Unit 04

Work, Energy and Power

STUDENT'S LEARNING OUTCOMES (SLO's)

After studying this unit, the students will be able to:

- Derive the formula for kinetic energy [using the equations of motion]
- Derive an expression for absolute potential energy of a body at a certain position in the gravitational field [including escape velocity]
- Deduce the work done from force-displacement graph.
- Differentiate between conservative and non-conservative forces.
- State and use the work - energy theorem in a resistive medium to solve p

4.1 WORK DONE BY A CONSTANT FORCE

Q 1. Define work and its SI unit.

Ans. Work

Definition

He dot product of force and displacement is called work.

Mathematically;

Work = Force × Displacement

or $W = \vec{F}.\vec{d}$

or $W = Fd\cos\theta$

The SI unit of work is joule (J).

Joule

If a force displaces a body in the direction of force to a distance of one metre, then work done will be on joule.

One joule = One newton × One metre

or $1\ J = 1\ N \times 1\ m = Nm$

Q 2. How is work done by a constant force? Explain.

Ans. Constant Force

Definition

A force that remains unchanged in magnitude and direction overtime while it is applied to an object is known as constant force.

Work done by a Constant Force

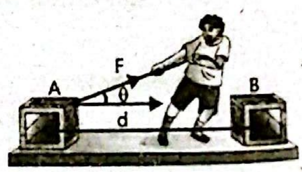
Consider an object pulled by a constant force **F**. The force displaces the object through a displacement **d** in the direction of force. In such a case, work *W* is defined as the product of the magnitude of the force **F** and magnitude of the displacement **d**.

Fig. 1

If the displacement is zero, then no work is done even if a large force is applied. For example, pushing on a wall may tire your muscles, but work done is zero; Fig. 1.

The force applied on a body may not always be in the direction of force; Fig. 2. If the force **F** makes an angle θ with the displacement **d**; Fig. 3, the work done is equal to the product of the component of force along the direction of the displacement and the magnitude of displacement. Then

$$W = (F\cos\theta)\,d = Fd\cos\theta$$



$W = F \cdot d$

When a constant force acts through a distance *d*, as plotted on a graph; Fig. 4, the distance is normally plotted along x-axis and the force along y-axis. As the force does not vary, in this case, the graph will be a horizontal straight line. If the constant force **F** and the displacement **d** are in the same direction, then the work done is *Fd*. Clearly shaded area in Fig. 4 is also *Fd*. Hence, the area under a force-displacement curve represent the work done by the constant force. In case if the force **F** is not in the direction of displacement, the graph is plotted between $F \cos\theta$ and *d*.

As work is a scalar quantity, therefore,

(i) if $\theta < 90°$, work is done and it is said to be positive work.

(ii) if $\theta = 90°$, no work is done.

(iii) if $\theta > 90°$, the work done is said to be negative.

(iv) the SI unit of work is Nm, also known as joule (J).

Fig. 2

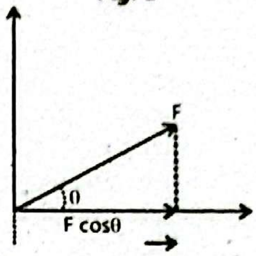


Fig. 3

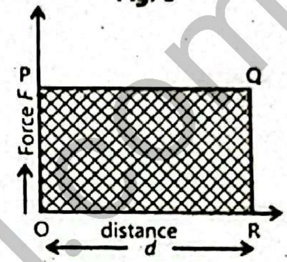


Fig. 4

4.2 WORK DONE BY A VARIABLE FORCE

3. How is work done by a variable force? Explain.

Ans. Variable Force

Definition

A force whose magnitude and / or direction changes with respect to time, position, or another variable is known as variable force.

Work done by a Variable Force

In many cases, the force does not remain constant during the process of doing work. For example, as a rocket moves away from the Earth, work is done against the force of gravity, which varies as the inverse square of the distance from the Earth's centre. Similarly, the force exerted by a spring increases with the amount of stretch. How can we calculate the work done in such situations?

Let us suppose the path of a particle in the xy-plane as it moves from point P to point Q; Fig. 5. The path has been divided into '*n*' short intervals of displacements $\mathbf{\Delta d_1}, \mathbf{\Delta d_2}, \ldots, \mathbf{\Delta d_n}$ and $\mathbf{F_1}, \mathbf{F_2}, \ldots, \mathbf{F_n}$ are the forces acting during these intervals, respectively.

During each small interval, the force is supposed to be approximately constant. So, the work done for the first interval can then be written as:

$$\Delta W_1 = F_1 . \Delta d_1 = F_1 \cos\theta_1 \Delta d_1$$

and in the second interval

$$\Delta W_2 = F_2 . \Delta d_2 = F_2 \cos\theta_2 \Delta d_2 \text{ and so on.}$$

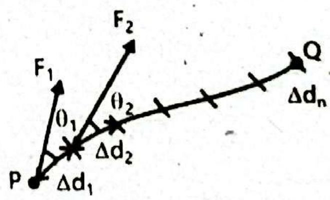


Fig. 5: A particle acted upon by a variable force, moves along the path shown from point P to point Q.

The total work done in moving the object can be calculated by adding all these terms.

$$W_{total} = \Delta W_1 + \Delta W_2 + \ldots\ldots + \Delta W_n$$

$$= F_1 \cos\theta_1 \Delta d_1 + F_2 \cos\theta_2 \Delta d_2 + \ldots\ldots + F_n \cos\theta_n \Delta d_n$$

or $$W_{total} = \sum_{i=1}^{n} F_i \cos\theta_i \Delta d_i \quad \ldots\ldots \text{(i)}$$

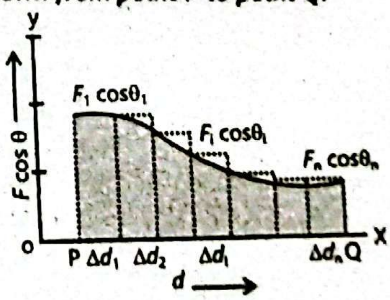


Fig. 6

We can examine this graphically by plotting $F \cos\theta$ verses *d*; Fig. 6. The displacement *d* has been sub-divided into the same '*n*' equal intervals. The value of $F \cos\theta$ at the beginning of each interval is indicated in the figure.

Now the i^{th} shaded rectangle has an area $F_i \cos\theta_i \Delta d$ which is the work done during the i^{th} interval. Thus, the work done given by Eq. (i) equals the sum of the areas of all the rectangles. If we sub-divide the distance into a large number of intervals so that each Δd becomes very small, the work done given by Eq. (i) becomes more accurate. If each Δd approach to zero, then we obtain an exact result for the work done, such as:

$$W_{total} = \lim_{\Delta d \to 0} \sum_{i=1}^{n} F_i \cos\theta_i \, \Delta d_i \qquad \text{.....(iii)}$$

If this limit Δd approaches zero, the total area of all the rectangles, Fig. 6 approaches the area between the F cos θ verses *d* curve and x-axis from P to Q as shown shaded in Fig. 7.

Thus, the work done by a variable force in moving a particle between two points is equal to the area under the *F* cos θ verses *d* curve between the two points P and Q as shown in Fig. 7.

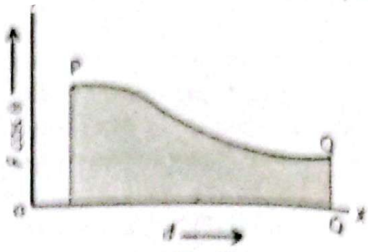


Fig. 7

Example 4.1: A force F acting on an object varies with distance d as shown in Fig. 8. Calculate the work done by the force as the object moves from d = 0 to d = 6 m.

Solution: The work done by the force is equal to the total area under the curve from

d = 0 to d = 6 m. This area is equal to the area of the rectangular section from d = 0 to d = 4 m, plus the area of triangular section from d = 4 m to d = 6 m.

Hence

Work done represented by the area of rectangle = 4 m × 5 N = 20 N m = 20 J

Work done represented by the area of triangle $= \frac{1}{2} \times 2\ \text{m} \times 5\ \text{N} = 5\ \text{N m} = 5$ J

Therefore, the total work done = 20 J + 5 J = 25 J **Ans.**

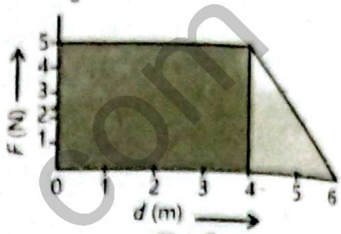


Fig. 8

4.3 CONSERVATIVE AND NON-CONSERVATIVE FORCES

4. Define conservative field and conservative force. Prove that; work done by gravitational force is independent of the path followed.

or Define conservative field. Show that gravitational field is conservative in nature.

Ans. Conservative Field

Definition

The space around the Earth in which its gravitational force acts on a body is called the gravitational field.

Explanation

This is an interesting property of the gravitational force that when an object is moved from one place to another, the work done by the gravitational force does not depend on the choice of the path. Let us explore it.

> **For Your Information**
>
> When an object is moved in the gravitational field, the work is done by the gravitational force. If displacement is in the direction of gravitational force, the work is positive. If the displacement is against the gravitational force, the work is said to be negative.

Consider an object of mass *m* being displaced with constant speed from point A to B along various paths in the presence of a gravitational force. In this case, the gravitational force is equal to the weight *mg* of the object.

The work done by the gravitational force along the path-1 (ADB) can be split into two parts (path AD and path DB). The work done along AD is zero, because the weight *mg* is perpendicular to this path, whereas the work done along DB is (−*mgh*) because the direction of *mg* is opposite to that of the displacement i.e., θ = 180°. Hence, the work done in displacing a body from A to B through path 1 is:

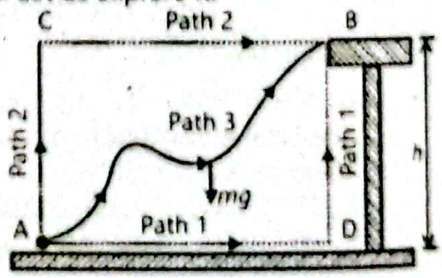


Fig. 9

$$W_{ADB} = 0 + (-mgh) = -mgh$$

If we consider the path-2 (ACB), the work done along AC is also (−*mgh*). Since the work done along CB is zero therefore,

$$W_{ACB} = -mgh + 0 = -mgh$$

Now consider path-3, i.e., a curved one. Imagine the curved path is broken down into a series of horizontal and vertical steps; Fig. 10. There is no work done along the horizontal steps, because *mg* is perpendicular to the displacement for these steps. Work is done by the force of gravity only along the vertical displacements. During the segment CD, *mg* is not negative; it is positive. But here all Δy elements are negative, so the products of *mg* and Δy for all the elements will again be negative. Therefore, we can write:

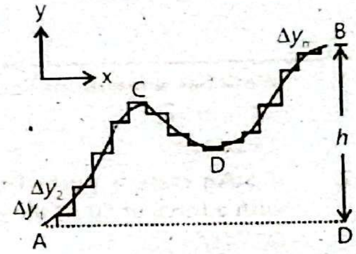


Fig. 10

$$W_{AB} = -mg\,(\Delta y_1 + \Delta y_2 + \Delta y_3 + \ldots\ldots + \Delta y_n)$$

As $\Delta y_1 + \Delta y_2 + \Delta y_3 + \ldots\ldots + \Delta y_n = h$

Hence $W_{AB} = -mgh$

The net amount of work done along the curved path AB is still $(-mgh)$. We conclude from the above discussion that:

Work done by gravitational force is independent of the path followed.

