UNIT 13

# Probability

## Students' learning outcomes

**At the end of the unit, the students will be able to:**

- Calculate the probability of a single event and the probability of an event not occurring.
- Solve real life problems involving probability.
- Calculate relative frequency as an estimate of probability.
- Calculate expected frequencies.
- Solve real life problems involving relative and expected frequencies.

$$\text{Probability} = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

It is written as: $P(A) = \dfrac{n(A)}{n(S)}$

$P(A)$ = Probability of an events

$n(A)$ = Number of favourable outcomes

$n(S)$ = Total number of possible outcomes

**Example 1:** Abdul Raheem rolls a fair dice, what is the probability of getting the number divisible by 3?

***Sol:*** When a dice is rolled, the sample space will be:

$S = \{1, 2, 3, 4, 5, 6\}$; $n(S) = 6$

Let "$A$" be the event of getting the number divisible by 3.

$A = \{3,6\}$; $n(A) = 2$

$$P(A) = \frac{n(A)}{n(S)} = \frac{2}{6} = \frac{1}{3}$$

The probability of getting the number divisible by 3 is $\frac{1}{3}$.

**Keep in mind**

The range of probability for an event is:

$0 \le P(A) \le 1$

**Teachers' note:**

Clear the concept of all the types of events by using different colours of balls or pencils etc.

**Example 2:** If Zeeshan rolled two fair dice, find the probability of getting:

(i) Even numbers on both dice.

(ii) Multiples of 3 on both dice.

(iii) Even number on the first dice and the number 3 on the second dice.

(iv) At least the number 3 on the first dice and number 4 on the second dice.

***Sol:*** When a pair of fair dice is rolled, the sample space will be:

| 1st / 2nd | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 1, 1 | 1, 2 | 1, 3 | 1, 4 | 1, 5 | 1, 6 |
| 2 | 2, 1 | 2, 2 | 2, 3 | 2, 4 | 2, 5 | 2, 6 |
| 3 | 3, 1 | 3, 2 | 3, 3 | 3, 4 | 3, 5 | 3, 6 |
| 4 | 4, 1 | 4, 2 | 4, 3 | 4, 4 | 4, 5 | 4, 6 |
| 5 | 5, 1 | 5, 2 | 5, 3 | 5, 4 | 5, 5 | 5, 6 |
| 6 | 6, 1 | 6, 2 | 6, 3 | 6, 4 | 6, 5 | 6, 6 |

**Try Yourself!**

Can you find out the sample space when 3 dice are rolled

(i) Even numbers on both dice.

Let "$A$" be the event of getting even numbers on both dice.

$A = \{(2, 2), (2, 4), (2, 6), (4, 2) (4, 4), (4, 6), (6, 2), (6, 4), (6, 6)\}$

$n(A) = 9$; $n(S) = 36$

$$P(A) = \frac{n(A)}{n(S)} = \frac{9}{36} = \frac{1}{4}$$

Thus, the probability of getting even numbers on both dice is $\frac{1}{4}$.

(ii) Multiple of 3 on both dice.

Let "$B$" be the event of getting multiples of 3 on both dice.

$B = \{(3, 3), (3, 6), (6, 3), (6, 6)\}$

$n(B) = 4$; $n(S) = 36$

$$P(B) = \frac{n(B)}{n(S)} = \frac{4}{36} = \frac{1}{9}$$

Thus, the probability of getting multiples of 3 on both dice is $\frac{1}{9}$.

(iii) Even number on the first dice and the number 3 on the second dice.

Let "C" be the event of getting even numbers on the first dice and the number 3 on the second dice.

$C = \{(2,3), (4,3), (6,3)\}$

$n(C) = 3; n(S) = 36$

$P(C) = \frac{n(C)}{n(S)} = \frac{3}{36} = \frac{1}{12}$

Thus, the probability of getting an even number on the first dice and the number 3 on the second dice is $\frac{1}{12}$.

(iv) At least the number 3 on the first dice and number 4 on the second dice.

Let "D" be the event of getting at least the number 3 on the first dice and number 4 on the second dice.

$D = \{(3, 4), (4, 4), (5, 4), (6, 4)\}$

$n(D) = 4; n(S) = 36$

$P(D) = \frac{n(D)}{n(S)} = \frac{4}{36} = \frac{1}{9}$

Thus, the probability of getting at least the number 3 on the first dice and number 4 on the 2nd dice is $\frac{1}{9}$.

**Example 3:** Zubair rolls a dice, what is the probability of not getting the number 6?

***Sol:*** Let "A" be the event of getting the number 6.

The sample space while rolling a dice is: $S = \{1, 2, 3, 4, 5, 6\}$

$n(S) = 6$

$A = \{6\}; n(A) = 1$

$P(A) = \frac{n(A)}{n(S)} = \frac{1}{6}$

To find out probability of not getting the number 6, we have

$P(A') = 1 - P(A)$

$= 1 - \frac{1}{6} = \frac{6-1}{6} = \frac{5}{6}$

Thus, the probability of not getting the number 6 is $\frac{5}{6}$.

**Example 4:** If two fair dice are rolled. What is the probability of getting:

(i) not a double six (ii) not the sum of both dice is 8

***Sol:*** Sample space of two fair dice is given by:

$S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$

$n(S) = 36$

(i) not a double six.

Let "A" be the event that a double six occurs.

$A = \{(6, 6)\}; n(A) = 1$

$P(A) = \frac{n(A)}{n(S)} = \frac{1}{36}$

Let "A′" be the event that not a double six occurs

As we know that

$P(A') = 1 - P(A)$

$= 1 - \frac{1}{36} = \frac{36-1}{36} = \frac{35}{36}$

Thus, the probability of not getting the double six is $\frac{35}{36}$.

(ii) not the sum of both dice is 8.

Let "B" be the event that the sum of both dice is 8.

$B = \{(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)\}$

$n(B) = 5$

$P(B) = \frac{n(B)}{n(S)} = \frac{5}{36}$

Let "B′" be the event not sum of both dice is 8.

$P(B') = 1 - P(B)$

$= 1 - \frac{5}{36} = \frac{36-5}{36} = \frac{31}{36}$

Thus, the probability of not the sum of both dice be 8 is $\frac{31}{36}$.

