ratio of rotational and translational K.E. of a hoop is:**

(A) 1 : 2 (B) $1 : \sqrt{2}$

(C) 1 : 1 ✓ (D) $\sqrt{2} : 1$

29. **If orbital velocity of a satellite is 7.9 $km\ s^{-1}$ which is 700 km above the Earth and *R* is the radius of Earth, then time required to complete one rotation will be:**

(A) 84 min ✓ (B) 84 s

(C) 6050 s (D) 24 hours

30. **Artificial gravity can be created in the spaceship by**

(A) Revolving around the earth

(B) Spinning around its own axis ✓

(C) Increased its velocity

(D) Decreasing its velocity

31. **Orbital speed of a satellite can be determined by the equation:**

(A) $\sqrt{2gR}$ (B) $\sqrt{\frac{2GM}{R}}$

(C) $\sqrt{gR}$ (D) $\sqrt{\frac{GM}{r}}$ ✓

32. **The ratio of escape velocity to the critical orbital velocity is:**

(A) 1 (B) $\frac{1}{2}$

(C) $\sqrt{\frac{1}{2}}$ (D) $\sqrt{2}$ ✓

33. **Orbital velocity of a satellite of mass "m_s" orbiting around earth of mass "*M*" is:**

(A) $\sqrt{\frac{GM}{r}}$ ✓ (B) $\sqrt{\frac{GM_s}{r}}$

(C) $\sqrt{\frac{GM}{r}}$ (D) $\sqrt{gR}$

34. **Artificial gravity like earth is obtained, if space ship rotate with frequency.**

(A) $\frac{1}{2\pi}\sqrt{gR}$ (B) $\frac{1}{2\pi}\sqrt{2gR}$

(C) $\frac{1}{4\pi}\sqrt{gR}$ (D) $\frac{1}{2\pi}\sqrt{\frac{g}{R}}$ ✓

35. **Dimensions of $\sqrt{\frac{R}{g}}$ are same as:**

(A) angular frequency (B) Force

(C) Torque (D) time period ✓

36. **The number of satellites in global positioning system is:**

(A) 3 (B) 12

(C) 24 ✓ (D) 36

37. **A body of 1Kg moving up with a = g then its apparent weight is:**

(A) 19.6 N (B) 4.9 N

(C) 9.8 N ✓ (D) 0 N

38. **The low flying earth satellites have acceleration:**

(A) 9.8 m/s^2 ✓ (B) 4.9 m/s^2

(C) 10 m/s^2 (D) 7.9 m/s^2

39. **The time period of artificial satellite close to the Earth is given by**

(A) $\frac{2\pi R}{v}$ ✓ (B) $2\pi Rv$

(C) $\frac{2\pi v}{R}$ (D) $\frac{\pi R}{v}$

ADDITIONAL SHORT ANSWER QUESTIONS

SQ

1. **What is the direction of angular displacement?**

Ans. It is perpendicular to the plane of rotation, following the right hand rule.

2. **Can angular displacement be negative?**

Ans. Yes, if the rotation is in the clockwise direction.

3. **What causes angular acceleration in a rotating object?**

Ans. A net torque acting on the object.

4. **How many radians are there in 2 degree?**

Ans. In one complete revolution

$$360° = 2\pi \text{ rad}$$

or $$1° = \frac{2\pi}{360} \text{ rad}$$

or $1^\circ = \frac{\pi}{180}$ rad

or $2 \times 1^\circ = 2 \times \frac{\pi}{180}$ rad

or $2^\circ = \frac{\pi}{90}$ rad

$= \frac{3.14}{90}$ rad

$\boxed{2^\circ = 0.035 \text{ rad}}$

5. Banked tracks are needed for turns on highway. Why?

Ans. Banked tracks are needed for safe turns because friction alone cannot provide enough centripetal force to keep the car moving in a circle.

6. Calculate moment of inertia of sphere of radius 0.5m and mass 10 kg.

Ans. r = 0.5m, m = 10kg

Moment of inertia of sphere = I = ?

$I = \frac{2}{5} mr^2$

$I = \frac{2}{5} (10)(0.5)^2$

$I = 4 (0.25)$

$I = 1 \text{ kgm}^2$

7. Describe what should be maximum velocity for a satellite to orbit close to the earth around it?

Ans. Orbital velocity = $v = \sqrt{\frac{Gm}{r}}$

GM are constant,

Hence $v \propto \frac{1}{r}$

Radius can be reduced to Re

So; $v = \sqrt{\frac{Gm}{R_e}}$ .... (i)

$g = \frac{Gm}{R_e^2}$

$Gm = g R_e^2$

$\sqrt{Gm} = \sqrt{gR_e^2}$

Put in (i)

$\sqrt{\frac{gR_e^2}{R_e}} = \sqrt{gR_e}$

$v = \sqrt{gR_e}$

Is the maximum velocity for a satellite to orbit close to earth around it.

8. Explain the difference between tangential velocity and the angular velocity. If one of these is given for a wheel of known radius, how will you find the other?

Ans. Difference between tangential and angular velocities is given below:

Tangential velocity	Angular velocity
The is the linear velocity of a particle moving along a circle or curve. **Formula:** $v = r\omega$	The rate of change of angular displacement is called angular velocity.

Symbol: It is denoted by v. **Direction:** The direction of linear velocity is always along the tangent on any point of the circle. **Unit:** Its unit is m/s.	**Formula:** $\omega_{ave} = \frac{\Delta\theta}{\Delta t}$ **Symbol:** It is denoted by ω. **Direction:** The direction of angular velocity $\vec{\omega}$ is represented by a line drawn parallel to the axis of rotation. **Unit:** Its unit is rad s^{-1}.

Relation between tangential velocity and angular velocity

If v = magnitude of tangential velocity

ω = magnitude of angular velocity

r = radius of the circle

Then, $v = r\omega$

In the vector form, we can write $\vec{v} = \vec{\omega} \times \vec{r}$

Knowing the radius of the wheel r and angular velocity w, we can find tangential velocity v.