Conservative Force

Definition

It is a force for which the work done in moving an object between two points is independent of the path taken.

Examples of Conservation Force:

The gravitational force is a conservation force, other examples of conservative force are electrostatic force and elastic spring force, Magnetic force in certain contexts.

> **For Your Information**
>
> If the work done by a force in moving an object between two points is independent of the path followed or the work done in a closed path be zero, the force is called a conservative force.

5. Define non-conservative force. Also give examples.

Ans. Non-Conservative Force

A force is non-conservative if the work done by it in moving an object between two points or in a closed path depends on the path of motion.

Examples of Non-Conservative Force

(i) The kinetic frictional force is a non-conservative force. When an object slides over a surface, the kinetic frictional force always acts opposite to the motion and does negative work equal in magnitude to the frictional force multiplied by the length of the path. Thus, greater amount of work is done over a longer path between any two points. Hence, the work depends on the choice of path. Moreover, the total work done by a non- conservative force in a closed path is not zero.

(ii) Other examples of non-conservative force are air resistance, tension in a string, normal force and propulsion force of a rocket, applied forces by humans or machines in some cases.

6. Prove that the work done along a closed path by the gravitational force is zero.

Ans. Consider the closed path ADBA or ABDA; Fig. 10. Now total work done along this closed path is equal to the sum of the work done from point A to D, from point D to B and finally from point B to A.

$$W_{Total} = W_{ACBDA} = W_{AD} + W_{DB} + W_{BC} + W_{CA}$$

$$= \vec{F}.\vec{d} + \vec{F}.\vec{h} + \vec{F}.\vec{d} + \vec{F}.\vec{h}$$

$$= Fd\cos 90^\circ + Fh\cos 180^\circ + Fd\cos 90^\circ + Fh\cos 0^\circ$$

$$= 0 - mgh + 0 + mgh = 0$$

$$W_{Total} = W_{ACBDA} = 0$$

Thus, work done along a closed path by the gravitational force is zero.

MCQ's

1. **Work has dimensions like:**
(A) Torque✓ (B) Momentum
(C) Velocity (D) Power

2. **If 50kg crate is pushed through 2*m* across the floor with a force of 50 *N*, the work done will be.**
(A) 245*J* (B) 150 *J*
(C) 200*J* (D) 100 *J*✓

3. **Work done is maximum when angle between force and displacement is:**
(A) 90° (B) 0°✓
(C) 180° (D) 45°

4. **The work is zero if the angle between force and displacement is:**
(A) 45° (B) 90°✓
(C) 0° (D) 180°

5. **The dimensions of work are:**
(A) $[MLT^{-1}]$ (B) $[MLT^{-2}]$
(C) $[ML^2T^{-2}]$✓ (D) $[MLT]$

6. **Work is negative when angle between $\vec{F}$ and $\vec{d}$ is:**
(A) 0° (B) 90°
(C) 45° (D) 180°✓

7. **When a force $\vec{F}$ makes angle 60° with displacement $\vec{d}$, then work done on the body will be:**
(A) Fd (B) $\frac{Fd}{2}$✓
(C) 0.866 Fd (D) $\frac{Fd}{3}$

8. **The SI unit of work is:**
(A) newton (B) watt
(C) pascal (D) joule✓

9. **Area under force-displacement graph gives:**
(A) Velocity (B) Power
(C) Work done✓ (D) Acceleration

10. **When the finite force is parallel to the direction of motion of the body, the work done is:**
(A) Minimum (B) Maximum✓
(C) Infinity (D) Varies

11. **The work done is said to be negative if:**
(A) Work is always positive (B) $\theta < 90°$
(C) $\theta > 90°$✓ (D) $\theta = 90°$

12. **Which of the following statements is true?**
(A) Work is a vector quantity
(B) Work can never be negative
(C) Work is always done when a force is applied
(D) Work is a scalar quantity✓

13. **The area under a force versus displacement graph represents:**
(A) Power (B) K.E.
(C) Work done✓ (D) Force

14. **If a force acts in the opposite direction to displacement, the work done is:**
(A) Positive (B) Negative✓
(C) Zero (D) Infinite

15. **The total work done by a gravitational force in a closed path is:**
(A) Maximum (B) Equal to change in K.E.
(C) Zero✓ (D) Negative

16. **Which of the following is true for a non-conservative force?**
(A) Work done is independent of the path
(B) Total mechanical energy is conserved
(C) Work done in a closed path is zero
(D) Work done depends on the path taken✓

17. **The work done by a conservative force results in a change in:**
(A) Total energy
(B) P.E. and K.E. with no loss✓
(C) K.E. only (D) P.E. only

18. **The work done by a gravitational force when an object is dropped from a height is converted into:**
(A) Heat energy (B) Electrical energy
(C) P.E. (D) K.E.✓

4.4 POWER

Q 7. Define and explain the term power.

Ans. Power

Definition

The rate of doing work is called power.

Explanation

If work ΔW is done in a time interval Δt, then the average power P_{av} during the interval Δt is given as:

$$P_{av} = \frac{\Delta W}{\Delta t}$$

If work is expressed as a function of time, the instantaneous power P at any instant is given as:

$$P = \underset{\Delta t \to 0}{\text{Limit}} \frac{\Delta W}{\Delta t}$$

where ΔW is the work done in short interval of time Δt.

Since $\quad \Delta W = \mathbf{F}.\Delta \mathbf{d}$

Hence $\quad P = \dfrac{\mathbf{F}.\Delta \mathbf{d}}{\Delta t} = \mathbf{F}.\dfrac{\Delta \mathbf{d}}{\Delta t}$

Since $\quad \dfrac{\Delta \mathbf{d}}{\Delta t} = \mathbf{v}$

Hence $\quad P = \mathbf{F}.\mathbf{v}$

Unit: The SI unit of power is watt (W).

Watt: If one joule of work is done in one second, then power will be equal to one watt.

Commercial Unit of Electrical Energy

A commercial unit of electrical energy is kilowatt-hour.

Kilowatt hour: It is the work done in one hour by an agency whose power is one kilowatt.

Therefore $\quad 1\ \text{kW h} = 1000\ \text{W} \times 3600\ \text{s}$

or $\quad 1\ \text{kW h} = 3.6 \times 10^6\ \text{J} = 3.6\ \text{M J}$

For You Information
Approximate Powers

Device	Power (W)
Jumbo Jet Aircraft	1.3×10^8
Car at 90 km h^{-1}	1.1×10^5
Electric heater	2×10^3
Coloured TV	120
Flash light (two cells)	1.5
Pocket calculator	7.5×10^{-4}

Example 4.2: A 70 kg man runs up a long flight of stairs in 4.0 s. The vertical height of the stairs is 4.5 m. Calculate his power output in watts.

Solution:

Given that;

$m = 70\ \text{kg}\ ,\ t = 4.0\ \text{s}\ ,\ h = 4.5\ \text{m}$

To Find:

$P = ?$

Calculations:

Work done $= mgh$

Power $= \dfrac{mgh}{t}$

Putting the values

$$P = \frac{70\ \text{kg} \times 9.8\ \text{ms}^{-2} \times 4.5\ \text{m}}{4\ \text{s}}$$

$$P = 7.7 \times 10^2\ \text{kg m}^2\ \text{s}^{-3} = 7.7 \times 10^2\ \text{W}\ \ \textbf{Ans.}$$

Do you know?
It takes about 9×10^9 J of energy to make a car and the car then uses about 1×10^{12} J of energy from petrol in its life time.

4.5 ENERGY

Q8. Define K.E. and P.E. Derive an expression for K.E.

Ans. Potential Energy

Definition

The energy of a body due to its position or height is known as potential energy.

Potential Energy = P.E. = mgh

Kinetic Energy

Definition

Kinetic energy is the energy possessed by a body due to its motion.

$$\text{Kinetic Energy} = \text{K.E.} = \frac{1}{2}mv^2$$

Derivation of Expression for K.E.

Consider a car running with a constant speed on a road. If its engine is switched OFF, it will still cover some distance before stopping. As long as it is moving, it is doing work against the force of friction of the road. In other words, during this interval, it will exert a force equal in magnitude to the force of friction f.

Let the distance travelled before coming to rest be d, then the work done by the car would be $f\,d$. This work is done by the car due to its motion. The ability of a body to do work due to its motion is its kinetic energy. Therefore, kinetic energy of the car is equal to $f\,d$. The acceleration can be found by using Newton's second law of motion, i.e.,

$$F = ma$$

As the car slows down and finally stops, its acceleration **a** is negative because it is produced by force of friction f acting apposite to the direction of motion. Thus,

$$f = -ma \qquad \text{or} \qquad a = -\frac{f}{m}$$

We can now determine the value of ($f\,d$) by using the third equation of motion, i.e.,

$$2aS = v_f^2 - v_i^2 \qquad \text{...... (i)}$$

Initial velocity $v_i = v$

Final velocity $v_f = 0$

Distance $S = d$

Acceleration $a = -\dfrac{f}{m}$

Putting the values in Eq. (i)

$$2 \times \left(\frac{-f}{m}\right) d = 0 - v^2$$

$$f\,d = \frac{1}{2} mv^2$$

As $f\,d$ is equal to the kinetic energy of the body, therefore,

$$\text{Kinetic energy} = \text{K.E.} = \frac{1}{2} mv^2$$

Unit: Since, kinetic energy is equal to work which the body is capable of doing, so the unit of kinetic energy must be that of work, *i.e.*, joule (J).

Example 4.3: A car weighing 18620 N is running with a speed of 16 m s^{-1}. Brakes are applied and it is brought to rest in a distance of 80 m. Determine the average force of friction acting on it.

Solution:

Given that;

v = 16 m s^{-1}, d = 80 m, w = 18620 N

To Find:

f = ?