**Example 5:** Let $A$, $B$ and $C$ are three missiles and they are fired at a target. If the probabilities of hitting the target are $P(A) = \frac{1}{4}$, $P(B) = \frac{3}{7}$, $P(C) = \frac{5}{9}$ respectively.

Find the probabilities of

(i) missile $A$ does not hit the target.

(ii) missile $B$ does not hit the target,

(iii) missile $C$ does not hit the target.

***Sol:*** (i) missile $A$ does not hit the target.

Since, $P(A) = \frac{1}{4}$

Let '$A'$' be the event that missile $A$ does not hit the target

$$P(A') = 1 - P(A)$$

$$= 1 - \frac{1}{4} = \frac{4-1}{4} = \frac{3}{4}$$

Thus, the probability of missile '$A$' does not hit the target is $\frac{3}{4}$.

(ii) missile '$B$' does not hit the target.

Since, $P(B) = \frac{3}{7}$

Let '$B'$' be the event missile $B$ does not hit the target

$$P(B') = 1 - P(B)$$

$$= 1 - \frac{3}{7}$$

$$= \frac{7-3}{7} = \frac{4}{7}$$

Thus, the probability of missile '$B$' does not hit the target is $\frac{4}{7}$.

(iii) missile '$C$' does not hit the target.

Since, $P(C) = \frac{5}{9}$

Let '$C'$' be the event missile $C$ of not hitting the target

$$P(C') = 1 - P(C)$$

$$= 1 - \frac{5}{9} = \frac{9-5}{9} = \frac{4}{9}$$

Thus, the probability of missile 'C' does not hit the target is $\frac{4}{9}$.

**Example 6:** A bag contains 5 blue balls and 8 green balls. Find the probability of selecting at random:

(i) a blue ball (ii) a green ball. (iii) not a green ball.

***Sol:*** (i) a blue ball

Let '$A$' be the event that the ball is blue

Blue balls $= n(A) = 5$

Total balls $= n(S) = 5 + 8 = 13$

$$P(A) = \frac{n(A)}{n(S)}$$

$$= \frac{5}{13}$$

Thus, the probability of selecting a blue ball is $\frac{5}{13}$.

(ii) a green ball

a green ball

Let '$B$' be the event that ball is green

Green balls $= n(B) = 8$

Total balls $= n(S) = 5 + 8 = 13$

$$P(B) = \frac{n(B)}{n(S)} = \frac{8}{13}$$

Thus, the probability of selecting green ball is $\frac{8}{13}$.

(iii) not a green ball

Let '$B'$' be the event that the ball is not green.

$$P(B') = 1 - P(B)$$

$$= 1 - \frac{8}{13}$$

$$= \frac{13-8}{13} = \frac{5}{13}$$

Thus, the probability of not selecting a green ball is $\frac{5}{13}$.

**Example 7:** A card is drawn at random, from a pack of 52 playing cards. What is the probability of getting:

(i) a card of heart (ii) neither spade nor heart

Total number of cards = 52 ; $n(S) = 52$

Let '$A$' be the event of selecting a card of heart.

Number of heart cards=13; $n(A) = 13$

$$P(A) = \frac{n(A)}{n(S)}$$

$$= \frac{13}{52} = \frac{1}{4}$$

Total cards = 52
↓
4 suits
↓

| 13 Spades (Black) | 13 Clubs (Black) | 13 Hearts (Red) | 13 Diamonds (Red) |
|---|---|---|---|
| 1 king | 1 king | 1 king | 1 king |
| 1 Queen | 1 Queen | 1 Queen | 1 Queen |
| 1 Jack | 1 Jack | 1 Jack | 1 Jack |
| 1 Ace | 1 Ace | 1 Ace | 1 Ace |
| (2 – 10) cards | (2 – 10) cards | (2 – 10) cards | (2 – 10) cards |

Thus, the probability of getting a card of heart is $\frac{1}{4}$.

(ii) neither spade nor heart

Let '$B$' be the event of selecting a card of spade or heart

Number of spade and heart cards = 26 ; $n(B) = 26$

$$P(B) = \frac{n(B)}{n(S)}$$

$$= \frac{26}{52} = \frac{1}{2}$$

Let '$B'$' be the event of selecting neither spade nor heart card.

$$P(B') = 1 - P(B)$$

$$= 1 - \frac{1}{2}$$

$$= \frac{1}{2}$$

Thus, the probability of getting neither spade nor heart cards is $\frac{1}{2}$.

# EXERCISE 13.1

**1. Arshad rolls a dice, with sides labelled L, M, N, O, P, U. What is the probability that the dice lands on consonant?**

***Sol:*** The sides of the dice are labeled: L, M, N, O, P, U.

Step 1: Identify the consonants and vowels

- **Consonants:** L, M, N, P
- **Vowels:** O, U

Step 2: Count the consonants and vowels

- Total consonants = 4 (L, M, N, P)
- Total sides on the dice = 6

Step 3: Calculate the probability

The probability of landing on a consonant is the ratio of the number of consonants to the total number of sides:

$$P(\text{consonant}) = \frac{\text{Number of consonants}}{\text{Total sides}} = \frac{4}{6} = \frac{2}{3}$$

**Final Answer:**

The probability that the dice lands on a consonant is $\frac{2}{3}$.

**2. Shazia throws a pair of fair dice. What will be the probability of getting:**

**(i) sum of dots is at least 4.**

**(ii) product of both dots is between 5 to 10.**

**(iii) the difference between both the dots is equal to 4.**

**(iv) number at least 5 on the first dice and the number at least 4 on the second dice.**

***Sol:*** To solve these probability problems, we'll consider that Shazia is throwing **two fair six-sided dice**. Each die has faces numbered from 1 to 6, and the total number of outcomes is:

$$6 \times 6 = 36$$

Now, let's evaluate each part:

(i) **The sum of dots is at least 4**

The sum of dots $S$ ranges from 2 (1+1) to 12 (6+6). For $S \geq 4$, we exclude $S = 2$ and $S = 3$.

- $S = 2$: (1,1) → 1 outcome.
- $S = 3$: (1,2), (2,1) → 2 outcomes.
- Total outcomes where $S < 4 = 1 + 2 = 3$.
- Favorable outcomes where $S \geq 4 = 36 - 3 = 33$.