9. Is centripetal force a real force?

Ans. No, it is not a separate force; it is the name given to the net force causing circular motion (e.g., tension, gravity, friction).

10. What happens if centripetal force is removed?

Ans. The object will move in a straight line tangential to the circle due to inertia.

11. What provides centripetal force in case of a car turning in a circle?

Ans. Friction between the tyres and the road.

12. Can gravity acts as a centripetal force?

Ans. Yes, for example, in planetary orbits, gravity acts as the centripetal force.

13. Explain what is meant by centripetal force and why it must be furnished to an object if the object is to follow a circular path?

Ans. Centripetal Force

The force required to bend a straight path into circular path is called centripetal force.

This force always directed towards the centre of circular path.

$$F_c = ma_c = \frac{mv^2}{r}$$

The direction of a body moving in a circular path is always changing.

Without centripetal force, body will move along the tangent to circle.

To bend the normally straight path into circular path, a perpendicular force is needed. This force is called the centripetal force.

14. What is moment of inertia?

Ans. It is the measure of an object's resistance to changes in its rotational motion.

15. On what factors does the moment of inertia depend?

Ans. It depends on the mass of the object and the distribution of that mass relative to the axis of rotation.

16. Which has greater moment of inertia, a ring or a disc of same mass and radius?

Ans. The ring, because all its mass is farther from the axis.

17. What is the physical significance of moment of inertia?

Ans. It determine how much torque is needed for a desired angular acceleration.

18. What is meant by moment of inertia? Explain its significance.

Ans. Moment of Inertia

The product of mass of the particle and square of its perpendicular distance from the axis of rotation is called moment of inertia.

Its SI unit is kg m^2.

It is denoted by "I" and is given by the relation.

$$I = mr^2$$

Physical Significance

Moment of inertia "I" plays the same role in angular motion as the mass in linear motion.

It helps to determine angular acceleration of a body.

19. What is angular momentum?

Ans. It is the rotational equivalent of linear momentum. It is the product of moment of inertia and angular velocity.

20. Give an example of conservation of angular momentum.

Ans. A spinning ice skater pulls in his / her arms and spins faster to reduce moment of inertia.

21. What causes a change in angular momentum?

Ans. An external torque.

22. Can angular momentum be conserved in collisions?

Ans. Yes, if no external torque acts on the system.

23. Why does a diver spin faster when they curl their body mid-air?

Ans. By reducing their moment of inertia, angular velocity increases to conserve angular momentum.

24. How does conservation of angular momentum apply to planetary motion?

Ans. As a planet moves closer to the sum (smaller radius), its speed increases to conserve angular momentum.

25. What is meant by angular momentum? Explain the law of conservation of angular momentum.

Ans. Angular Momentum (L)

The momentum of a rotating body is called angular momentum.

Angular momentum is given by

$$\vec{L} = \vec{r} \times \vec{p}$$

$$\vec{L} = \vec{r} \times m\vec{v}$$

Where also $L = I\omega$ = Moment of inertia × Angular velocity

r = Radius of circle

$\vec{v}$ = Linear velocity

m = mass of the body and

$\vec{p}$ = Linear momentum of the body

Law of Conservation of Angular Momentum:

According to this law, if no external torque acts on a body, then total angular momentum of a body about its axis of rotation always remains constant.

$$L_{total} = \text{constant}$$

$$L_1 + L_2 + L_2 + \ldots\ldots = \text{constant}$$

26. Show that orbital angular momentum $L_o = mvr$.

Ans. We know that when a satellite of mass m is moving along a circle of radius $\vec{r}$ with velocity $\vec{v}$, the angular momentum of the satellite orbiting around the earth is given by

$$\vec{L_o} = \vec{r} \times \vec{p}$$

$$\vec{L_o} = \vec{r} \times m\vec{v}$$

$$\vec{L_o} = m \times (\vec{r} \times \vec{v})$$

$$|\vec{L_o}| = = m\,(r\,v \sin\theta)$$

But $\vec{v}$ and $\vec{r}$ are perpendicular to each other *i.e.*, $\theta = 90°$

$\therefore$ $L_o = m\,v\,r \sin 90°$

$$L_o = m\,v\,r \times 1$$

$$\boxed{L_o = m\,v\,r}$$

27. **What is an artificial satellite?**

Ans. It is a man-made object placed in orbit around a planet or moon.

28. **What keeps an artificial satellite in orbit?**

Ans. The gravitational force of the Earth provides the centripetal force needed to keep the satellite in circular motion.

29. **What is angular velocity of a satellite?**

Ans. It is the rate at which the satellite sweeps out an angle around the Earth, measured in radians per second.

30. **What is the direction of angular velocity in satellite motion?**

Ans. It is along the axis of rotation, determined by using the right hand rule.

31. **Why do not artificial satellites fall back to the Earth?**

Ans. Because their tangential velocity creates enough centripetal force to keep them in stable orbit.

32. **What factors affect the angular velocity of a satellite?**

Ans. The radius of the orbit and gravitational force of the Earth.

33. **What is orbital velocity?**

Ans. It is the minimum velocity needed for an object to stay in a stable orbit around a planet or another celestial body.

34. **Does orbital velocity depend on the mass of the orbiting object?**

Ans. No, it only depends on the mass of the central body and the radius of the orbit.

35. **What happens if an object moves faster than orbital velocity?**

Ans. It may escape the orbit and follow a different path, possibly escaping the gravitational pull if it reaches escape velocity.

36. **How does altitude affect orbital velocity?**

Ans. As altitude increases, orbital velocity decreases because the object is father from the centre of the Earth.

37. **What is the difference between orbital velocity and escape velocity?**

Ans. Orbital velocity keeps an object in circular motion, while escape velocity allows it to become free from the gravitational field.

38. **Describe what should be the minimum velocity, for a satellite, to orbit close to the Earth around it?**

Ans. **Critical Velocity**

The minimum velocity which is required to put a satellite into an orbit close to the earth is known as critical velocity.

Calculation

Consider a satellite of mass m is moving with velocity v in a circle of radius R (i.e radius of the Earth).