Calculations:

The kinetic energy of the car is equal to the work done by it before stopping, i.e.,

$$\frac{1}{2} mv^2 = f\,d$$

Here $m = \dfrac{w}{g} = \dfrac{18620\ \text{N}}{9.8\ \text{m s}^{-2}} = 1900\ \text{kg}$

Putting the value in the above equation:

$$\frac{1}{2} \times 1900\ \text{kg} \times (16\ \text{m s}^{-1})^2 = f \times 80$$

or $f = 3040$ N **Ans.**

For You Information

Approximate Energy Values

Source	Energy (J)
Burning 1 ton coal	30×10^9
Burning 1 litre petrol	5×10^7
K.E. of a car at 90 km h^{-1}	1×10^6
Running Person at 10 km h^{-1}	3×10^2
Fission of one atom of uranium	1.8×10^{11}
K.E. of a molecule of air	6×10^{21}

9. What is meant by absolute potential energy? Derive an expression for absolute P.E.

Ans. Absolute Potential Energy

Definition

The absolute gravitational potential energy of an object at a certain position is the work done by the gravitational force in displacing the object from that position to infinity where the force of gravity becomes zero.

Expression for Absolute P.E.

The relation for the calculation of the work done by the gravitational force or potential energy is *mgh*, which is true only near the surface of the Earth where the gravitational force is nearly constant. But if the body is displaced through a large distance in space, let it be from point 1 to N; Fig. 11 in the gravitational field, then the gravitational force will not remain constant, since it varies inversely to the square of the distance.

In order to overcome this difficulty, we divide the distance between points 1 and N into small steps each of length Δr so that the value of the force remains constant for each small step. Hence, the total work done can be calculated by adding the work done during all these steps. If r_1 and r_2 re the distances of points 1 and 2 respectively, from the centre O of the Earth; Fig. 11, the work done during the first step i.e., displacing a body from point 1 to point 2 can be calculated as below. The distance between the centre of this step and centre of the Earth will be:

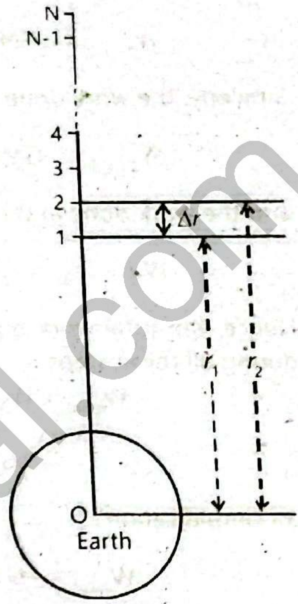


Fig. 11

$$r = \frac{r_1 + r_2}{2}$$

As $\quad r_2 - r_1 = \Delta r \quad$ then $\quad r_2 = r_1 + \Delta r$

Hence $$r = \frac{r_1 + r_1 + \Delta r}{2} = r_1 + \frac{\Delta r}{2} \qquad \text{...... (i)}$$

The gravitational force F at the centre of this step is:

$$F = G\frac{Mm}{r^2} \qquad \text{...... (ii)}$$

where $\quad m$ = mass of an object , M = mass of the Earth

and $\quad G$ = Gravitational constant

Squaring Eq. (i)

$$r^2 = \left(r_1 + \frac{\Delta r}{2}\right)^2$$

$$r^2 = r_1^2 + 2r_1\frac{\Delta r}{2} + \left(\frac{\Delta r}{2}\right)^2$$

As $(\Delta r)^2 << r_1^2$, so $(\Delta r)^2$ can be neglected as compared to r_1^2.

Hence $\quad r^2 = r_1^2 + r_1 \Delta r$

Putting the value of $\Delta r = r_2 - r_1$

$$r^2 = r_1^2 + r_1(r_2 - r_1) = r_1 r_2$$

Hence, Eq. (ii) becomes:

$$F = G\frac{Mm}{r_1 r_2}$$

As this force is assumed to be constant during the interval Δr, so the work done is:

$$W_{1\to 2} = \mathbf{F}.\mathbf{\Delta r} = F\Delta r \cos 180° = -GMm\frac{\Delta r}{r_1 r_2}$$

The negative sign indicates that the work has to be done on the body from point 1 to 2 because displacement is opposite to gravitational force. Putting the value of Δr, we have

$$W_{1\rightarrow 2} = -GMm\frac{r_2 - r_1}{r_1 r_2}$$

or $$W_{1\rightarrow 2} = -GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

Similarly, the work done during the second step in which the body is displaced from point 2 to 3 is:

$$W_{2\rightarrow 3} = -GMm\left(\frac{1}{r_2} - \frac{1}{r_3}\right)$$

and the work done in the last step is:

$$W_{N-1\rightarrow N} = -GMm\left(\frac{1}{r_{N-1}} - \frac{1}{r_N}\right)$$

Hence, the total work done in displacing a body from point 1 to N is calculated by adding up the work done during all these steps.

$$W_{total} = W_{1\rightarrow 2} + W_{2\rightarrow 3} + \ldots\ldots\ldots + W_{N-1\rightarrow N}$$

$$= -G\,Mm\left[\left(\frac{1}{r_1} - \frac{1}{r_2}\right) + \left(\frac{1}{r_2} - \frac{1}{r_3}\right) + \ldots\ldots\ldots + \left(\frac{1}{r_{N-1}} - \frac{1}{r_N}\right)\right]$$

On simplification:

$$W_{total} = -G\,Mm\left(\frac{1}{r_1} - \frac{1}{r_N}\right)$$

If the point N is situated at an infinite distance from the Earth, then

$$r_N = \infty \qquad \text{so} \qquad \frac{1}{r_N} = \frac{1}{\infty} = 0$$

Hence $$W_{total} = -\frac{GMm}{r_1}$$

This total work by definition is the absolute potential energy (P.E) as stated earlier represented by U.

$$U = -\frac{GMm}{r}$$

This is also known as the absolute value of gravitational potential energy of a body at a distance r from the centre of the Earth.

Note that when r increases, U becomes less negative i.e., U increases. It means when we raise a body above the surface of the Earth, its P.E. increases. Therefore, if we want to raise the body up to infinite distance, we will have to do work on it equal to $\frac{GMm}{R}$, so that its P.E. becomes zero.

Now the absolute potential energy on the surface of the Earth is found by putting $r = R$ (Radius of the Earth), so

$$\text{Absolute potential energy} = U_g = -\frac{GMm}{R}$$

The negative sign shows that the Earth's gravitational field for mass m is attractive.

The above expression gives the work or the energy required to take the body out of the Earth's gravitational field, where its potential energy with respect to Earth is zero.

MCQs

1. The ratio of dimensions of K.E and Power is:

(A) 1 : 1 (B) [T] : 1✓

(C) 1 : [T] (D) [M] : [T]

2. The dimensions of power are:

(A) $[ML^2T^{-1}]$ (B) $[ML^2T^{-2}]$

(C) $[ML^2T^{-3}]$✓ (D) $[ML^2T^0]$

3. The power is equal to dot product of force and:

(A) Displacement (B) Acceleration

(C) Velocity✓ (D) Position vector

4. If 6 joule of work is done in 3 seconds, then power is:

(A) 6 watt (B) 3 watt

(C) 18 watt (D) 2 watt✓

5. **Which one is non-conservative force?**
(A) Electric force (B) Elastic force
(C) Gravitational force (D) Frictional force ✓

6. **Which one is a conservative force?**
(A) Elastic spring force (B) Frictional force
(C) Air resistance (D) Tension in the string ✓

7. **Which of the following is the example of conservative force?**
(A) Resorting force in compressed spring
(B) Tension in string
(C) Propulsion force of rocket
(D) Gravitational field ✓

8. **The Dimensions of work are:**
(A) $[ML^2T^{-2}]$ (B) $[ML^0T^{-1}]$
(C) $[M^0L^0T^{-2}]$ ✓ (D) $[ML^0T^{-1}]$

9. **The power of an electric heater is (approximately):**
(A) 1*kW* (B) 2*kW*
(C) 3*kW* (D) 4*kW* ✓

10. **One watt-hour is equal to:**
(A) 3.6 *MJ* (B) 3.6 *kJ* ✓
(C) 36 *kJ* (D) 36 *MJ*

11. **One kilowatt-hour (kWh) is equal to:**
(A) 3.6 *MJ* ✓ (B) 3.6 *kJ*
(C) 36 *MJ* (D) 3.6 *J*

12. **Power is also defined as:**
(A) $\vec{F}.\vec{m}$ (B) $\vec{F}.\vec{d}$
(C) $\vec{F}.\vec{v}$ ✓ (D) $\vec{F}.\vec{t}$

13. **One kilowatt is equal to:**
(A) 1000 J s^{-1} ✓ (B) 10^6 watt
(C) 10^6 Js^{-1} (D) 100 Js^{-1}

14. **The unit of electrical energy commonly used in household is:**
(A) watt (B) watt-hour ✓
(C) kilowatt-hour (D) joule

15. **Which of the following is a correct example of a unit of power?**
(A) watt (B) Js^{-1}
(C) horsepower (D) all of these ✓

16. **Which of the following appliances uses the most power?**
(A) 100 W bulb (B) 200 W iron
(C) 1500 W microwave oven ✓
(D) 500 W heater

17. **A machine with higher power rating means:**
(A) It consumes less power
(B) It works more slowly
(C) It does more work in less time ✓
(D) It cannot be used continuously

18. **A machine does 500 J work in 10 s. What is its power output?**
(A) 5 W (B) 50 W ✓
(C) 5000 W (D) 0.02 W

19. **What is horsepower (hp) approximately equal to:**
(A) 100 W (B) 746 W ✓
(C) 1000 W (D) 500 W

4.6 ESCAPE VELOCITY

Q10. Define escape velocity. Show that an expression for escape velocity can be expressed as $\sqrt{2gR}$.

Ans. Escape Velocity

Definition

The initial velocity of an object with which it goes out of the Earth's gravitational field, is known as escape velocity.

Expression for Escape Velocity

The escape velocity corresponds to the initial kinetic energy gained by the body, which carries it to an infinite distance from the surface of the Earth.

$$\text{Initial K.E.} = \frac{1}{2} m v_{esc}^2$$

We know that the work done in lifting a body from Earth's surface to an infinite distance is equal to the increase in potential energy.

$$\text{Increase in P.E.} = 0 - \left(-G\frac{Mm}{R}\right) = G\frac{Mm}{R}$$

where *M* and *R* are the mass and radius of the Earth respectively. The body will escape out of the gravitational field if the initial K.E. of the body is equal to increase in P.E. Then

$$\frac{1}{2} m v_{esc}^2 = G\frac{Mm}{R}$$

or

$$v_{esc} = \sqrt{\frac{2GM}{R}}$$

For You Information

Some Escape speeds (km s^{-1})

Heavenly body	Escape speed
Moon	2.4
Mercury	4.3
Mars	5.0
Venus	10.4
Earth	11.2
Uranus	22.4
Neptune	25.4
Saturn	37.0
Jupiter	61

As $g = \frac{GM}{R^2}$ or $gR = \frac{GM}{R}$

Hence $v_{esc} = \sqrt{2gR}$

The value of v_{esc} comes out to be approximately 11 km s^{-1}.