**Probability:**

$$P(S \geq 4) = \frac{\text{Favorable outcomes}}{\text{Total outcomes}} = \frac{33}{36} = \frac{11}{12}.$$

(ii) **The product of both dots is between 5 and 10**

We look for outcomes where $5 \leq P \leq 10$. Let the dots on the dice be $(x, y)$, and the product is $P = x \cdot y$.

- $P = 5$: (1,5), (5,1).
- $P = 6$: (1,6), (2,3), (3,2), (6,1).
- $P = 8$: (2,4), (4,2).
- $P = 9$: (3,3).
- $P = 10$: (2,5), (5,2).

**Total favorable outcomes:**

(1,5), (5,1), (1,6), (2,3), (3,2), (6,1), (2,4), (4,2), (3,3),
(2,5), (5,2) (11 outcomes).

**Probability:**

$$P(5 \leq P \leq 10) = \frac{11}{36}.$$

(iii) **The difference between both dots is equal to 4**

Let the dots on the dice be $(x, y)$, and the difference is $|x - y| = 4$.

- Possible pairs:
  - $x - y = 4$: (5,1), (6,2).
  - $y - x = 4$: (1,5), (2,6).

**Total favorable outcomes:**

(5,1), (6,2), (1,5), (2,6) (4 outcomes).

**Probability:**

$$P(|x - y| = 4) = \frac{4}{36} = \frac{1}{9}.$$

(iv) **Number at least 5 on the first die and the number at least 4 on the second die**

Let the dots on the dice be $(x, y)$, where $x \geq 5$ and $y \geq 4$.

- For $x = 5$: $y = 4,5,6$ → 3 outcomes.
- For $x = 6$: $y = 4,5,6$ → 3 outcomes.

**Total favorable outcomes:**

$$3 + 3 = 6.$$

**Probability:**

$$P(x \geq 5 \text{ and } y \geq 4) = \frac{6}{36} = \frac{1}{6}.$$

3. **One alphabet is selected at random from the word "MATHEMATICS". Find the probability of getting:**

| | | | | | |
|---|---|---|---|---|---|
| **(i)** | **vowel** | **(ii)** | **consonant** | **(iii)** | **an E** |
| **(iv)** | **an A** | **(v)** | **not M** | **(vi)** | **not T** |

***Sol:*** The word "**MATHEMATICS**" contains 11 letters. Let's break down the problem:

**Step 1: Identify vowels and consonants**

- Vowels: A, A, E, I → Total vowels = 4.
- Consonants: M, T, H, M, T, C, S → Total consonants = 7.

**Step 2: Solve each part**

(i) **Probability of getting a vowel**

The favorable outcomes are A, A, E, I (4 vowels). Total letters = 11.

$$P(\text{vowel}) = \frac{\text{Number of vowels}}{\text{Total letters}} = \frac{4}{11}.$$

(ii) **Probability of getting a consonant**

The favorable outcomes are M, T, H, M, T, C, S (7 consonants).
Total letters = 11.

$$P(\text{consonant}) = \frac{\text{Number of consonants}}{\text{Total letters}} = \frac{7}{11}.$$

(iii) **Probability of getting an E**

The letter E appears **once** in the word.

$$P(E) = \frac{\text{Number of E's}}{\text{Total letters}} = \frac{1}{11}.$$

(iv) **Probability of getting an A**

The letter A appears **twice** in the word.

$$P(A) = \frac{\text{Number of A's}}{\text{Total letters}} = \frac{2}{11}.$$

(v) **Probability of not getting an M**

The letter M appears **twice** in the word, so the remaining letters = $11 - 2 = 9$.

$$P(\text{not M}) = \frac{\text{Number of non-M letters}}{\text{Total letters}} = \frac{9}{11}.$$

*(vi)* **Probability of not getting a T**

The letter T appears **twice** in the word, so the remaining letters = $11 - 2 = 9$.

$$P(\text{not T}) = \frac{\text{Number of non-T letters}}{\text{Total letters}} = \frac{9}{11}.$$

4. **Aslam rolled a dice. What is the probability of getting the numbers 3 or 4? Also find the probability of not getting the numbers 3 or 4.**

***Sol:*** Aslam rolls a standard six-sided die, where each face is numbered 1,2,3,4,5,6.

**Step 1: Total possible outcomes**

The total number of outcomes when rolling a die is:

$$6$$

**Step 2: Probability of getting the numbers 3 or 4**

- Favorable outcomes: $\{3, 4\} \rightarrow$ **2 outcomes.**
- Total outcomes: 6.

The probability of getting 3 or 4 is:

$$P(3 \text{ or } 4) = \frac{\text{Favorable outcomes}}{\text{Total outcomes}} = \frac{2}{6} = \frac{1}{3}.$$

**Step 3: Probability of not getting the numbers 3 or 4**

If the outcomes 3 or 4 are excluded, the remaining outcomes are $\{1, 2, 5, 6\} \rightarrow$ **4 outcomes.**

The probability of not getting 3 or 4 is:

$$P(\text{not 3 or 4}) = \frac{\text{Remaining outcomes}}{\text{Total outcomes}} = \frac{4}{6} = \frac{2}{3}.$$

5. **Abdul Hadi labelled cards from 1 to 30 and put them in a box. He selects a card at random. What is the probability that selected card containing:**

(i) **the number 25**

(ii) **number between 17 to 22**

(iii) **number at least 20**

(iv) **number not 27 and 29**

(v) **number not between 12 to 15**

***Sol:*** Abdul Hadi has **30 cards labeled from 1 to 30**, so the total number of outcomes is:

$$\text{Total outcomes} = 30$$

Let's calculate each part:

**(i) Probability that the selected card contains the number 25**

The number 25 appears on **1 card.**

$$P(\text{number } 25) = \frac{\text{Number of favorable outcomes}}{\text{Total outcomes}} = \frac{1}{30}.$$