Since gravitational force provides the necessary centripetal force

So, $mg = \frac{mv^2}{r}$

or $v^2 = gR$ r = R (radius of the Earth)

or $v = \sqrt{gR}$

$v = \sqrt{9.8 \times 6.4 \times 10^6}$

$v = 7900$ m/sec

or $\boxed{v = 7.9 \text{ km s}^{-1}}$

This means that minimum 7.9 km s^{-1} velocity is required to put a satellite to orbit close to the earth.

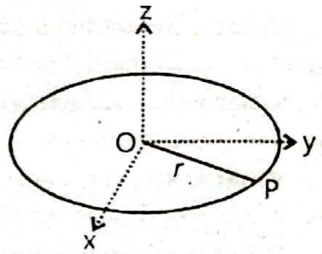
39. **State the direction of the following vectors in simple situations; angular momentum and angular velocity.**

Ans. **(i) Direction of Angular Momentum**

Angular momentum is given by

$$\vec{L} = \vec{r} \times \vec{p}$$

According to right hand rule, if we curl the finger of our right hand from $\vec{r}$ to $\vec{p}$, then the thumb indicates the



direction of $\vec{L}$ which is along the axis of rotation.

(ii) Direction of Angular Velocity: We know that

$$\vec{v} = \vec{\omega} \times \vec{r}$$

If we curl the fingers of the right hand from $\vec{\omega}$ towards $\vec{r}$, then the thumb indicates the direction of $\vec{v}$.

$\vec{r}$ is along radius, $\vec{v}$ is perpendicular to radius, $\vec{\omega}$ is along the axis of rotation out of the plane of paper.

If $\vec{r}$ along x-axis $\vec{v}$ along y-axis, then $\vec{\omega}$ along z-axis perpendicularly out of the plane of paper.

Also if we curl the fingers of our right hand along the direction of rotation, the thumb indicates the direction of angular velocity $\vec{\omega}$.

40. What is weightlessness?

Ans. It is a sensation or condition where a person or object experiences no net force of gravity, falling as if they have no weight.

41. Do astronauts in satellites experience zero gravity?

Ans. No, gravity still acts on them, but they feel weightless because they are in free fall along with the satellite.

42. Why do astronauts feel weightless in a satellite?

Ans. Because both the satellite and the astronauts are falling toward the Earth at the same rate, creating a condition of free fall.

43. Is the force of gravity zero inside a satellite?

Ans. No, the force of gravity exists, but since everything is falling together, the effect of weight is not felt.

44. What is the actual cause of weightlessness in orbiting satellites?

Ans. Continuous free fall around the Earth due to orbital motion.

45. Can weightlessness be experienced on the Earth?

Ans. Yes, for example; in a falling elevator or a parabolic flight path (vomit comet).

46. What is the effect of weightlessness on the human body?

Ans. It can be lead to muscle weakening and bone loss over time due to lack of resistance.

47. Why is artificial gravity needed in space?

Ans. To prevent health problems like muscle atrophy and bone loss caused by long-term weightlessness.

48. How can artificial gravity be created in a spacecraft?

Ans. By rotating the space craft or a section of it to use centrifugal force to simulate gravity.

49. Explain, why an object orbiting the Earth is said to be freely falling? Use your explanation to point out why objects appear weightless under certain circumstances?

Ans. When an object is put into orbit around the earth, then as a result of tangential velocity and velocity due to force of gravity, the object starts moving along a curved path. For a suitable tangential orbital velocity, the curvature of this path becomes equal to the curvature of earth. Because in this case.

Centripetal force = Force of gravity

or $$\frac{mv^2}{r} = mg$$

or $$= \frac{v^2}{r}$$

$\therefore$ $$a_c = \frac{v^2}{r} = g$$

Under this condition, the object falls freely under the action of gravity. When a body is freely falling, it is moving with acceleration equal to acceleration g due to gravity.

Suppose a body of mass m is moving downward with acceleration a and its apparent weight is T, then we can write

$$ma = mg - T$$

or $$T = mg - ma$$

If $a = g$, then apparent weight is given by

$$T = mg - mg = 0$$

50. When mud flies off the tyre of a moving bicycle? In what direction does it fly? Explain.

Ans. The mud flies off along the tangent to the tyre.

Reason

When speed of bicycle increases then adhesive force (sticking force) between the mud and the tyre is not sufficient to provide the necessary centripetal force so the mud leaves the tyre and moves along tangent to tyre because the direction of linear velocity in a rotating body is along tangent.

51. Why does a diver change his body positions before and after diving in the pool?

Ans. Angular momentum of the diver is kept constant

Initial angular momentum = Final angular momentum

or $$L_1 = L_2$$

or $$I_1\omega_1 = I_2\omega_2$$

or $$mr_1^2\,\omega_1 = mr_2^2\,\omega_2$$

When the diver lifts off the diving board, his legs and arms are fully extended. R_1 is large. Moment of inertia I_1 is large. Angular velocity ω_1 is small. When the diver pulls his legs and arms inwards, r_2 decreases, moment of inertia I_2 decreases and to conserve angular momentum, angular velocity ω_2 of the diver increases. In this way, the diver can make more somersaults.

52. A student holds two dumb-bells with stretched arms while sitting on a turntable. He is given a push until he is rotating at certain angular velocity. The student then pulls the dumb-bells towards his chest fig (b). what will be the effect on rate of rotation?

Ans. Initial angular momentum = Final angular momentum

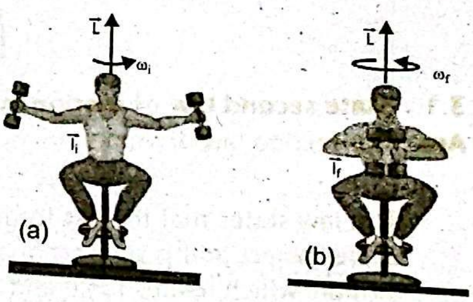
or $$L_1 = L_2$$

or $$I_1\omega_1 = I_2\omega_2$$

or $$mr_1^2\,\omega_1 = mr_2^2\,\omega_2$$

Initially, the student holds the two dumb-bells in his stretched arms r_1 is large. Therefore, moment of inertia I_1 is large and angular velocity ω_1 is small.