4.7 WORK-ENERGY THEOREM

11. State and explain work-energy theorem in a resistive medium.

Ans. Work Energy Theorem

Statement

The network done by all the forces acting on an object is equal to the change in its kinetic energy.

Explanation

If a force F acts on a body of mass m, initially moving with velocity v_i, through a distance d and increases its velocity to v_f, then the acceleration produced will be:

$$2ad = v_f^2 - v_i^2$$

or $$a = \frac{1}{2d}(v_f^2 - v_i^2) \quad \text{...... (i)}$$

From the second law of motion:

$$F = ma$$

or $$a = \frac{F}{m} \quad \text{...... (ii)}$$

Tidbits

All the food you eat in one day has about the same energy as 1/3 litre of petrol.

Comparing Eqs. (i) and (ii):

$$\frac{F}{m} = \frac{1}{2d}(v_f^2 - v_i^2)$$

or $$Fd = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

This expression is the work-energy theorem. It states that:

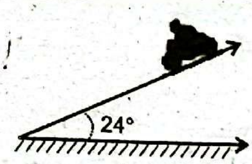
The change in kinetic energy of an object is equal to the work done on it by a net force.

$$W = \text{Change in kinetic energy} = (K.E)_f - (K.E)_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

The work-energy theorem is applicable in a wide range of physical situations. For example;

(i) for any system with varying velocity
(ii) for both constant and variable forces
(iii) for linear and Curvilinear motion
(iv) for friction and non-conservative forces
(v) for rotational motion
(vi) for particle system and rigid bodies.

Example 4.4: A motorcycle rider weighing 60 kg is coasting down a 24° slope. The weight of motorcycle is 30 kg. At the top of the slope, the speed of motorcycle is 3.2 m s^{-1}. If the kinetic frictional force is 100 N, what will be the speed of the motorcycle 72 m downhill?



Solution:

Given that;

$m = 60$ kg , $\theta = 24°$, $v = 3.2$ ms^{-1} , $m = 30$ kg

To Find:

$v_f = ?$

Calculations:

The normal force F_N is balanced by the component of weight (mg cos 24°) perpendicular to the slope, Let the kinetic frictional force is f, then the net force F is:

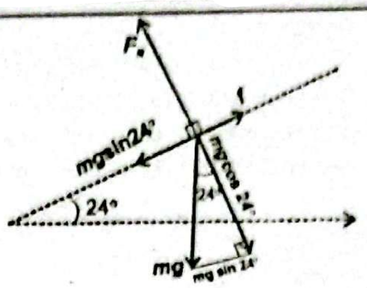
$F = mg \sin 24° - f$ where m = total mass = 60 kg + 30 kg = 90 kg or $F = (90\text{ kg} \times 9.8\text{ ms}^{-2} \times 0.4) - 100\text{ N}$

$F = 252.8\text{ N}$

Work done $W = Fd = 252.8\text{ N} \times 72\text{ m} = 18201.6\text{ J}$

As work is positive, so applying work – energy theorem,

$$W = (K.E.)_f - (K.E.)_i$$



Form here,

$$(K.E.)_f = W + (K.E.)_i$$

Putting the values, we have

$$\frac{1}{2}mv_f^2 = W + \frac{1}{2}mv_i^2$$

$$\frac{1}{2} \times 90\text{ kg} \times v_f^2 = 18201\text{ J} + \frac{1}{2} \times 90\text{ kg} \times (3.2\text{ m s}^{-1})^2$$

This gives, $v_f^2 = 414.7\text{ m}^2\text{ s}^{-2}$

or $v_f = \sqrt{414.7\text{ m}^2\text{ s}^{-2}} = 20.4\text{ m s}^{-1}$ **Ans.**

4.8 INTER-CONVERSION OF POTENTIAL ENERGY AND KINETIC ENERGY

12. Explain, how do P.E. and K.E. are inter-convertable.

Ans. Consider a body of mass m at rest, at a height h above the surface of the Earth, Fig. 12. At position A, the body has $P.E. = mgh$ and $K.E. = 0$. We release the body and as it falls, we can examine how kinetic and potential energies associated with it interchange.

Let us calculate $P.E.$ and $K.E.$ at the position B when the body has fallen a distance x, ignoring air friction.

$$P.E. = mg(h - x)$$

and
$$K.E. = \frac{1}{2}mv_B^2$$

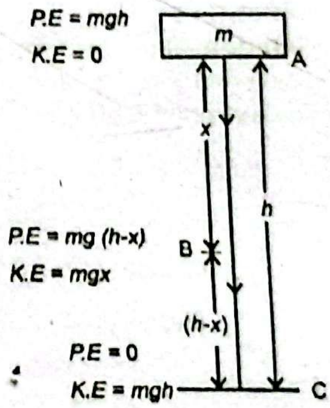


Fig. 12

Velocity v_B, at position B, can be calculated from the relation,

$$v_f^2 = v_i^2 + 2gS$$

As $v_f = v_B$, $v_i = 0$, $S = x$

$$v_B^2 = 0 + 2gx$$

$$v_B^2 = 2gx$$

Therefore
$$K.E. = \frac{1}{2}m(2gx)$$
$$= mgx$$

Total energy at position B = P.E. + K.E.

$$\text{Total energy} = mg(h - x) + mgx = mgh$$

At position C, just before the body strikes the Earth, $P.E. = 0$ and $K.E. = \frac{1}{2}mv_C^2$, where v_C can be found out by the following expression.

$$v_C^2 = v_i^2 + 2gh = 2gh \quad \text{as} \quad v_i = 0$$

i.e.,
$$K.E. = \frac{1}{2}mv_C^2 = \frac{1}{2}m \times 2gh = mgh$$

Thus, at point C, kinetic energy is equal to the original value of the potential energy of the body. Actually, when a body falls, its velocity increases i.e., the body is being accelerated under the action of gravity. The increase in velocity results in the increase in its kinetic energy. On the other hand, as the body falls, its height decreases and hence, its potential energy also decreases. Thus,

Loss in *P.E.* = Gain in *K.E*

$$mg\,(h_1 - h_2) = \frac{1}{2} m\,(v_2^2 - v_1^2)$$

where v_1 and v_2 are he velocities of the body at the heights h_1 and h_2 respectively. This result is true only when frictional force is not considered.

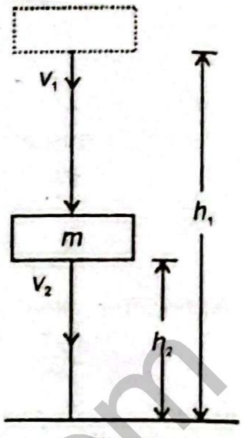


Fig. 13

If we assume that a frictional force *f* is present during the downward motion, then a part of P.E. is used in doing work against friction equal to *f h*. The remaining *P.E.* = *mgh* – *f h* is converted into K.E.

Hence $$mgh - f\,h = \frac{1}{2} mv^2$$

or $$mgh = \frac{1}{2} mv^2 + f\,h$$

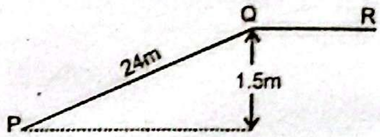
Thus Loss in P.E. = Gain in K.E. + Work done against friction

Conversely,

Loss of K.E. = Gain in P.E. + Work done against friction

Example 4.5: A car weighing 1100 kg is moving with a velocity of 12 m s^{-1}. When it is at point P, its engine stops. If the frictional force is 120 N, what will be its velocity at point Q? How far beyond Q will it go before coming to rest?

Solution: The kinetic energy possessed by the car at point P will partly be converted into P.E. and partly used up in doing work against friction as it reaches point Q. Therefore,



Loss of K.E. = Gain in P.E. + Work against friction

$$\frac{1}{2} m\,(v_i^2 - v_f^2) = wh + fd$$

$$\times\ 1100\text{ kg}\,(144\text{ m}^2\text{ s}^{-2} - v_f^2) = (1100\text{ kg} \times 9.8\text{ m s}^{-2} \times 1.5\text{ m}) + 120\text{ N} \times 24\text{ m}$$

$$550\text{ kg}\,(144\text{ m}^2\text{ s}^{-2} - v_f^2) = 16170\text{ kg m}^2\text{ s}^{-2} + 2880\text{ kg m}^2\text{ s}^{-2}$$

$$(144\text{ m}^2\text{ s}^{-2} - v_f^2) = \frac{16170\text{ kg m}^2\text{ s}^{-2} + 2880\text{ kg m}^2\text{ s}^{-2}}{550\text{ kg}} = 34.6\text{ m}^2\text{ s}^{-2}$$

$$v_f^2 = 144\text{ m}^2\text{ s}^{-2} - 34.6\text{ m}^2\text{ s}^{-2} = 109.4\text{ m}^2\text{s}^{-2}$$

Velocity at point Q, $v_f = \sqrt{109.4\text{ m}^2\text{ s}^{-2}} = 10.5\text{ m s}^{-1}$

Now if the car stops at point R, then using the formula:

$$\frac{1}{2} mv^2 =, fS$$

$$\frac{1}{2} \times 1100\text{ kg} \times 109.4\text{ m}^2\text{ s}^{-2} = 120\text{ kg m s}^{-2} \times S$$

S = 501 m approximately **Ans.**

Example 4.6: An object of mass 3 kg falls from a height of 15 m. If it strikes the ground with a velocity of 16 m s^{-1}, calculate the average frictional force of the air.