**(ii) Probability that the selected card contains a number between 17 and 22**

The numbers between 17 and 22 are: **17, 18, 19, 20, 21, 22** $\rightarrow$ **6 cards.**

$$P(\text{number between 17 and 22}) = \frac{\text{Number of favorable outcomes}}{\text{Total outcomes}}$$

$$= \frac{6}{30} = \frac{1}{5}$$

**(iii) Probability that the selected card contains a number at least 20**

The numbers that are at least 20 are: **20, 21, 22, ..., 30 → 11** cards.

$$P(\text{number at least 20}) = \frac{\text{Number of favorable outcomes}}{\text{Total outcomes}} = \frac{11}{30}.$$

**(iv) Probability that the selected card is not 27 and not 29**

The numbers 27 and 29 are excluded, so the remaining numbers are 30 − 2 = 28 cards.

$$P(\text{not 27 and not 29}) = \frac{\text{Number of favorable outcomes}}{\text{Total outcomes}} = \frac{28}{30}$$

$$= \frac{14}{15}.$$

**(v) Probability that the selected card is not between 12 and 15**

The numbers between 12 and 15 are: **12, 13, 14, 15 → 4 cards.**

The remaining numbers are 30 − 4 = 26 cards.

$$P(\text{not between 12 and 15}) = \frac{\text{Number of favorable outcomes}}{\text{Total outcomes}}$$

$$= \frac{26}{30} = \frac{13}{15}.$$

6. **The probability that Ayesha will pass the examination is 0.85. What will be the probability that Ayesha will not pass the examination?**

***Sol:*** The probability that Ayesha will **not pass** the examination is the complement of the probability that she will pass. The formula for the complement is:

$$P(\text{not passing}) = 1 - P(\text{passing})$$

Given:

$$P(\text{passing}) = 0.85$$

Calculation:

$$P(\text{not passing}) = 1 - 0.85 = 0.15$$

**Final Answer:**

The probability that Ayesha will not pass the examination is **0.15**.

7. **Taabish tossed a fair coin and rolled a fair dice once. Find the probability of the following events:**

(i) **tail on coin and at least 4 on dice.**

(ii) **head on coin and the number 2,3 on dice.**

(iii) **head and tail on coin and the number 6 on dice.**

(iv) **not tail on coin and the number 5 on dice.**

(v) **not head on coin and the number 5 and 2 on dice.**

Taabish tosses a fair coin and rolls a fair six-sided die, so we calculate probabilities based on the total outcomes and the favorable outcomes.

**Step 1: Total possible outcomes**

- Outcomes of the coin: {Head (H), Tail (T)} → 2 outcomes.
- Outcomes of the dice: {1, 2, 3, 4, 5, 6} → 6 outcomes.
- Total outcomes:

$$2 \times 6 = 12$$

**Step 2: Solve each part**

(i) **Tail on the coin and at least 4 on the dice**

- **Tail on the coin:** 1 outcome (T).
- **At least 4 on the dice:** Numbers 4, 5, 6 → 3 outcomes.
- Favorable outcomes:

$(T,4), (T,5), (T,6)$ (3 outcomes).

**Probability:**

$$P(\text{Tail and at least 4}) = \frac{\text{Favorable outcomes}}{\text{Total outcomes}} = \frac{3}{12} = \frac{1}{4}.$$

(ii) **Head on the coin and the number 2 or 3 on the dice**

- **Head on the coin:** 1 outcome (H).
- **Number 2 or 3 on the dice:** Numbers 2, 3 → 2 outcomes.
- Favorable outcomes:

$(H,2), (H,3)$ (2 outcomes).

**Probability:**

$$P(\text{Head and number 2 or 3}) = \frac{\text{Favorable outcomes}}{\text{Total outcomes}} = \frac{2}{12} = \frac{1}{6}.$$

**(iii) Head and Tail on the coin and the number 6 on the dice**

Let "C" be an event of occurring head and tail on coin and number 6 on dice.

$C = \{(H, 6), (T, 6)\}, n(C) = 2$

Probability of event C:

$$P(C)\frac{n(C)}{n(S)} = \frac{2}{12} = \frac{1}{6}$$

**(iv) Not tail on the coin and the number 5 on the dice**

Let 'A' be an event of occurring a tail on coin and number 5 on dice.

$A = \{(T,5)\}, n(A) = 1$

Probability event $A$:

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{12}$$

The probability of not tail on coin and number 5 on dice is:

$$P(A') = 1 - P(A)$$

$$= 1 - \frac{1}{12}$$

$$= \frac{12-1}{12}$$

$$= \frac{11}{12}$$

**(v) Not head on the coin and the number 5 and 2 on the dice**

Let 'B' be an event of occurring head on coin and number 5 and 2 on dice.

$B = \{(H,2),(H,5)\}, n(B) = 2$

Probability event $B$:

$$P(B) = \frac{n(B)}{n(S)} = \frac{2}{12} = \frac{1}{6}$$

The probability of not occurring head on coin and number 5 and 2 one dice.

$$P(B') = 1 - P(B)$$

$$= 1 - \frac{1}{6} = \frac{6-1}{6} = \frac{5}{6}$$

8. **A card is selected at random from a well shuffled pack of 52 plying cards. What will be the probability of selecting:**

**(i) a queen (ii) neither a queen nor a jack**

***Sol:*** A standard deck of 52 playing cards consists of 4 suits (hearts, diamonds, clubs, spades), with 13 cards in each suit. Each suit contains exactly one queen and one jack.

**(i) Probability of selecting a queen**

There are **4 queens** in the deck (one in each suit).

The total number of cards is 52.

**Probability:**

$$P(\text{queen}) = \frac{\text{Number of queens}}{\text{Total cards}} = \frac{4}{52} = \frac{1}{13}.$$

**(ii) Probability of selecting neither a queen nor a jack**

- Total cards: 52.
- Total queens: 4.
- Total jacks: 4.
- Cards that are either a queen or a jack: $4 + 4 = 8$.
- Cards that are neither a queen nor a jack: $52 - 8 = 44$.

**Probability:**

$$P(\text{neither queen nor jack}) = \frac{\text{Number of favorable outcomes}}{\text{Total cards}} = \frac{44}{52} = \frac{11}{13}.$$

9. **A card is chosen at random from a pack of 52 playing cards. Find the probability of getting:**

**(i) a jack (ii) no diamond**

***Sol:*** **(i) Probability of getting a jack**

In a standard deck of 52 playing cards:

- There are **4 jacks** (one in each suit: hearts, diamonds, clubs, spades).
- Total number of cards = 52.