Finally, the student pulls the dumb-bells towards his chest, r_2 becomes small, moment of inertia I_2 decreases and to conserve angular momentum. Angular velocity ω_2 of the student becomes large and he spins faster.



SOLVED EXERCISE

MULTIPLE CHOICE QUESTIONS

Tick (✓) the correct answer.

3.1 The ratio of angular speed of minute's hand and hour's hand of a watch is:

(a) 1 : 6 (b) 6 : 1 (c) 1 : 12 (d) 12 : 1✓

3.2 A body travelling in a circle at constant speed:

(a) has constant velocity (b) has an inward radial acceleration✓

(c) is not accelerated (d) has an outward radial acceleration

3.3 A stone at the end of long string is whirled in vertical circle at a constant speed. The tension in the string will be minimum when the stone is:

(a) at the top of the circle✓ (b) half way down

(c) at the bottom of circle (d) any where in the circle

3.4 Every point of rotating rigid body has:
(a) same angular velocity✓ (b) same linear velocity
(c) same linear acceleration (d) same linear distance

3.5 The minimum velocity necessary to put a satellite into the orbit is called:
(a) terminal velocity (b) critical velocity✓ (c) artificial velocity (d) angular velocity

3.6 An astronaut is orbiting around the Earth in a large capsule. Then,
(a) he will be in a state of weightlessness with respect to capsule
(b) he is freely falling towards the Earth✓
(c) a ball projected at an angle has a straight line path as observed by him
(d) all the above

3.7 An object in uniform circular motion makes 10 revolutions in 2 seconds. Which of the following statement is true?
(a) Its period is 2.0 s (b) Its period is 20 s (c) Its frequency is 5 Hz✓ (d) Its frequency is 0.2 Hz

3.8 A man inside the artificial satellite feels weightlessness because the force of attraction due to the Earth is:
(a) zero at pole (b) balanced by the force of attraction due to the moon
(c) equal to the centripetal force✓
(d) non-effective due to some particular design of the satellite

3.9 A bottle of soda water is grasped from the neck and swung briskly in a vertical circle. Near which portion of the bottle do the bubbles collect?
(a) Near the bottom (b) In the middle of bottle
(c) Bubbles remain distributed throughout the volume of the bottle.
(d) Near the neck of the bottle✓

3.10 The moment of inertia of body depends upon:
(a) mass of the body and its distribution about axis of rotation✓
(b) volume of the body
(c) kinetic energy of the body (d) angular momentum of the body

SHORT ANSWER QUESTIONS

3.1 State second law of motion in case of rotation.

Ans. The second law of motion in the case of rotation is expressed as:

$$\tau = I\alpha$$

This law states that the net torque acting on a rotating object is directly proportional to the angular acceleration of the object and is proportional to the moment of inertia. It is the rotational analog of Newton's second law of motion, which relates force and acceleration in linear motion.

3.2 What is the effect of changing the position of a diver while diving in the pool?

Ans. When a diver changes his position during a dive, such as shifting from a tucked position to a pike or layout position, it affects their body's dynamics, especially in terms of rotational speed and control. This principle is rooted in conservation of angular momentum, which states that if no external torques are acting on the system, the total angular momentum remains constant.

Let jus understand how different positions affect the dive:

1. **Tucked Position:** The diver curls their body tightly, which decreases their moment of inertia (the distribution of mass relative to the center of rotation). Since angular momentum is conserved, a decrease in the moment of inertia causes the diver to spin faster.
2. **Pike Position:** The diver bends at the hips while keeping their legs together. This is a compromise between the tucked and layout positions. The moment of inertia is larger than in the tucked position, so the diver spins slower than in the tuck but faster than in the layout.
3. **Layout Position:** The diver keeps their body straight and extended. This increases their moment of inertia, meaning they rotate more slowly than in the tucked or pike positions.

Thus, by changing their body position during the dive, a diver can control their speed of rotation and adjust their movements to complete twists or flips more precisely. The more compact the position (like tucked), the faster the spin, and the more extended (like layout), the slower the spin.

3.3 How do we get butter from the milk by using centrifuge?

Ans. To explain the process of making butter from milk using circular motion and rotational motion, we can think of the churning process as a physical application of these motions.

1. **Separation of Cream (Initial Step):** First, fresh milk is left to sit so that the cream rises to the top due to gravity. This is a natural separation, not involving rotational motion, but it creates the cream that will be churned later.

2. **Churning and Rotational Motion:** Once you have the cream, the process of turning it into butter involves churning, where rotational motion plays a key role. In traditional butter-making, this is done using a butter churn, which consists of a container and a paddle or dasher.

   Rotational Motion: The churn is either rotated manually or mechanically, causing the paddle to spin inside the container. This spinning motion applies centrifugal force to the cream, which causes the fat molecules to move and separate from the liquid (buttermilk).

   Circular Motion: The cream molecules start to move in circular paths as they are forced by the rotational motion. This circular motion increases the contact between the fat molecules, causing them to clump together and form solid butter.

3. **Formation of Butter:** As the cream is churned, the rotational motion causes the fat globules to collide and stick together, forming larger clumps. Over time, these clumps of fat form into solid butter, and the remaining liquid (buttermilk) is separated out.

4. **Further Kneading (Optional):** After the butter has formed, it may still have some residual buttermilk in it. Kneading the butter further (sometimes using circular hand motions) helps to squeeze out this buttermilk and smoothen the butter into a uniform texture.

   In summary, circular and rotational motions are crucial in the churning process, where rotational motion causes the separation of fat and buttermilk, and circular motion helps in forming the butter by causing fat particles to collide and bind together.

3.4 Mass is a measure of inertia in linear motion. What is its analogue in rotational motion? Describe briefly.

Ans. In rotational motion, the analogous quantity to mass (which measures inertia in linear motion) is the moment of inertia.