Solution:

Given that;

$m = 3$ kg , $h = 15$ m , $v = 16$ ms^{-1}

To find:

$f = ?$

Calculations:

Loss of P.E = Gain in K.E. + Work done against friction

$\because v_i = 0,\ mgh = \frac{1}{2}mv^2 + f\,h$

$3\text{ kg} \times 9.8\text{ m s}^{-2} \times 15\text{ m} = \frac{1}{2} \times 3\text{ kg }(16\text{ m s}^{-1})^2 + f \times 15\text{ m}$

$441\text{ kg m}^2\text{ s}^{-2} = 384\text{ kg m}^2\text{ s}^{-2} + 15\text{ m} \times f$

or $f = \frac{441\text{ kg m}^2\text{ s}^{-2} - 284\text{ kg m}^2\text{ s}^{-2}}{15\text{ m}} = 3.8\text{ kg ms}^{-2} = 3.8\text{ N}$ **Ans.**

MCQ's

1. **The value of escape velocity is maximum for:**
(A) Moon (B) Earth
(C) Jupiter✓ (D) Mercury

2. **Which heavenly body had maximum escape velocity:**
(A) Moon (B) Mercury
(C) Mars✓ (D) Earth

3. **Escape velocity of object depends upon:**
(A) Mass of object (B) Size of object
(C) Shape of object (D) Radius of planet✓

4. **Absolute P.E of an object at an infinite height w.r.t Earth is taken as:**
(A) Zero✓ (B) Negative
(C) Minimum (D) Maximum

5. **If velocity and mass of a moving object are doubled, then K.E becomes:**
(A) Double (B) 4 times
(C) 6 times^2 (D) 8 times✓

6. **In Earth's gravitational field, work done in a closed path is:**
(A) Maximum (B) Positive
(C) Negative (D) Zero✓

7. **In work-Energy principle work done on a body is equal to:**
(A) Kinetic energy (B) Potential energy
(C) Plastic potential energy (D) Change in energy✓

8. **Gravity performs zero work when body moves:**
(A) Vertically (B) Horizontally✓
(C) at 60° with vertical (D) at 60° with horizontal

9. **The K.E of an object of mass "m" is "E", its momentum will be:**
(A) 2Em (B) $\sqrt{\frac{2E}{m}}$
(C) $\sqrt{\frac{1}{2}Em}$✓ (D) $\sqrt{2\,mE}$

10. **Escape velocity of an object is independent of:**
(A) Mass of the object✓ (B) Mass of planet
(C) Radius of planet (D) Type of planet

11. **K.E of a body with constant mass 'm' has the relation with momentum:**
(A) $K.E \propto p$ (B) $K.E \propto p^2$✓
(C) $K.E \propto \frac{1}{p}$ (D) $K.E \propto \frac{1}{p^2}$

12. **The value of 'g' at the centre of the Earth is:**
(A) Infinite (B) 2g
(C) 3g (D) Zero✓

13. **Pull of the Earth on 20 kg body on surface of earth is:**
(A) 20 N (B) 196 N
(C) 19.6 N✓ (D) 1960 N

14. **Escape velocity is independent of:**
(A) Mass of the planet
(B) Radius of the planet
(C) Mass of the escaping object✓
(D) Gravitational constant

15. **What happens if an object is launched at exactly escape velocity?**
(A) It escapes the planet's gravity and slows to a stop at infinity✓
(B) It orbits the planet
(C) It returns to the surface
(D) It falls into the planet's core

16. **If a spacecraft is launched with a velocity greater than escape velocity, it will:**
(A) Return to the Earth
(B) Enter a circular orbit
(C) Move into an elliptical orbit
(D) Leave the gravitational influence of the Earth permanently✓

17. **According to work-energy theorem, if no net work is done on a system, then:**
(A) The P.E. changes
(B) The K.E. remains constant✓
(C) The mechanical energy increases
(D) The system must be at rest

18. **The work-energy theorem is applicable:**
(A) Only for conservative forces
(B) Only when friction is absent
(C) For all types of forces✓
(D) Only in vertical motion

19. **If the reference level is taken at ground level, the absolute P.E. of a body at height h is:**
(A) Zero (B) mgh✓
(C) Negative (D) Constant

20. **Which of the following effects the absolute P.E. of an object?**
(A) Its speed
(B) Its height from reference level✓
(C) Its temperature
(D) Its density

21. **The absolute P.E. of an object becomes negative, when:**
(A) It is moving downward
(B) It is an a frictionless surface
(C) It is at the ground level
(D) It is below the chosen reference level✓

22. **What happens to the absolute P.E. as an object moves higher above the Earth's surface?**
(A) It decreases (B) It stays the same
(C) It increases✓ (D) It becomes zero.

23. **If the reference level is changed, the absolute P.E. of a body:**
(A) Remains the same (B) Increases exponentially
(C) Changes✓ (D) Becomes negative

24. **A ball is thrown upward. As it rises, its:**
(A) K.E. increases, P.E. decreases
(B) K.E. decreases, P.E. increases✓
(C) Both K.E. and P.E. increases
(D) Both K.E. and P.E. remain constant

25. **During free fall, the loss in P.E. is equal to:**
(A) Loss in K.E. (B) Gain in K.E.✓
(C) Total energy of the body
(D) Work done by friction

ADDITIONAL SHORT ANSWER QUESTIONS

SQ

1. What is the work done by gravity when an object falls vertically downward?
Ans. Positive work, since force (gravity) and displacement are in the same direction.

Q.2 What is meant by a variable force?
Ans. A variable force is a force that changes in magnitude or direction over time or distance.

Q.3 Can the formula W = F.d be used for variable force?
Ans. No. This formula is is only valid for constant force. For variable force, work is found by using calculus or graphical methods.

Q.4 How can we find work done by a variable force graphically?
Ans. By calculating the area under the force versus displacement graph.

Q.5 Is work done by a variable force always positive?
Ans. No, it can be positive, negative, or zero depending on the direction of force relative to displacement.

Q.6 What is a conservative force? Give two examples.
Ans. Conservation Force
A conservative force is a force for which the work done is independent of the path taken and depends only on the initial and final positions.
Example: Gravitational force, electrostatic force.

Q.7 What is non-conservative force? Give two examples.
Ans. Non-Conservative Force
A non-conservative force is a force for which the work done depends on the path taken.
Examples: Friction and air resistance.

Q.8 Can work done by a conservative force around a closed path be non-zero?
Ans. No, the work done by a conservative force in a closed path is zero.

Q.9 Can work done by a non-conservation force around a closed path be zero?
Ans. No, it is not zero; energy is lost or gained depending on the path.

Q.10 What is work done by gravitational force?
Ans. It is the work done when an object is moved in the gravitational field of the Earth.

Q.11 Does the path taken affect the work done by gravity?
Ans. No, it depends only on the vertical displacement.

Q.12 What happens to the work done by gravity if height increases?
Ans. Work done increases proportionally with height.

Q.13 What is average power and instantaneous power?

Ans. Average power is the total work done divided by the total time taken. Whereas instantaneous power is the power at a specific moment.

Q.14 On What factors does G.P.E. depend?

Ans. It depends on mass, gravitational acceleration, and height.

Q.15 What is absolute P.E.?

Ans. It is the total P.E. of a body measured from a reference point, usually taken as zero at infinity or ground level.

Q.16 What is the most common reference point for absolute P.E.?

Ans. The ground or infinity is commonly taken as the reference point where P.E. is considered zero.

Q.17 Is absolute P.E. always positive?

Ans. No, it can be positive or negative depending on the choice of the reference point.

Q.18 How is absolute P.E. different from relative P.E.?

Ans. Absolute P.E. is measured from a fixed zero level, while relative P.E. is measured between two points.

Q.19 Can absolute P.E. be negative?

Ans. Yes, if the object is below the reference level, the P.E. is negative.

Q.20 Does escape velocity depend on the mass of the object?

Ans. No, it depends only on the mass and radius of the planet or celestial body.

Q.21 What is the escape velocity?

Ans. It is the minimum velocity needed for an object to escape the gravitational pull of a planet or celestial body without returning.

The escape velocity from the surface of the Earth is approximately 11.2 kms^{-1} or 11,200 ms^{-1}.

Q.22 How is escape velocity related to orbital velocity?

Ans. Escape velocity is $\sqrt{2}$ times the orbital velocity for a circular orbit at the same altitude.

Q.23 Is the work-energy theorem valid for variable forces?

Ans. Yes, it is valid for both constant and variable forces.

Q.24 Can the work done be negative in the work-energy theorem?

Ans. Yes, if the force acts opposite to the motion, the work is negative and K.E. decreases.

Q.25 What is the significance of work-energy theorem?

Ans. It links force, displacement and energy, and simplifies solving problems involving motion.

Q.26 Can the work-energy theorem be applied to a falling object?

Ans. Yes, the gravitational force does work on the object, increasing its K.E.

Q.27 Give an example of interconversion of P.E. and K.E.

Ans. When a ball is dropped from a height, its P.E. converts to K.E. as it falls.

Q.28 Can K.E. be converted back into P.E.?

Ans. Yes, for example, when a roller coaster climbs a hill, its K.E. changes into P.E.

Q.29 Does friction affect the interconversion of energy?

Ans. Yes, friction converts some mechanical energy into heat, reducing total usable mechanical energy.

Q.30 Can energy be lost during interconversion?

Ans. Ideally, no but in real-life situations, some energy may be lost as heat, sound, or air resistance.

Q.31 A car is moving along a circle of radius r. It completes four revolutions and terminates its journey at starting point. How much work is done by the car? Explain.

Ans. Work done is given by

$$W = \vec{F} \cdot \vec{d}$$

Since total displacement is zero, because the car completes four revolutions and terminating and starting points are coincided.

So, $\vec{d} = 0$

Hence, $W = \vec{F} . (0)$

$\boxed{W = 0}$

So, total work done by the car is zero.

Q.32 What will be the velocity of the particle if its momentum and kinetic energy are equal in magnitude?

Ans. Let v be the velocity of the particle.

Momentum of the particle = p = mv

K.E. of the particle = k.E. = $\frac{1}{2}mv^2$

Conditions: K.E. = P

or $\frac{1}{2}mv^2 = mv$

or $\frac{v^2}{v} = \frac{2m}{m}$

or v = 2

So $\boxed{v = 2\ ms^{-1}}$

Q.33 Convert 1.4kW into joule per second.

Ans. $1.4\ kW = 1.4 \times 10^3\ W$ $(\because\ k = 10^3)$

$\boxed{1.4\ kW = 1400\ Js^{-1}}$ $(\because\ W = J\ s^{-1})$

Q.34 Does work done in raising a box on platform depend upon how fast it is raised up? If not why?

Ans. No, work done in raising a box does not depend upon its speed.

Reason: As we known; W = F.d = Fd cos θ ...... (i)

It is clear from Eq. (i) that work done depends on:

(i) Force-acted on the body

(ii) Distance covered by the body

(iii) Angle between force and displacement

Q.35 Point out positions where gravitational potential energy is taken as zero.

Ans. Gravitational potential energy is zero:

(i) When object is on the ground.

(ii) At centre of the Earth

(iii) When separation between two increasing mases is infinity.

Q.36 A person holds a bag of groceries while standing still, talking to a friend. A car is stationary with its engine running. From the stand point of work, how are these two situations similar?

Ans. In both cases, displacement d = 0

Work = F × d = F × 0

$\boxed{Work = 0}$

Hence, in both cases, no work is done.

Q.37 Calculate the work done in kilo joules in lifting a mass of 10 kg (at a steady velocity) through a vertical height of 10 m.

Ans. Here $m = 10\ kg$, $h = 10\ m,$

$g = 9.8\ ms^{-2}$, Work done = $W = ?$

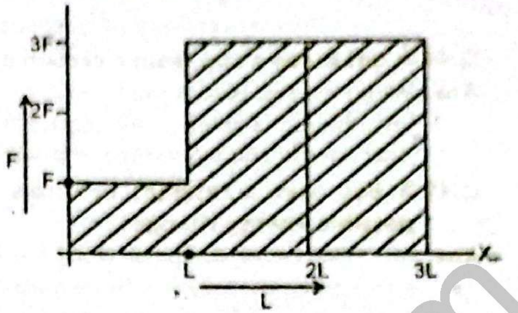
$W = P.E = mgh$

$= 10 \times 9.8 \times 10 = 980\ J$

$= \frac{980}{1000}\ kJ$

$\boxed{W = 0.980\ kJ}$

Q.38 A force F acts through a distance L. The force is then increased to 3 F, and acts through a further distance of 2 L. Draw the work diagram to scale.