**Probability:** $P(\text{jack}) = \frac{\text{Number of jacks}}{\text{Total cards}} = \frac{4}{52} = \frac{1}{13}.$

**(iii) Head and Tail o... ...e coin and the number 6 on the dice**

Let "C" be an event of occurring head and tail on coin and number 6 on dice.

$C = \{(H, 6), (T, 6)\}, n(C) = 2$

Probability of event C:

$$P(C)\frac{n(C)}{n(S)} = \frac{2}{12} = \frac{1}{6}$$

**(iv) Not tail on the coin and the number 5 on the dice**

Let 'A' be an event of occurring a tail on coin and number 5 on dice.

$A = \{(T,5)\}, n(A) = 1$

Probability event $A$:

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{12}$$

The probability of not tail on coin and number 5 on dice is:

$$P(A') = 1 - P(A)$$

$$= 1 - \frac{1}{12}$$

$$= \frac{12-1}{12}$$

$$= \frac{11}{12}$$

**(v) Not head on the coin and the number 5 and 2 on the dice**

Let 'B' be an event of occurring head on coin and number 5 and 2 on dice.

$B = \{(H,2),(H,5)\}, n(B) = 2$

Probability event $B$:

$$P(B) = \frac{n(B)}{n(S)} = \frac{2}{12} = \frac{1}{6}$$

The probability of not occurring head on coin and number 5 and 2 one dice.

$$P(B') = 1 - P(B)$$

$$= 1 - \frac{1}{6} = \frac{6-1}{6} = \frac{5}{6}$$

8. **A card is selected at random from a well shuffled pack of 52 plying cards. What will be the probability of selecting:**
**(i) a queen (ii) neither a queen nor a jack**

***Sol:*** A standard deck of 52 playing cards consists of 4 suits (hearts, diamonds, clubs, spades), with 13 cards in each suit. Each suit contains exactly one queen and one jack.

**(i) Probability of selecting a queen**

There are **4 queens** in the deck (one in each suit).

The total number of cards is 52.

**Probability:**

$$P(\text{queen}) = \frac{\text{Number of queens}}{\text{Total cards}} = \frac{4}{52} = \frac{1}{13}.$$

**(ii) Probability of selecting neither a queen nor a jack**

- Total cards: 52.
- Total queens: 4.
- Total jacks: 4.
- Cards that are either a queen or a jack: $4 + 4 = 8$.
- Cards that are neither a queen nor a jack: $52 - 8 = 44$.

**Probability:**

$$P(\text{neither queen nor jack}) = \frac{\text{Number of favorable outcomes}}{\text{Total cards}} = \frac{44}{52} = \frac{11}{13}.$$

9. **A card is chosen at random from a pack of 52 playing cards. Find the probability of getting:**
**(i) a jack (ii) no diamond**

***Sol:*** **(i) Probability of getting a jack**

In a standard deck of 52 playing cards:

- There are **4 jacks** (one in each suit: hearts, diamonds, clubs, spades).
- Total number of cards = 52.

**Probability:** $P(\text{jack}) = \frac{\text{Number of jacks}}{\text{Total cards}} = \frac{4}{52} = \frac{1}{13}.$

**(ii) Probability of getting no diamond**

In a standard deck of 52 playing cards:

- There are **13 diamonds** (one-quarter of the deck).
- Cards that are not diamonds $= 52 - 13 = 39$.

**Probability:**

$$P(\text{no diamond}) = \frac{\text{Number of non-diamond cards}}{\text{Total cards}} = \frac{39}{52} = \frac{3}{4}$$

$$\text{Relative frequency} = \frac{\text{Frequency of specific event}}{\text{Total frequency}} = \frac{x}{N}, \text{ where } N = \Sigma f$$

**Example 8:** Find the relative frequency of the given date.

| $X$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|
| $f$ | 3 | 5 | 6 | 9 | 10 | 8 | 2 |

***Sol:***

| $X$ | $f$ | Relative frequency |
|---|---|---|
| 2 | 3 | $\frac{3}{43} = 0.07$ |
| 3 | 5 | $\frac{5}{43} = 0.12$ |
| 4 | 6 | $\frac{6}{43} = 0.14$ |
| 5 | 9 | $\frac{9}{43} = 0.21$ |
| 6 | 10 | $\frac{10}{43} = 0.23$ |
| 7 | 8 | $\frac{8}{43} = 0.19$ |
| 8 | 2 | $\frac{2}{43} = 0.04$ |
| **Total** | $\Sigma f = 43$ | |

**Example 9:** A survey was conducted on 80 students of Grade - IX and asked about their favourite colour. The responses are:

(i) Red colour = 23 students (ii) Green colour = 15 students

(iii) Pink colour = 25 students(iv) Blue colour = 10 students

(v) White colour = 7 students.

Find the relative frequency for each colour.

***Sol:*** Total number of students = 80

(i) Relative frequency for red colour $= \frac{23}{80} = 0.29$

It means that 29% students prefer red colour.

(ii) Relative frequency for green colour $= \frac{15}{80} = 0.19$

It means that 19% students prefer green colour.

(iii) Relative frequency for pink colour $= \frac{25}{80} = 0.31$

It means that 31% students prefer pink colour.

(iv) Relative frequency for blue colour $\frac{10}{80} = 0.12$

It means that 12% students prefer blue colour.

(v) Relative frequency for white colour $= \frac{7}{80} = 0.09$

It means that 9% students prefer white colour.

**Example 10:** Abdul Rehman obtained different marks in different subjects out of 100 marks. The detail is as under:

| Subject | Urdu | English | Islamiyat | Mathematics | Science | Computer Science |
|---|---|---|---|---|---|---|
| Marks Obtained | 75 | 80 | 72 | 95 | 81 | 85 |

Find the relative frequency of above given data.