Moment of inertia (I) is a measure of an object's resistance to change in its rotational motion about an axis. It depends not only on the total mass of the object but also on how that mass is distributed relative to the axis of rotation.

Mathematically, for a system of particles:

$$I = \Sigma m_i r_i^2$$

where m_i is the mass of ith particles and r_i, its perpendicular distance from the axis of rotation.

3.5 Why is it harder for a car to take turn at higher speed than at lower speed?

Ans. When a car turns, it needs to follow a curved path, which requires the tires to exert a force called centripetal force to keep the car moving along that curve. The force is proportional to the square of the car's speed and inversely proportional to the radius of the curve. In simpler terms, the faster the car goes, the more force is needed to keep it on the curved path.

At higher speeds, the centripetal force required for a turn increases, which means the tires need more grip to prevent slipping. If the car is going too fast, the available friction between the tires and the road may not be enough to provide the necessary centripetal force, causing the car to lose traction and slide outward, or even skid.

At lower speeds, the required centripetal force is smaller, so the car can take turns more easily without the risk of losing traction. Thus, higher speeds make it harder for the car to safely take a turn due to the increased need for friction and the limited capacity of the tires to provide it.

3.6 What are the benefits of using rear wheels of heavy vehicles consisted of double tyres?

Ans. Using double tyres (also known as dual wheels or twin tyres) on the rare axles of heavy vehicles offers several important benefits.

1. **Increased load carrying capacity:**

- Dual tyres distribute the vehicle's weight over a large area, reducing the load on each individual tyre.
- This allows the vehicle to carry heavier loads safety without exceeding the maximum load rating on each tyre.

2. **Improved stability:**

- The wider footprint from two tyres provides greater lateral stability, especially important where cornering or during high winds.
- This helps prevent rollovers and improves overall vehicle control.

3. **Redundancy in case of tyre failure:**
If one tyre in a pair fails, the other can temporarily support the load, giving the driver more control and time to stop safety.

4. **Reduced ground pressure:**
Dual tyres spread the vehicle's weight over a large surface area, which reduces ground pressure.

5. **Better traction:**
More tyre surface area means more grip, which is essential for heavy vehicles in off-road or slippery conditions.

3.7 When a moving car turns around a corner to the left, in what direction do the occupants tend to fall? Explain briefly.

Ans. When a moving car turns left, the occupants tend to fall to the right.
This happens due to inertia. As the car turns left, your body wants to keep moving in a straight line (which would be toward the right side of the car from the car perspective). Since the car is turning left under you, it feels like you are "pushed" to the right. In reality, it is your body trying to continue its straight-line motion while the car moves out from under you.

3.8 Why is the acceleration of a body moving uniformly in a circle, directed towards the centre?

Ans. When a body moves uniformly in a circle, it means its speed is constant, but its direction is continuously changing. This constant change in direction means,the velocity in changing, and any change in velocity means there is acceleration, even if the speed is the same. This type of acceleration is called centripetal acceleration. That is why the acceleration is towards the centre.
As the object moves around the circle, the magnitude of speed stays the same, but the direction of velocity vector changes. This change in direction always happens in such a way that the new velocity vector "leans in" toward the centre of the circle.

3.9 How does an astronaut feel weightlessness while orbiting from the Earth in a spaceship?

Ans. An astronaut feels weightlessness while orbiting the Earth in a spaceship because both the astronaut and the spaceship are in a state of free fall towards the Earth. Let uş know how does it work. Inside the spaceship, everything including the astronaut is falling at the same rate, so there is no normal force. With no normal force acting on the astronaut's body, they feel weightless.

CONSTRUCTED RESPONSE QUESTIONS

3.1 If angular velocity of different particles of a rigid body is constant, will the linear velocity of these particles be also constant?

Ans. Yes, if the angular velocity of different particles of a rigid body is constant, the linear velocity of these particles will also be constant, but with some important details.
For a rigid body, each particle's linear velocity is given by the equation:

$$v = \omega \times r$$

If the angular velocity ω is constant (in both magnitude and direction), and the position vector **r** does not change (because it's a rigid body), then the linear velocity of each particle will also be constant in magnitude and direction.
This assumes that there are no external forces or torques acting on the rigid body, and that the rigid body is not experiencing any deformation. The particles will continue to move in their respective circular paths at a constant speed.

3.2 A loaf of bread is lying on rotating plate. A crow takes away the loaf of bread and the rotation of the plate increases. Why?

Ans. The increase in the rotation speed of the plate can be explained by the conservation of angular momentum.
When the loaf of bread is on the rotating plate, the system (plate + loaf of bread) has a certain amount of angular momentum, which is the product of the moment of inertia and the angular velocity. The moment of inertia depends on the distribution of mass relative to the axis of rotation.

When the crow takes the loaf of bread, it removes the mass from the system. Since angular momentum must be conserved (if no external torques are acting on the system), the plate's angular velocity increases to compensate for the loss of mass. The reduction in mass results in a decrease in the moment of inertia, so in order to keep angular momentum constant, the plate must rotate faster. This is similar to how an ice skater spins faster when pulling their arms in close to their body.

3.3 Why do we tumble when we take the sharp turn with large speed?

Ans. When you take a sharp turn at high speed, you experience what is known as "centrifugal force." This is the apparent force that pushes you away from the center of the turn. When you turn, your body wants to continue moving in a straight line due to inertia, but the vehicle is changing direction.

If the turn is sharp and you're going too fast, the friction between your body and the vehicle may not be enough to keep you safely in your seat or control your movement. This can cause you to lose your balance or tumble.

The faster you go, the stronger the centrifugal force becomes, and the more likely you are to be thrown off balance or even ejected from your seat if you don't have proper restraints (like a seatbelt or a firm grip). This is why it's important to slow down when approaching sharp turns, especially at high speeds, to prevent tumbling or losing control.

3.4 What will be time period of a simple pendulum in an artificial satellite at a certain height?

Ans. In an artificial satellite orbiting the Earth, the time period of a simple pendulum would be essentially zero. This is because the satellite and everything inside it, including the pendulum, are in free fall due to the gravitational pull of the Earth. This creates a microgravity or weightless environment.