Ans. Work done = Area under the curve

$= (F \times L) + (3F \times 2L) = FL + 6\ FL = 7\ FL$

Q.39 In which case is more work done? When a 50 kg bag of books is lifted through 50 cm, or when a 50 kg crate is pushed through 2 m across the floor with a force of 50 N?

Ans. For the bag of books:

Case-1: $m = 50\ kg$

$h = 50\ cm = \frac{50}{100}\ m$

$= 0.5\ m$

Work $= W = mgh$

$= 50 \times 9.8 \times 0.5$

$\boxed{W = 245\ J}$

For the crate:

Case-2: $F = 50\ N$, $d = 2\ m$

Work $= W = F \times d$

$= 50 \times 2$

$\boxed{W = 100\ J}$

Hence, work done on the bag of books is greater than work done on the crate. i.e., more work is done in **case-1**.

Q.40 An object has 1 J of potential energy. Explain, what does it mean?

Ans. An object has a potential energy of 1 joule, when a force of 1 newton is applied to lift the body through a height of 1m.

$F \times h = W$

1 newton × 1 m = 1 joule

This work has been stored in the object and the object has ability of doing 1 joule of work.

Q.41 A ball of mass m is held at a height h_1 above a table. The table top is at a height h_2 above the floor. One student says that the ball has potential energy mgh1 but another says that it is mg (h_1 + h_2). Who is correct?

Ans. Both the students are correct because P.E with respect to table = mgh_1

P.E with respect to floor = $mg(h_1 + h_2)$

One student has chosen table top as the point of reference and the second student has taken the floor as the point of reference.

Q.42 When a rocket re-enters the atmosphere, its nose cone becomes very hot. Where does this heat energy come from?

Ans. When a rocket re-enters the atmosphere, a part of its kinetic energy is converted in to heat energy, in doing work against the frictional forces of the atmosphere and does particles. That is why nose cone of the rocket becomes very hot.

Q.43 What sort of energy is in the following?

(a) Compressed spring

(b) Water in a high dam

(c) A moving car

Ans. (a) Elastic potential energy is stored in a compressed spring.

(b) Gravitational potential energy is stored in water in a high dam.

(c) A moving car has kinetic energy.

Q.44 A girl drops a cup from a certain height, which breaks into pieces. What energy changes are involved?

Ans. When the cup was in the hands of the girl, it had gravitational P.E. When the cup is dropped, its P.E. is converted into the K.E. + work done against friction. On striking the ground, this energy is converted into sound energy, heat energy and work done in breaking the cup into pieces.

Q.45 A boy uses a catapult to throw a stone which accidentally smashes a greenhouse window. List the possible energy changes.

Ans. (i) Before throwing the stone has elastic potential energy due to the pulled catapult.

(ii) When the stone is thrown elastic potential energy of catapult is converted in to kinetic energy of stone + work done against friction.

(iii) When the stone smashes the greenhouse window, a part of kinetic energy of stone is used in smashing the window.

(iv) A part of kinetic energy of stone is converted into heat energy and sound energy.

(v) A part of kinetic energy of stone is converted into the kinetic energy of the broken pieces of glass of the window.

SOLVED EXERCISE

MULTIPLE CHOICE QUESTIONS

Tick (✓) the correct answer.

4.1 A 1 kg mass has potential energy of 1 joule relative to the ground when it is at a height of:
(a) 0.102 m✓ (b) 1 m (c) 9.8 m (d) 32 m

4.2 An iron sphere whose mass is 30 kg has the same diameter as an aluminium sphere whose mass is 10.5 kg. The spheres are simultaneously dropped from a cliff. When they are 10 m from the ground, they have identical:
(a) accelerations✓ (b) momentums (c) potential energies (d) kinetic energies

4.3 A body at rest may have:
(a) speed (b) velocity (c) momentum (d) energy✓

4.4 The height above the ground of a child on a swing varies from 0.5 m of his lowest point to 1.5 m at his highest point. The maximum speed of the child is approximately:
(a) 1.5 m s^{-1} (b) 4.4 m s^{-1}✓ (c) 9.8 m s^{-1} (d) Depends upon child's mass

4.5 When a ball is thrown vertically upward and then falls back to the ground, which force can be considered conservative in this scenario?
(a) Air resistance (b) Gravity✓
(c) Friction between ball and air (d) Contact force with hand

4.6 According to work-energy principle in linear motion, the work done on body is equal to:
(a) change in K.E.✓ (b) change in P.E. (c) zero (d) sum of K.E. and P.E.

4.7 Power of a lamp is 6 W. How much energy does a lamp give out in 2 min?
(a) 12 J (b) 20 J (c) 3 J (d) 720 J✓

4.8 A dry battery can deliver 3000 J of energy to a 2 W small electric motor before the battery is exhausted. For how many minutes does the battery run?
(a) 1500 min (b) 100 min (c) 50 min (d) 25 min✓

4.9 The kinetic energy acquired by a mass m after travelling a fixed distance from rest under the action of a constant force is directly proportional to:
(a) $\sqrt{m}$ (b) $1/\sqrt{m}$ (c) m (d) independent of m✓

4.10 A body moves a distance of 10 m along a straight line under the action of 5 N force. If the work done is 25 J, the angle which the force makes with the direction of motion of the body is:
(a) 0° (b) 30° (c) 60°✓ (d) 90°

SHORT ANSWER QUESTIONS

4.1 Why is electrical power required at all when the elevator is descending? Why should be there a limit on the number of passengers in this case?

Ans. Let us break this question into two parts:

1. Why is electrical power required when the elevator is descending?
Even when an elevator is going down, electrical power is still needed for a few reasons:

(i) Control and safety systems: The elevators motor, brakes and control systems need power to ensure smooth and safe descent.

(ii) Counterweight system: Most elevators use a counterweight that balances the elevator car. When the elevator descends, the motor may actually act as a brake or generator, controlling the descent.
If too much weight is in the car, the system could become imbalanced, and power is needed to manage that.

(iii) Regenerative braking: Some modern elevators are regenerative drives that generate electricity when descending. But even in those systems, control electronics and safety systems still need power.

2. Why should there be a limit on the number of passengers when descending?
Even though you are going down (which might seems easier than going up), limiting the number of passengers is still important.

(i) Weight limits: Elevators are designed to carry a maximum load. Exceeding it stresses the cables, brakes, and motor. On descent, if the elevator is too heavy compared to the counterweight, it might descent too quickly-making it harder to brake safely.

(ii) Braking system: The brakes need to handle the extra K.E. from a heavier load during descent. Overloading can make it harder to stop safely and could lead to dangerous situations.

(iii) System balance: The elevator system is finally balanced. Too much weight throws off the balance, making it harder for the motor and control system to manage the motion, even during descent.

4.2 A body is being raised to a height H from surface of the Earth. What is the sign of work done by both (body and the Earth)? Justify.

Ans. When a body is raised to a height H from the surface of the Earth, two main forces are involved:
- Gravitational force
- Applied force

1. Work done by the applied force:
The applied force acts upward, in the same direction as the displacement of the body.
Since force and displacement are in the same direction, therefore, work done by the applied force is positive.
Justification: $W_{applied} = Fd\cos(0^\circ) = Fd$

2. Work done by gravity:
The gravitational force acts downward, while the displacement is upward.
Since force and displacement are in opposite directions, work done by gravity is negative.
Justification: $W_{gravity} = mg\,H\cos(180^\circ) = -mg\,H$

4.3 A body falls towards the Earth in air. Will its total mechanical energy be conserved during fall? Justify.

Ans. No, the total mechanical energy of a body falling through air is not conserved during the fall.
Justification: Mechanical energy is the sum of P.E. and K.E. In an ideal vacuum, where there is no air resistance, the only force acting on the falling body is gravity (a conservative force). In such a case, the loss in P.E. is exactly equal to the gain in K.E., so mechanical energy is conserved. However, when a body falls through air, air resistance (drag) acts on it. Air resistance is non-conservative force.
This force does negative work on the body, converting a part of the mechanical energy into thermal energy (heat) and sound. As a result, the body loses mechanical energy over time.
Conclusion: Because of air resistance, some of the mechanical energy is dissipated, and total mechanical energy is not conserved during the fall in air.

4.4 Calculate power of a crane in kilowatt which lifts a mass of 1000 kg to a height of 100 m in 20 second.

Ans. Using the formula:

$$\text{Power} = \frac{\text{Work done}}{\text{Time}} \quad \text{.......... (i)}$$

Work done = Force × Distance where Force = Weight

Therefore, weight = mg = 1000 kg × 9.8 ms^{-2}

= 9800 N

Thus Work done = 9800 N × 100 m = 980,000 J

Putting the values in Eq. (i)

$$\text{Power} = \frac{980{,}000\ J}{20\ s}$$

Power = 49,000 Js^{-1} = 49 kW

4.5 A trolley of mass 1500 kg carrying sand bags of 500 kg is moving uniformly with a speed of 40 km h^{-1} on a frictionless track. After some time, sand starts leaking out of whole sand bags on the road at a rate of 0.05 kg s^{-1}. What is the speed of the trolley after entire sand bags are empty?

Ans. Mass of trolley = 1500 kg

Mass of sand bags = 500 kg

Total mass = 2,000 kg

$$\text{Initial velocity} = 40\ km\ h^{-1} = \frac{40 \times 1000}{3600}\ ms^{-1} = 11.11\ ms^{-1}$$

Now, sand leaks vertically, and since the track is frictionless and no external horizontal force acts, horizontal momentum is conserved.

Momentum Conservation:

Initial momentum = Final momentum

Initial momentum = 2000 kg × 11.11 ms^{-1} = 22220 kg ms^{-2}

After all the sand has leaked out, only the trolley remains:

Final mass = 1500 kg and final speed = v_f, then

$$1500\ kg \times v_f = 22220\ kg\ ms^{-1}$$

$$v_f = \frac{22220\ kg\ ms^{-1}}{1500\ kg}$$

v_f = 14.81 ms^{-1} or 53.3 km h^{-1}

So, the trolley speeds up because the leaking sand takes no horizontal momentum with it.