***Sol:***

| Subject | Marks obtained | Relative frequency |
|---|---|---|
| Urdu | 75 | $\frac{75}{488} = 0.15$ |
| English | 80 | $\frac{80}{488} = 0.16$ |
| Islamiyat | 72 | $\frac{72}{488} = 0.15$ |
| Mathematics | 95 | $\frac{95}{488} = 0.19$ |
| Science | 81 | $\frac{81}{488} = 0.17$ |
| Computer Science | 85 | $\frac{85}{488} = 0.17$ |
| **Total** | $\Sigma f = 488$ | |

**Example 11:** Six fair dice are rolled 50 times. The probability of occurrence of different number of sixes are given below. Find the expected frequency of the following data:

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| $P(x)$ | 0.09 | 0.10 | 0.12 | 0.24 | 0.10 | 0.20 | 0.15 |

Find the expected frequency of occurrence of each six.

***Sol:***

| No. of Sixes ($x$) | $P(x)$ | Expected frequency $= N \times P(x) = 50 \times P(x)$ |
|---|---|---|
| 0 | 0.09 | $50 \times 0.09 = 4.5$ |
| 1 | 0.10 | $50 \times 0.10 = 5$ |
| 2 | 0.12 | $50 \times 0.12 = 6$ |
| 3 | 0.24 | $50 \times 0.24 = 12$ |
| 4 | 0.10 | $50 \times 0.10 = 5$ |
| 5 | 0.20 | $50 \times 0.20 = 10$ |
| 6 | 0.15 | $50 \times 0.15 = 7.5$ |

**Example 12:** Find the average number of times getting 1 or 6, when a fair dice is rolled 300 times.

> **Remember!**
> Sum of all expected frequencies is always equal to or approximately equal to a fixed number of trials.

***Sol:*** Let "$S$" be the sample space when a fair dice is rolled:

$S = \{1, 2, 3, 4, 5, 6\}$ ; $n(S) = 6$ Let "$B$" be the event that 1 or 6 comes up.

$$B = \{1, 6\} \; ; \; n(B) = 2$$

So, $$P(B) = \frac{n(B)}{n(S)} = \frac{2}{6} = \frac{1}{3}$$

Therefore, $E(B) = N \times P(B)$

$$= 300 \times \frac{1}{3} = 100$$

Thus, the average number of times 1 or 6 comes up is 100.

**Example 13:** If the probability of a defective bolt is 0.3. Find the number of non-defective bolts in a total to 800.

***Sol:*** The probability of defective bolt is = 0.3

Probability of non-defective bolt = 1 − 0.3 = 0.7

Number of non-defective bolts = 0.7 × 800 = 560

Thus, the non-defective bolts will be 560.

## EXERCISE 13.2

1. **A researcher collected data on number of deaths from Horse-Ricks in Russian Army crops over to years. The table is as follows:**

| No. of death | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| Frequency | 60 | 50 | 87 | 40 | 32 | 15 | 10 |

***Sol:*** **Find the relative frequency of the given data.**

We already have the **total frequency** as:

Total frequency = 60 + 50 + 87 + 40 + 32 + 15 + 10 = 294.

Now, let's compute the relative frequencies for each category and simplify the fractions.

(1) **Number of Deaths = 0**

**Frequency = 60**

$$\frac{60}{294} = \frac{30}{147}$$

(2) **Number of Deaths = 1**

**Frequency = 50**

$$\frac{50}{294} = \frac{25}{147}$$

(3) **Number of Deaths = 2**

**Frequency = 87**

$$\frac{87}{294} = \frac{29}{98}$$

(4) **Number of Deaths = 3**

**Frequency = 40**

$$\frac{40}{294} = \frac{20}{147}$$

(5) **Number of Deaths = 4**

**Frequency = 32**

$$\frac{32}{294} = \frac{16}{147}$$

(6) **Number of Deaths = 5**

**Frequency = 15**

$$\frac{15}{294} = \frac{5}{98}$$

(7) **Number of Deaths = 6**

**Frequency = 10**

$$\frac{10}{294} = \frac{5}{147}$$

**Final Summary Table:**

| Number of Deaths | Frequency | Relative Frequency (Simplified Fraction) |
|---|---|---|
| 0 | 60 | $\frac{30}{147}$ |
| 1 | 50 | $\frac{25}{147}$ |
| 2 | 87 | $\frac{29}{98}$ |
| 3 | 40 | $\frac{20}{147}$ |
| 4 | 32 | $\frac{16}{147}$ |
| 5 | 15 | $\frac{5}{98}$ |
| 6 | 10 | $\frac{5}{147}$ |
| Total | $\Sigma f = 294$ | |

2. **The frequency of defective products in 750 samples are shown in the following table. Find the relative frequency for the given table.**

| No. of defectives per sample | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| No. of sample | 120 | 140 | 94 | 85 | 105 | 50 | 40 | 66 | 50 |

***Sol:*** **Step 1: Data Table**

The data provided is:

| No. of Defectives per Sample | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|
| No. of Samples | 120 | 140 | 94 | 85 | 105 | 50 | 40 | 66 | 50 |

**Step 2: Total Number of Samples**

The total number of samples is:

Total Number of Samples

$$= 120 + 140 + 94 + 85 + 105 + 50 + 40 + 66 + 50 = 750$$

**Step 3: Compute Relative Frequencies**

Now, we'll compute the relative frequencies for each category and express them as simplified fractions:

0. **Relative Frequency of 0 defectives:**

$$\frac{120}{750} = \frac{4}{25}$$

1. **Relative Frequency of 1 defective:**

$$\frac{140}{750} = \frac{14}{75}$$

2. **Relative Frequency of 2 defectives:**

$$\frac{94}{750} = \frac{47}{375}$$

3. **Relative Frequency of 3 defectives:**

$$\frac{85}{750} = \frac{17}{150}$$

4. **Relative Frequency of 4 defectives:**

$$\frac{105}{750} = \frac{21}{150}$$

5. **Relative Frequency of 5 defectives:**

$$\frac{50}{750} = \frac{1}{15}$$

6. **Relative Frequency of 6 defectives:**

$$\frac{40}{750} = \frac{4}{75}$$

7. **Relative Frequency of 7 defectives:**

$$\frac{66}{750} = \frac{11}{125}$$

8. **Relative Frequency of 8 defectives:**

$$\frac{50}{750} = \frac{1}{15}$$

**Final Summary Table**

| No. of Defectives per Sample | Frequency | Relative Frequency (Simplified Fraction) |
|---|---|---|
| 0 | 120 | $\frac{4}{25}$ |
| 1 | 140 | $\frac{14}{75}$ |
| 2 | 94 | $\frac{47}{375}$ |
| 3 | 85 | $\frac{17}{150}$ |
| 4 | 105 | $\frac{21}{150}$ |
| 5 | 50 | $\frac{1}{15}$ |
| 6 | 40 | $\frac{4}{75}$ |
| 7 | 66 | $\frac{11}{125}$ |
| 8 | 50 | $\frac{1}{15}$ |
| Total | $\Sigma f = 750$ | |