In such a situation, the pendulum would not experience any restoring force, as it would be in a state of continuous free fall along with the satellite. Hence, the pendulum would not swing, and the concept of a "time period" does not apply in the traditional sense.

In short, the time period of a simple pendulum in an artificial satellite is effectively zero.

3.5 Is the motion of a satellite in its orbit, uniform or accelerated?

Ans. The motion of a satellite in its orbit is accelerated, not uniform, even if it moves with constant speed.

This is because the satellite is constantly changing direction as it orbits.

3.6 What are the advantages that radian has been preferred as SI unit to degree?

Ans. Radian is preferred over degree as the SI unit of angular measurement for several key reasons:

1. **Natural Relationship to Circular Motion:** Radians provide a direct relationship to the geometry of a circle. An angle of 1 radian corresponds to the angle subtended by an arc whose length is equal to the radius of the circle. This makes radians inherently tied to the properties of a circle, making them more natural in mathematical and physical calculations.
2. **Simplification in Mathematical Formulas:** Many mathematical formulas, particularly in calculus, become simpler when angles are measured in radians. For instance, the derivative of the sine function is straightforward when angles are in radians. In degrees, additional conversion factors are needed, complicating the formulas.
3. **Consistency in SI Units:** Radians are dimensionless, meaning they are a pure number and don't introduce extra units into calculations. Since the SI system is based on consistency and dimensionless quantities, radians fit perfectly into the system, unlike degrees, which are not dimensionless.
4. **Trigonometric and Physical Functions:** Many physical laws and trigonometric functions, such as those in oscillatory motion (e.g., angular frequency in simple harmonic motion), are most naturally expressed in radians. Using radians ensures that these functions have the correct units and scaling, leading to more accurate and direct interpretations.
5. **Ease of Conversion:** Although degrees are more intuitive in everyday life, radians are easier to work with in higher-level mathematics and physics because they avoid the need for frequent conversions (e.g., 360 degrees = radians). This simplification makes complex calculations more straightforward.
6. **International Standardization:** The International System of Units (SI) emphasizes radians for angular measurement to maintain consistency in scientific research and technical applications worldwide. This ensures that mathematical models, engineering designs, and scientific experiments are universally understood and standardized.

These advantages contribute to the widespread adoption of radians in science, engineering, and mathematics.

3.7 In uniform circular motion, what are the average velocity and average acceleration for one revolution? Explain.

Ans. In uniform circular motion, an object moves in a circular path with constant speed. However, its direction changes continuously, meaning both velocity and acceleration are constantly changing.

1. Average Velocity for One Revolution

Average velocity is defined as the total displacement divided by the time taken.

For one full revolution, the displacement is zero because the object ends up at the same point it started.

Therefore, the average velocity for one revolution is zero.

2. Average Acceleration for One Revolution

Average acceleration is defined as the change in velocity divided by the time taken.

Since velocity is a vector quantity, its direction changes continuously in uniform circular motion. Over one full revolution, the change in velocity is equal to the initial velocity but in the opposite direction.

The object experiences a change in direction, but because the magnitude of the velocity (speed) is constant, the total change in velocity over one complete revolution is non-zero, and its average acceleration is non-zero.

However, calculating average acceleration exactly requires considering the specific change in velocity over the time interval, which results in a value of zero when considering direction alone.

3.8 In a rainstorm with a strong wind, what determines the best position to hold an umbrella?

Ans. In a rainstorm with strong wind, the best position to hold an umbrella is determined by the direction of the wind and the rain. Here are some tips:

1. **Angle the umbrella into the wind:** Tilt the umbrella so that the wind hits it at a slight angle, rather than directly from behind or in front. This helps to reduce the risk of the umbrella turning inside out. The top of the umbrella should be facing into the wind.
2. **Hold it slightly forward:** If the rain is blowing at an angle, hold the umbrella slightly forward from your body, angling it to block both the rain and wind. This will help shield you from the wettest part of the storm.
3. **Firm grip:** Hold the umbrella with both hands if possible to maintain control, as strong winds can make it difficult to keep the umbrella steady. Be prepared to adjust your grip if the wind picks up.
4. **Keep it low to the ground:** Hold the umbrella closer to your body to minimize exposure to the wind. A lower, more controlled position can prevent the umbrella from being blown away or flipped inside out.
5. **Use a wind-resistant umbrella:** If possible, choose an umbrella designed to withstand wind, such as those with reinforced frames or vented designs. These allow wind to pass through the umbrella without causing damage.

This position will give you better control and protect you from both the rain and the strong wind.

3.9 A ball is just supported by a string without breaking. If it is whirled in a vertical circle, it breaks. Explain why.

Ans. When the ball is just hanging by the string, the only force acting on the string is the weight of the ball (its gravitational force). This tension is relatively low and with the string's strength.

However, when the ball is whirled in a vertical circle, the tension in the string increases significantly at the bottom of the circle. At that point, the string must support both:

(i) the weight of the ball (mg), and

(ii) the centripetal force $\left(\frac{mv^2}{r}\right)$ needed to keep the ball moving in a circle.

So, the total tension becomes:

$$T = mg + \frac{mv^2}{r}$$

If this combined tension exceeds the string's breaking strength, the string will snap. That is why, it breaks when the ball is whirled, but not when it is simply hanging.

3.10 How is the centripetal force supplied in the following cases:

(a) a satellite orbiting around the Earth?

(b) a car taking a turn on a level road?

Ans. In each of these situations, the centripetal force is supplied by a different source. Let us know how?

(a) The gravitational force between the Earth and the satellite provides the necessary centripetal force.

Centripetal force = Gravitational attraction by the Earth

(b) The frictional force between the tyres of the car and the road supplies the centripetal force that allows the car to follow a curved path.