4.6 Give absolute and gravitational units of work in M.K.S and C.G.S systems.

Ans. 1. M.K.S system (Metre-kilogram-second)

- **Absolute unit of work is Joule (J)**

  One joule (J) is equal to the work done when a force of one newton moves an object one metre in the direction of the force.

  i.e., $1\ J = 1\ N \times 1\ m = 1\ kg\ m^2s^{-2}$

- **Potential unit of work is kilojoule (kJ) or Megajoule (MJ)**

  $1\ kJ = 1000\ J = 10^3\ J$

  $1\ MJ = 1000{,}000\ J = 10^6\ J$

2. C.G.S system (Centimetre-gram-second)

- **Absolute unit of work is erg:**

  One erg is equal to the work done when a force of one dyne moves an object one centimetre in the direction of the force.

i.e., One erg = One dyne × 1 cm = 1 g cm^2 s^{-2}

Potential unit of work is kilo erg, mega erg

$$1 \text{ kerg} = 1000 \text{ erg} = 10^3 \text{ erg}$$

$$1 \text{ Merg} = 1000{,}000 \text{ erg} = 10^6 \text{ erg}$$

Conversion between units:

$$1 \text{ joule} = 10^7 \text{ erg}$$

$$1 \text{ erg} = 10^{-7} \text{ joule}$$

4.7 A body dropped from a height of H reaches the ground with a speed of 1.2 $\sqrt{gH}$. Calculate work done by air friction.

Ans. To calculate the work done by air friction, we will compare the mechanical energy, the body should have (in vacuum or no air resistance) with the mechanical energy, it actually has (with air friction).

1. **Ideal case (No air resistance)**
If the body falls from a height H and there is no air resistance, the P.E. at the top is fully converted into K.E. at the bottom.

$$\text{Initial P.E} = mgh$$

$$\text{Final K.E} = \frac{1}{2}mv^2 = mgh$$

So, in an ideal case:

$$v = \sqrt{2g\,H}$$

2. **Actual case (with air resistance)**
As the body reaches the ground with a speed of $v = 1.2\sqrt{gH}$.
So, the actual K.E at the bottom is:

$$K.E.' = \frac{1}{2}mv'^2 = \frac{1}{2}m\,(1.2\sqrt{gH})^2 = \frac{1}{2}m\,(1.44\,gH)$$

$$= 0.72\,mgH$$

3. **Work done by air friction:**
This is the loss in mechanical energy due to air friction.

$$W_{friction} = \text{Initial P.E} - \text{Final K.E} = mg\,H - 0.72\,mgH = 0.28\,mgH$$

Thus, work done by air friction is 0.28 mgH.

4.8 A bicycle has a K.E. of 150 J. What K.E. would the bicycle have if it had:
(i) same mass but has speed double?
(ii) three times mass and was moving with one half of the speed?

Ans. Given that; original K.E. = 150 J

(i) Same mass, but speed is doubled

As $K.E. = \frac{1}{2}mv^2$

If speed is doubled, then

$$K.E.' = \frac{1}{2}m\,(2v)^2 = \frac{1}{2}m\,.\,4v^2 = 4\left(\frac{1}{2}mv^2\right)$$

$$K.E.' = 4 \times 150\text{ J} = 600\text{ J}$$

(ii) Three times the mass and half the speed:

So $m' = 3m$ and $v' = \frac{v}{2}$

Thus, K.E. becomes as:

$$K.E.' = \frac{1}{2}(3m)\left(\frac{v}{2}\right)^2 = \frac{1}{2} \cdot 3m \cdot \frac{v^2}{4} = \frac{3}{8}mv^2$$

$$K.E.' = \frac{3}{8} \times 2 \times 150\ J = 56.25\ J$$

4.9 What will be the effect on K.E. of the body having mass m, moving with velocity v when its momentum becomes double? Justify.

Ans. As $K.E. = \frac{1}{2}mv^2$ and $P = mv$

If the momentum becomes double, then

$$P' = 2P = 2mv$$

Let the new velocity be v', and since the mass m is constant, so

$$P' = mv' = 2\ mv \quad \text{or} \quad v' = 2v$$

The new K.E. wil be

$$K.E.' = \frac{1}{2}m\,(v')^2 = \frac{1}{2}m\,(2v)^2 = \frac{1}{2}m \cdot 4v^2 = 4 \cdot \frac{1}{2}mv^2 = 4\ K.E.$$

Conclusion: When the momentum of a body becomes double, its K.E. becomes 4 times greater. This is because K.E. depends on square of velocity, while momentum is directly proportional to velocity.

4.10 Does the international space station have gravitational P.E. or kinetic energy or both? Explain.

Ans. The international space station (ISS) has both G.P.E and K.E.

1. **Gravitational Potential Energy (G.P.E.)**
   Even though it is in orbit, the ISS is still within Earth's gravitational field. G.P.E depends on the mass of the object, the mass of the Earth, and the distance from the centre of the Earth.

$$G.P.E = -\frac{GMm}{r}$$

   The negative sign indicates that the ISS is in a bound gravitational system.

2. **Kinetic Energy (K.E.)**
   The ISS is moving very fast (about 28,000 km h^{-1} or 7.66 km s^{-1}) to stay in the orbit. It constantly falls towards the Earth, but its forward velocity keeps it in a circular path.

$$K.E. = \frac{1}{2}mv^2$$

In orbit:

The ISS maintains a balance between these two energies. In fact, for a stable circular orbit.

$$K.E. = -\frac{1}{2} \times P.E.$$

So, yes the ISS has both G.P.E and K.E., and they are crucial for keeping it in orbit.

CONSTRUCTED RESPONSE QUESTIONS

4.1 When will you say that a force is conservative? Give two conditions.

Ans. A force is said to be conservative if it satisfies the following two conditions:

1. **Path Independence**
   The work done by the force in moving an object between two points is independent of the path taken. It depends only on the initial and final positions.

2. **Zero work in a closed path**
   The total work done by the force when moving an object around any closed path is zero.

4.2 A light and a heavy body have same linear momentum, which one has greater K.E.?

Ans. The lighter body will have greater kinetic energy if both bodies have the same linear momentum.

Explanation: The kinetic energy K.E. is given by

$$K.E = \frac{1}{2}mv^2$$

If the two bodies have the same momentum (p), the kinetic energy is inversely proportional to their mass (m). Thus, the lighter body (smaller m) will have a larger K.E., while the heavier body (larger m) will have a smaller K.E.

4.3 A motorcycle is running with constant speed on a horizontal track. Is any work being done on the motorcycle, if no net force is acting on it?

Ans. If the motorcycle is moving at a constant speed on a horizontal track and no net force is acting on it, no net work is being done on the motorcycle.

Reasons:

1. Work is done when a force acts on an object and causes displacement in the direction of the force. Mathematically, work is given by

   $$W = Fd\cos\theta$$

2. The problem states there is no net force acting on the motorcycle. This implies that all forces (e.g., engine thrust, air resistance, rolling friction) are perfectly balanced. According to Newton's first law, an object in motion at constant velocity does not require additional force to maintain its motion if the forces are balanced.
3. Since there is no net force acting on the motorcycle, there is no net work being done on it. However, internal work is being done by the engine of the motorcycle to overcome friction, air resistance, and other opposing forces. This internal energy is converted from the fuel to maintain constant speed, but it does not change the motorcycle's kinetic energy, as its speed is constant. Hence, the net work on the motorcycle is zero.

4.4 A force acts on a ball moving with 14 m s^{-1} speed and brings its speed to 6 m s^{-1}. Has the force done positive or negative work? Explain your answer.

Ans. The force has done negative work on the ball. That is why:

1. **Work and Kinetic Energy:** Work done on an object is related to its change in kinetic energy by the Work-energy Theorem:

   $$W = \Delta K.E. = K.E_{initial} - K.E_{final}$$

2. **Initial and Final Kinetic Energies:** The kinetic energy of the ball is given by:

   $$K.E. = \frac{1}{2}mv^2$$

- Initial kinetic energy: $K.E_{initial} = \frac{1}{2}m(14)^2 = 98\ m$
- Final kinetic energy: $K.E_{final} = \frac{1}{2}m(6)^2 = 18\ m$

3. **Change in Kinetic Energy:** $\Delta K.E = K.E_{final} - K.E_{initial} = 18\ m - 98\ m = -80\ m$
   Since the change in kinetic energy is negative, the work done is also negative.
4. **Interpretation:** Negative work means the force acting on the ball opposes its motion, causing it to lose energy and slow down.

4.5 A slow moving truck can have more kinetic energy than a fast moving car. How is this possible?

Ans. The kinetic energy of an object is determined by the formula: $K.E. = \frac{1}{2}mv^2$

For a slow-moving truck and a fast-moving car, their kinetic energies depend on both their masses and velocities. While the truck may be moving slowly, it has a much larger mass compared to the car. If the mass of the truck is significantly larger than the car's, it can compensate for its lower velocity, resulting in a higher overall kinetic energy.

For example:

- A truck with a mass of 10,000 kg moving at 5 ms^{-1}:

  $$K.E_{Truck} = \frac{1}{2} \times 10{,}000 \times 5^2 = 125{,}000\ J$$

- A car with a mass of moving at 20 ms^{-1}:

$$K.E_{Car} = \frac{1}{2} \times 1{,}000 \times 20^2 = 200{,}000\ J$$

In this case, the car has more kinetic energy. But if the truck moves slightly faster, say at 7 ms^{-1}, its K.E. becomes:

$$K.E. = \frac{1}{2} \times 10{,}000 \times 7^2 = 2{,}45{,}000\ J$$

Now, the truck's kinetic energy exceeds that of the car, despite moving slower. Hence, the truck's large mass can make its kinetic energy greater even at lower speeds.

4.6 Why work done against friction is non-conservative in nature? Explain briefly.

Ans. Work done against friction is non-conservative because the energy dissipated by friction cannot be fully recovered. In conservative forces, like gravity, the work done is path-independent, and the energy is conserved; it can be fully converted back into potential or kinetic energy. However, friction always converts some of the mechanical energy (kinetic or potential) into heat, which disperses into the environment, making it impossible to recover the same amount of energy. Therefore, the work done by friction depends on the path taken and results in a loss of mechanical energy.

4.7 Does wind contain kinetic energy? Explain.

Ans. Yes, wind contains K.E.

Explanation

Kinetic energy is the energy possessed by a body due to its motion. Wind is moving air, and since air has mass and is in motion, therefore, it possesses K.E.

The K.E. of the wind can be calculated by using the formula:

$$K.E = \frac{1}{2} mv^2$$

The K.E. of the wind is what wind turbines capture and convert into mechanical or electrical energy. The stronger and faster the wind, the more K.E. it has.