3. **A quiz competition on general knowledge is conducted. The number of corrected answers out of 5 questions for 100 sets of questions is given below.**

| $X$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $f$ | 10 | 23 | 15 | 25 | 18 | 9 |

**Find the relative frequencies for the given data.**

***Sol:*** ***Step 1: Data Table***

The data provided is:

| Number of Correct Answers (X) | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Frequency (f) | 10 | 23 | 15 | 25 | 18 | 9 |

***Step 2: Total Number of Sets of Questions***

The total number of sets is:

$$\text{Total Number of Sets} = 10 + 23 + 15 + 25 + 18 + 9 = 100$$

***Step 3: Compute Relative Frequencies***

Now, we'll compute the relative frequencies for each number of correct answers and express them as simplified fractions:

0. **Relative Frequency of 0 correct answers:**

$$\frac{10}{100} = \frac{1}{10}$$

1. **Relative Frequency of 1 correct answer:**

$$\frac{23}{100} = \frac{23}{100}$$

2. **Relative Frequency of 2 correct answers:**

$$\frac{15}{100} = \frac{3}{20}$$

3. **Relative Frequency of 3 correct answers:**

$$\frac{25}{100} = \frac{1}{4}$$

4. **Relative Frequency of 4 correct answers:**

$$\frac{18}{100} = \frac{9}{50}$$

5. **Relative Frequency of 5 correct answers:**

$$\frac{9}{100} = \frac{9}{100}$$

**Final Summary Table**

| Number of Correct Answers (X) | Frequency ($f$) | Relative Frequency (Simplified Fraction) |
|---|---|---|
| 0 | 10 | $\frac{1}{10}$ |
| 1 | 23 | $\frac{23}{100}$ |
| 2 | 15 | $\frac{3}{20}$ |
| 3 | 25 | $\frac{1}{4}$ |
| 4 | 18 | $\frac{9}{50}$ |
| 5 | 9 | $\frac{9}{100}$ |
| **Total** | $\Sigma f = 100$ | |

**4. A survey was conducted from the 50 students of a class and asked about their favourite food. The responses are as under:**

| Name of food item | Biryani | Fresh juice | Chicken | Bar. B.Q | Sweets |
|---|---|---|---|---|---|
| No. of students | 40 | 07 | 21 | 15 | 25 |

- **(i) how many percentages of students like biryani?**
- **(ii) how many percentages of students like chicken?**
- **(iii) which food is the least liked by the students?**
- **(iv) which food is the most preferred by the students?**

***Sol:***

**Data Table:** The data provided is:

| Name of Food Item | Number of Students |
|---|---|
| Biryani | 40 |
| Fresh Juice | 7 |
| Chicken | 21 |
| Bar B.Q | 15 |
| Sweets | 25 |

Total number of students = 50

| Name of food item | No. of students ($f$) | Relative frequency $\frac{f}{\Sigma f}$ | Percentage |
|---|---|---|---|
| Biryani | 40 | $\frac{40}{108} = 0.37$ | $0.37 \times 100 = 37\%$ |
| Fresh Juice | 07 | $\frac{7}{108} = 0.065$ | $0.065 \times 100 = 6.5\%$ |
| Chicken | 21 | $\frac{21}{108} = 0.194$ | $0.19 \times 100 = 19.4\%$ |
| Bar. B.Q. | 15 | $\frac{15}{108} = 0.14$ | $0.14 \times 100 = 14\%$ |
| Sweet | 25 | $\frac{25}{108} = 0.23$ | $0.23 \times 100 = 23\%$ |
| Total | $\Sigma f = 108$ | | |

i) 37% students like biryani.
ii) 19.4% students like chicken.
iii) Fresh juice is liked by least students.
iv) Biryani is most preferred by the students.

**5. In 500 trials of a thrown of two dice, what is expected frequency that the sum will be greater than 8?**

***Sol:*** To solve this, we first need to determine the probability of the sum of two dice being greater than 8. Then, we'll calculate the expected frequency based on the given number of trials.

Step 1: Possible outcomes

Two dice have 6 faces each, so there are $6 \times 6 = 36$ total outcomes.

Step 2: Outcomes where the sum is greater than 8

We list all combinations of two dice where the sum exceeds 8:

- **Sum = 9**: (3,6), (4,5), (5,4), (6,3) → 4 outcomes
- **Sum = 10**: (4,6), (5,5), (6,4) → 3 outcomes

- **Sum = 11**: (5,6), (6,5) → 2 outcomes
- **Sum = 12**: (6,6) → 1 outcome

Adding these outcomes: $4 + 3 + 2 + 1 = 10$.

Step 3: Probability

The probability of getting a sum greater than 8 is:

$$P(\text{sum} > 8) = \frac{\text{favorable outcomes}}{\text{total outcomes}} = \frac{10}{36} = \frac{5}{18}.$$

Step 4: Expected frequency

The expected frequency is given by:

$$\text{Expected Frequency} = P(\text{sum} > 8) \times \text{Total Trials}.$$

Substituting the values:

$$\text{Expected Frequency} = \frac{5}{18} \times 500 = \frac{2500}{18} \approx 138.89 \approx 139$$

6. **What is the expectation of a person who is to get Rs. 120 if he obtains at least 2 heads in single toss of three coins?**

***Sol:*** **Total Outcomes**: Tossing 3 coins gives $2^3 = 8$ total outcomes.

- **Favorable Outcomes**: You need at least 2 heads:
  - 2 heads: $HHT, HTH, THH \rightarrow 3$ outcomes.
  - 3 heads: $HHH \rightarrow 1$ outcome.
  - Total favorable outcomes $= 3 + 1 = 4$.
- **Probability of at least 2 heads:**

$$P(\text{at least 2 heads}) = \frac{\text{Favorable outcomes}}{\text{Total outcomes}} = \frac{4}{8} = \frac{1}{2}.$$