Centripetal force = Friction between tyres and the road

COMPREHENSIVE QUESTIONS

3.1 What is meant by angular momentum? Explain the law of conservation of angular momentum with daily life examples.

Ans. *See Q. 18, Q. 19 and Q. 20.*

3.2 Show that orbital angular momentum is; L = Iω.

Ans. *See Q. 18.*

3.3 Define moment of inertia. Prove that torque acting on rotating rigid body is equal to the product of its moment of inertia and angular acceleration.

Ans. *See Q. 16.*

3.4 What are artificial satellites? Calculate the minimum time period necessary to put a satellite into the orbit near the surface of the Earth.

Ans. *See Q. 12.*

3.5 Define orbital velocity and derive an expression for the same.

Ans. *See Q. 13.*

3.6 Write a note on artificial gravity. Derive an expression for frequency with which the spaceship rotates to provide artificial gravity.

Ans. *See Q. 15.*

3.7 Prove that; (i) $v = r\omega$ and (ii) $a = r\alpha$

Ans. *See Q. 6 and Q. 7*

NUMERICAL PROBLEMS

3.1 A laser beam is directed from the Earth to the Moon. If the beam is to have a diameter of 2.50 m at the Moon, how small must divergence angle be for the beam? The distance of Moon from the Earth is 3.8×10^8 m.

Solution:

Given Data:

Diameter = $S = 2.50\ m$

Distance = $r = 3.8 \times 10^8 m$

To Find:

Divergence angle $= \theta = ?$

Formula:

$$S = r\theta$$

Calculations:

As $S = r\theta$

or $\theta = \frac{S}{r} = \frac{2.5}{3.8 \times 10^8}$

$\boxed{\theta = 6.6 \times 10^{-9} radian}$ **Ans.**

3.2 A car is moving with a speed of 108 km h^{-1}. If its wheel has a diameter of 60 cm, find its angular speed in rad s^{-1} and rev s^{-1}.

Solution:

Given data:

Speed of car $v = 108$ km h^{-1}

$= 108 \times \frac{1000}{3600}$ ms^{-1}

$= 30$ ms^{-1}

Diameter of wheel $d = 60$ cm $= \frac{60}{10} = 0.6$ m

Radius of wheel $r = \frac{d}{2} = \frac{0.6 \text{ m}}{2} = 0.3$ m

To find:

(i) Angular speed in rad s^{-1} = ?

(ii) Angular speed in rev s^{-1} = ?

Conclusions:

(i) Relation between linear speed and angular speed is:

$$v = r\omega$$

or $\omega = \frac{v}{r}$

$$\omega = \frac{30 \text{ ms}^{-1}}{0.3 \text{ m}} = 100 \text{ rad s}^{-1}$$

(ii) Converting angular speed to revolution per second.

Since one revolution = 2π radian

Revolution per second $= \frac{\omega}{2\pi} = \frac{100 \text{ rad s}^{-1}}{2\pi}$

$= 16$ rev s^{-1} **Ans.**

3.3 An electric motor is running at 1800 rev min^{-1}. On switching off, it comes to rest in 20 s. If angular retardation is uniform, find the number of revolutions it makes before stopping.

Solution:

Given data:

Initial angular velocity $\omega_i = 1800 \text{ rev min}^{-1}$

$$= \frac{1800}{60} = 30 \text{ rev s}^{-1}$$

Final angular velocity $\omega_f = 0 \text{ rev s}^{-1}$

Time $t = 20 \text{ s}$

To find:

Angular displacement $\theta = ?$

Conclusions: Using the formula:

$$\theta = \frac{1}{2}(\omega_i + \omega_f) \times t$$

Putting the values

$$\theta = \frac{1}{2}(30 \text{ rev s}^{-1} + 0 \text{ rev s}^{-1}) \times 20 \text{ t}$$

$\boxed{\theta = 300 \text{ reV}}$ **Ans.**

3.4 A string 0.5 m long holding a stone can withstand maximum tension of 35.6 N. Find the maximum speed at which a stone of 0.5 kg mass can be whirled with it in a vertical circle.

Solution:

Given data:

Length of string (radius) = r = 0.5 m

Maximum tension = T_{max} = 35.6 N

Mass = m = 0.5 kg

Acceleration due to gravity = g = 9.8 ms^{-2}

Calculations: To find the maximum speed at which the stone can be whirled in a vertical circle, we need to consider the point in the motion where the tension in the string is greatest.

In a vertical circle, the tension is greatest at the bottom of the circle. At that point, the tension must support both the centripetal force and the weight of the stone.

At the bottom of the circle:

$$T = \frac{mv^2}{r} + mg$$

Putting the values

$$35.6 \text{ N} = \frac{0.5 \text{ kg} \times v^2}{0.5 \text{ m}} + 0.5 \text{ kg} \times 9.8 \text{ ms}^{-2}$$

$$35.6 \text{ N} = v^2 + 4.9 \text{ N}$$

$$v^2 = 35.6 \text{ N} - 4.9 \text{ N}$$

$$v^2 = 30.7$$

$\boxed{v = \sqrt{30.7} \approx 5.5 \text{ ms}^{-1}}$ **Ans.**

3.5 The flywheel of an engine is rotating at 2100 rev min^{-1} when the power source is shut off. What torque is required to stop it in 3 minutes? The moment of inertia of the flywheel is 36 kg m^2.

Solution:

Given data:

Initial angular velocity $\omega_i = 2100 \text{ rev min}^{-1}$

Final angular velocity $\omega_f = 0$

Time to stop $t = 3 \text{ min} = 180 \text{ s}$

Moment of inertia $I = 36 \text{ kg m}^2$

To find:

Torque required to stop the fly wheel $\tau = ?$

Calculations: To find the torque required to stop the flywheel, we can follow thee steps:

Step 1: Converting initial angular velocity to rad s^{-1}

$$2100 \text{ rev min}^{-1} \times \frac{2\pi \text{ rad}}{1 \text{ rev}} \times \frac{1 \text{ min}}{60 \text{ s}} = \frac{2100 \times 2\pi}{60\text{s}}$$

$$\approx 219.9 \text{ rad s}^{-1}$$

Step 2: Using angular declaration formula:

$$\alpha = \frac{\omega_f - \omega_i}{t} = \frac{0 - 219.9 \text{ rev s}^{-1}}{180 \text{ s}} \approx -1.22 \text{ rad s}^{-2}$$

Thus, torque τ can be calculated as:

$$\tau = I \times \alpha = 36 \text{ kg m}^2 \times 1.22 \text{ rad s}^{-2}$$

$\tau \approx -44$ Nm **Ans.**

Thus, a torque 44 N m is required to stop the flywheel.