COMPREHENSIVE QUESTIONS

4.1 Define K.E. and derive an expression for the same.

Ans. *See Q. 8*

4.2 How is work done by a:

(i) constant force **(ii) variable force?**

Ans. *See Q. 2 and Q. 3*

4.3 Define conservative field. Show that gravitational field is conservative in nature.

Ans. *See Q. 4*

4.4 What is meant by absolute P.E.? Derive an expression for absolute P.E.

Ans. *See Q. 9*

4.5 State and explain work-energy theorem in a resistive medium.

Ans. *See Q. 11*

4.6 Define escape velocity. Show that an expression for escape velocity can be expressed as $\sqrt{2Rg}$, where R and g denote radius of the Earth and acceleration due to gravity, respectively. Also find its numerical value near the surface of the Earth.

Ans. *See Q. 10*

NUMERICAL PROBLEMS

4.1 A machine gun fires 6 bullets per minute with a velocity of 700 m s^{-1}. If each bullet has a mass of 40 g, then find power developed by the gun?

Solution:

Given data:

Number of bullets fired per minute = 6

Mass of each bullet = m = 40 g = 0.04 kg

Velocity of each bullet = v = 700 ms^{-1}

To find:

Power developed by gun = P = ?

Calculations: Converting rate to bullets per second:

6 bullets per minute = $\frac{6}{60}$ = 0.1 bullet per second

Kinetic energy of one bullet is:

$$K.E. = \frac{1}{2}mv^2 = \frac{1}{2} \times 0.04\ kg \times (700\ ms^{-1})$$

$$= 0.02\ kg \times 49000\ m^2 s^{-2}$$

$$K.E. = 9800\ kg\ m^2 s^{-2} = 9800\ J$$

Now power developed by the gun is:

Power = Energy per second

= K.E. × bullets per second

= 9800 × 0.1

$\boxed{P = 980\ W}$ Ans.

4.2 A family uses 10 kW of power. Direct solar energy is incident on horizontal surface at an average rate of 300 W per square metre. If 75% of this energy can be converted into useful electrical energy, how large area is needed to supply 10 kW?

Solution:

Given data:

Power required = 10 kW = 10,000 W

Incident solar power = 300 Wm^{-2}

Efficiency = 75%

To Find:

Area = A = ?

Calculations:

$$Area = \frac{\text{Power required}}{\text{Incident solar power per unit area} \times \text{Efficiency}}$$

$$= \frac{10,000\ W}{300\ Wm^{-2} \times 75\%}$$

$$= \frac{10,000\ W}{225\ Wm^{-2}}$$

$$A = 44.44\ m^2$$

Thus, 44.44 m^2 of area is needed to supply 10 kW of power.

4.3 The mass of the Earth is 6.0×10^{24} kg and mass of the Sun is 1.99×10^{30} kg. The Sun is 160 million km away from the Earth. Find the value of gravitational P.E. of the Earth.

Solution:

Given Data:

Mass of Earth = m = 6.0×10^{24} kg

Mass of Sun = M = 1.99×10^{30} kg

Distance = r = 160 million km = 1.6×10^{11} m

Gravitational constant = G = $6.67 \times 10^{-11}\ Nm^2 kg^{-2}$

To Find:

$$\text{Gravitational P.E.} = G.P.E = -\frac{GMm}{r}$$

Putting the values

$$G.P.E = -\frac{6.67 \times 10^{-11}\ Nm^2 kg^{-2} \times 1.99 \times 10^{30}\ k \times 6.0 \times 10^{24}\ kg}{1.6 \times 10^{11}\ m}$$

$$= \frac{79.7 \times 10^{43}}{1.6 \times 10^{11}}$$

$$G.P.E = -4.97 \times 10^{33}\ J$$

So, the G.P.E of the Earth due to the Sun is -4.97×10^{33} J.

4.4 An object weighing 98 N is dropped from a height of 10 m. Its speed just before hitting the ground is 12 m s^{-1}. What is the frictional force acting on it?

Solution:

Given Data:

Weight = w = 98 N

Height = h = 10 m

Final speed = v = 12 ms^{-1}

Accelerate on due to gravity g = 9.8 ms^{-2}

To find:

Frictional force $F_{friction}$ = ?

Conclusions:

As $w = mg$ or $m = \frac{w}{g} = \frac{98\ N}{9.8\ ms^{-2}}$

= 10 kg

Initial P.E. = mgh

= 10 kg × 9.8 ms^{-2} × 10 cm

P.E. = 980 J

K.E. just before hitting the ground:

$$K.E. = \frac{1}{2}mv^2 = \frac{1}{2} \times 10\ kg \times (12\ ms^{-1})^2$$

= 720 J

Energy lost due to friction = P.E. – K.E.

= 980 J – 720 . 260 J

Thus, work done by friction = Frictiona force × Distance

$$W = F_{friction} \times d$$

or $F_{friction} = \frac{W}{d} = \frac{260\ J}{10\ m}$

$\boxed{W = 26\ N}$ **Ans.**

4.5 A 75 watt fan is used for 8 hours daily for 30 days. Find:

(i) energy consumed in electrical units

(ii) electricity bill if one unit costs Rs. 22.5?

Solution:

Given Data:

Power of the fan = P = 75 W = 0.075 kW

Time used per day = t = 8 h

Number of days = 30

Cost per unit = Rs. 22.5

(i) Energy consumed in electrical units (kWh):

Energy consumed = Power × Time

= 0.075 kW × 8 h / day × 30 days

= 0.075 kW × 240

Energy consumer = 18 kWh (units) Ans.

(ii) Electrical bill

Cost = Energy consumed × Cost per unit

= 18 kWh × 22.5

Cost = 405 **Ans.**

4.6 If an object of mass 2 kg thrown up from ground reaches a height of 5 m and fallsback to the Earth (neglecting air resistance), calculate:

(i) work done by gravity when the object reaches at 5 m height.

(ii) work done by gravity when the object comes back to the Earth.

(iii) total work done by gravity in upward and downward motion. Also mention physical significance of the result.

Solution:

Given Data:

Mass of the object = m = 2 kg

Max. height reached = h = 5 m

Acceleration due to gravity = g = 9.8 ms^{-2}

To Find:

Work done W = ?

Calculations:

(i) Work done by gravity when the object reaches 5 m height.

$W = Fd\cos\theta$

When the object is moving upward, displacement is up, but gravitational force is down, so

Angle (θ) = 180°, and cos (180°) = −1

So $W = mg \times h \times \cos(180^\circ)$

$= 2 \times 9.8\ ms^{-2} \times 5\ m \times (-1)$

($\because \cos 180^\circ = -1$)

W = −9.8 J

Thus, work is done by gravity.

(ii) Work done by gravity when the object comes back Earth:

Now the object is moving downward, and gravity also acts downward, so angle (θ) = 0°, and cos (0°) = 1

$W = mg \times h$

$= 2\ kg \times 9.8\ ms-2 \times 5\ m \times 1$ ($\because \cos 0^\circ = 1$)

W = 98 J

(iii) Total work done by gravity during upward and downward motion:

$W_{Total} = -98\ J + 98\ J = 0\ J$

Work done in a closed path in a conservative field is zero.

4.7 An electrical motor of one horse power is used to run a water pump. Water pumptakes 15 minutes to fill a tank of 400 litres at a height of 10 m (1 hp 746 watts). Find:

(a) actual input work done by electric motor to fill the tank

(b) actual output work done

Solution:

Given Data:

Power of motor P = 1 hp = 746 W

Time taken = t = 15 min = 15 × 60 s = 900 s

Volume of water = V = 400 L

Height = h = 10 m

Acceleration due to gravity = g = 9.8 ms^{-2}

Mass of water = m = 400 kg ($\because$ 1 L = 1 kg)

To Find:

(a) actual input work done to till the tank

(b) actual output work done

Calculations:

(a) Input work is calculated as:

Input work = Power × Time

= 746 W × 900 S= 671400 J

Input work = 671.4 kJ Ans.

(b) Actual output work is the work done to lift the water:

Output work = mgh

$= 400\ kg \times 9.8\ ms^{-2} \times 10\ m = 39{,}200\ J$

Output work = 39.2 kJ **Ans.**

4.8 A passenger just arrives at the airport and dragging his suitcase to luggage checks in at the desk. He pulls strap with a force of 200 N at an angle of 45° to the floor to displace it 50 m to the desk. Determine the value of work done by him on the suitcase.

Solution:

Given Data:

Force = F = 200 N

Angle = θ = 45°
Distance = d = 50 m

To Find:

Work done = W = ?

Calculations: Using the formula:

$$W = Fd \cos\theta$$

Putting the values

$$W = 200\ N \times 50\ m \times \cos 45°$$
$$= 10{,}000\ Nm \times 0.707$$
$$(\because \cos 45° = 0.707)$$
$$= 7070\ J$$
$$W = 7\ kJ$$ **Ans.**

Thus, the work done by the passenger on the suitcase is 7 kJ.

4.9 A 1200 kg car is running at a speed of 40 km h^{-1}. How much power will be expended by it to accelerate at 2 m s^{-2}?

Solution:

Given Data:

Mass of car = m = 1200 kg

Speed of car = $v = 40\ km\ h^{-1} = \frac{40 \times 1000}{60 \times 60}\ ms^{-1}$

$$v = 11.11\ ms^{-1}$$

Acceleration produced = a = 2 ms^{-2}

To Find:

Power = P = ?

Calculations: Using the formula:

$$P = F \times V \quad \text{...... (i)}$$

where $F = ma = 1200\ kg \times 2\ ms^{-2} = 2400\ N$

Putting the values in Eq. (i)

$$P = 2400\ N \times 11.11\ ms^{-1} = 26664\ W$$
$$P = 26.67\ kW$$ **Ans.**

4.10 A 200 g apple is lifted to 10 m and then dropped. What is its velocity when it hits the ground? Assume that 75% of work done in lifting the apple is transferred to K.E. by the time it hits the ground.

Solution:

Given Data:

Mass = m = 200 g = 0.2 kg
Height = h = 10 m
Gravitational acceleration = g = 9.8 ms^{-2}

To Find:

Velocity = v = ?

Conclusions: To calculate the velocity of the apple when it hits the ground, we can follow these steps:

1. Find the G.P.E., when the apple is at 10 m.

$$G.P.E = mgh$$
$$G.P.E = 0.2\ kg \times 9.8\ ms^{-2} \times 10\ m$$
$$G.P.E = 19.6\ J$$

2. Account for the efficiency (75% of work done is converted to K.E.)
Since 75% of the work done in lifting the apple is converted into K.E., at the point just before it hits the ground is:

$$K.E. = 0.75 \times 19.6\ J = 14.7\ J$$

3. For calculation of velocity, use the formula:

$$K.E. = \frac{1}{2}mv^2$$

or

$$v = \sqrt{\frac{2\,K.E}{m}}$$

Putting the values

$$v = \sqrt{\frac{2 \times 14.7}{0.2}} = \sqrt{\frac{29.4}{0.2}} = \sqrt{147}$$
$$v = 12.1\ ms^{-1}$$ **Ans.**

Thus, the velocity of the apple when it hits the ground is 12.1 ms^{-1}.