3.6 What is the moment of inertia of a 200 kg sphere whose diameter is 60 cm?

Solution:

Given data:

Mass = m = 200 kg

Diameter = d = 60 cm = 0.6 m

Radius $= r = \frac{0.6 \text{ m}}{2} = 0.3 \text{ m}$

To find:

Moment of inertia of a sphere $I = ?$

Calculations: To find the moment of inertia of a solid sphere about an axis through its centre, we use the formula:

$$I = \frac{2}{5} mr^2$$

Putting the values

Diameter = d = 60

$$I = \frac{2}{5} \times 200 \text{ kg} \times (0.3 \text{ m})^2$$

$$= \frac{2}{5} \times 200 \text{ kg} \times 0.09 \text{ m}^2$$

$I = 7.2 \text{ kg m}^2$ **Ans.**

3.7 A satellite is orbiting the Earth at an altitude of 200 km. Assuming the Earth's radius is 6400 km, calculate the orbital speed of the satellite.

Solution:

Given data:

Altitude $= h = 200\ km = 2.0 \times 10^5\ m$

Earth's radius $= r = 6400\ km = 6.4 \times 10^6\ m$

Mass of Earth $= M = 6.0 \times 10^{24}\ kg$

Total distance $= 6.4 \times 10^6\ m + 2.0 \times 10^5\ m$

$= 6.6 \times 10^6\ m$

Gravitational constant $= G = 6.67 \times 10^{-11}\ Nm^2\ kg^{-2}$

To find: Orbital speed v = ?

Calculations: Using the formula:

$$v = \sqrt{\frac{GM}{r}}$$

Putting the values

$$v = \sqrt{\frac{6.67 \times 10^{-11}\ Nm^2\ kg^{-2} \times 6.0 \times 10^{24}\ kg}{6.6 \times 10^6\ m}}$$

$$= \sqrt{\frac{3.986 \times 10^{14}\ Nm^2\ kg^{-1}}{6.6 \times 10^6\ m}} = \sqrt{6.04 \times 10^7}$$

$v \approx 7.78 \times 10^3\ ms^{-1}$

or $\boxed{v \approx 7.78\ km\ s^{-1}}$ **Ans.**

3.8 A space station has a radius of 20 m and rotates at an angular velocity of 0.5 rad s⁻¹. What is the artificial *gravity experienced by the astronauts on the space station?*

Solution:

Given data:

Radius $= r = 20\ m$

Angular velocity $\omega = 0.5\ rad\ s^{-1}$

To find:

Artificial gravity = ?

Calculations:

To find the artificial gravity (centripetal acceleration) experienced by the astronauts on the rotating space station, we use the formula:

$$a = \omega_r^2$$

Putting the values

$a = (0.5\ rad\ s^{-1})^2 \times 20\ m$

$a = 5\ ms^{-2}$ **Ans.**

The artificial gravity is $5\ ms^{-2}$. For comparison, Earth's gravity is about $9.8\ ms^{-2}$, so the astronauts would experience a little over half of Earth's gravity on this station.

3.9 A bicycle wheel has an angular momentum of 10 kg m² s⁻¹ and angular velocity of 2 rad s⁻¹. Find the value of its moment of inertia.

Solution:

Given data:

Angular momentum $L = 10\ kg\ m^2\ s^{-1}$

Angular velocity $\omega = 2\ rad\ s^{-1}$

To find:

Moment of inertia I = ?

Calculations: To find the moment of inertia (I) of a bicycle wheel, we can use the relationship between angular momentum L and angular velocity ω as:

$$L = I\omega$$

or

$$I = \frac{L}{\omega}$$

$$I = \frac{10\ kg\ m^2\ s^{-1}}{2\ rad\ s^{-1}}$$

$\boxed{I = 5\ kg\ m^2}$ **Ans.**

3.10 A diver comes off a board with arms straight up and legs straight down, giving him a moment of inertia of 18 kg m² about his rotation axis. Then tucks into a small ball, decreasing his moment of inertia to 3.6 kg m². While tucked, he makes two complete rotations in 1.0 second. If he had not tucked at all, how many revolutions would he have made in 1.5 s from board to water?

Solution:

The angular momentum is given by

$$L = I\omega$$

Finding angular velocity while tucked:

When the diver is tucked, his moment of inertia decreases to

$I_{tucked} = 3.6\ kg\ m^2$. He completes 2 revolutions in 1.0 s, so his angular velocity is:

$$\omega_{tucked} = \frac{\text{Total angle}}{\text{Time}} = \frac{2 \times 2\pi}{1.0\ s} = 4\pi\ rad\ s^{-1}$$

Calculating initial angular velocity (arm straight up)

By conservation of angular momentum:

Iinitialωinitial = Itucked ωtucked

Putting the values

$1.8\ \omega_{initial} = 3.6 \times 4\pi$

Solving for $\omega_{initial}$:

$$\omega_{initial} = \frac{3.6 \times 4\pi}{18} = 0.8\pi\ rad\ s^{-1}$$

Finding revolutions if he did not tuck:

If the diver had not tucked, be would have rotated with $\omega_{initial} = 0.8\pi\ rad\ s^{-1}$ for 1.5s. The total angle rotated is:

$$\theta = \omega_{initial} \times t = 0.8\pi \times 1.5 = 1.2\pi\ rad$$

Converting this angle into revolutions:

$$\text{Revolutions} = \frac{\theta}{2\pi} = \frac{1.2\pi}{2\pi} = 0.6\ rev$$ **Ans.**

Thus, if the diver had not tucked, he would have made 0.6 revolutions in 1.5 seconds.