- **Reward**: The person gets Rs. 120 if they get at least 2 heads.
- **Expectation**: Multiply the probability by the reward:

$$\text{Expectation} = \frac{1}{2} \times 120 = 60.$$

**Final Answer:**

The expectation of the person is **Rs. 60.**

7. **Find the expected frequencies of the given data if the experiment is repeated 200 times.**

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| $P(x)$ | 0.11 | 0.21 | 0.17 | 0.18 | 0.09 | 0.17 | 0.07 |

***Sol:*** To calculate the **expected frequencies** for the given data when the experiment is repeated 200 times, use the formula:

$$\text{Expected Frequency} = P(x) \times \text{Total Number of Trials}.$$

Given:

- Total trials $(n) = 200$
- $P(x)$ values for $x = 0,1,2,3,4,5,6$:

$0.11, 0.21, 0.17, 0.18, 0.09, 0.17, 0.07.$

Calculation:

$$\text{Expected Frequency for each } x = P(x) \times 200$$

- For $x = 0$: $0.11 \times 200 = 22$
- For $x = 1$: $0.21 \times 200 = 42$
- For $x = 2$: $0.17 \times 200 = 34$
- For $x = 3$: $0.18 \times 200 = 36$
- For $x = 4$: $0.09 \times 200 = 18$
- For $x = 5$: $0.17 \times 200 = 34$
- For $x = 6$: $0.07 \times 200 = 14$.

| X | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| P(X) | 0.11 | 0.21 | 0.17 | 0.18 | 0.09 | 0.17 | 0.07 |
| Expected Frequency | 22 | 42 | 34 | 36 | 18 | 34 | 14 |

**Results:**

The expected frequencies are:

$$x = 0: 22,\ x = 1: 42,\ x = 2: 34,\ x = 3: 36,\ x = 4: 18,\ x = 5: 34,\ x = 6: 14.$$

8. **The probability of getting 5 sixes while tossing six dice is $\frac{2}{5}$. How many times would you expect it to show 5 sixes?**

**Sol:** Let "A" be an event of getting 5 sixes while tossing dice.

Probability of getting 5 sixes = $P(A) = \frac{2}{5}$

Number of trials $= N = 200$

We know that

Expected frequency $= N \times P(A)$

Expected frequency $= 200 \times \frac{2}{5}$

$= 40 \times 2$

$= 80$ times

## REVIEW EXERCISE 13

1. **Four options are given against each statement. Encircle the correct option.**

(i) Each element of the sample space is called:
- (a) event
- (b) experiment
- (c) sample point
- (d) outcomes

(ii) An outcome which represents how many times we expect the things to be happened is called:
- (a) outcomes
- (b) favourable outcome
- (c) sample space
- (d) sample point

(iii) Which one tells us how often a specific event occurs relative to the total number of frequency event or trials?
- (a) expected frequency
- (b) sum of relative frequency
- (c) relative frequency
- (d) frequency

(iv) Estimated probability of an event occurring is also known as:
- (a) relative frequency
- (b) expected frequency
- (c) class boundaries
- (d) sum of expected frequency

(v) The sum of all expected frequencies is equal to the fixed number of:
- (a) trials
- (b) relative frequencies
- (c) outcomes
- (d) events

(vi) The chance of occurrence of a particular event is called:
- (a) sample space
- (b) estimated probability
- (c) probability
- (d) expected frequency

(vii) An event which will probably occur. It has greater chance to occur is called:
- (a) equally likely event
- (b) likely event
- (c) unlikely event
- (d) certain event

(viii) Find out the total number of possible sample space when 4 dice are rolled:
- (a) $6^2$
- (b) $6^3$
- (c) $6^4$
- (d) $6^6$

(ix) While rolling a pair of dice, what will be the probability of double 2?
- (a) $\frac{1}{6}$
- (b) $\frac{1}{3}$
- (c) $\frac{5}{6}$
- (d) $\frac{1}{36}$

(x) A card is chosen from a pack of 52 playing cards, find the probability of getting no jack and king:
- (a) $\frac{2}{13}$
- (b) $\frac{11}{13}$
- (c) $\frac{2}{52}$
- (d) $\frac{11}{52}$

***Answers:***

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| (i) | c | (ii) | b | (iii) | c | (iv) | a | (v) | a |
| (vi) | c | (vii) | b | (viii) | c | (ix) | d | (x) | b |

2. **Define the following:**
**(i) relative frequency**
**(ii) expected frequency**

**Ans. (i) Relative Frequency**

Relative frequency is the ratio of the number of times an event occurs to the total number of trials or observations in an experiment. It is used to estimate the likelihood of an event based on experimental data.

**Formula**:

$$\text{Relative Frequency} = \frac{\text{Frequency of the Event}}{\text{Total Number of Trials}}$$

**Example**: If a die is rolled 50 times and the number 6 appears 10 times, the relative frequency of rolling a 6 is:

$$\frac{10}{50} = 0.2.$$

**(ii) Expected Frequency**

Expected frequency is the predicted number of times an event should occur based on its theoretical probability in a given number of trials. It is calculated by multiplying the probability of the event by the total number of trials.

**Formula**:

**Expected Frequency = Probability of the Event × Total Number of Trials**

**Example**: If a coin is tossed 100 times, the probability of getting heads is 0.5. The expected frequency of heads is:

$$0.5 \times 100 = 50.$$

3. **An urn contains 10 red balls, 5 green balls and 8 blue balls. Find the probability of selecting at random.**
**(i) a green ball (ii) a red ball**
**(iii) a blue ball (iv) not a red ball**
**(v) not a green ball**

***Sol:*** Given Data:

- Number of red balls: 10
- Number of green balls: 5
- Number of blue balls: 8
- Total balls in the urn:

$$10 + 5 + 8 = 23.$$

**(i) Probability of selecting a green ball:**

The probability is:

$$P(\text{green ball}) = \frac{\text{Number of green balls}}{\text{Total balls}} = \frac{5}{23}.$$

**(ii) Probability of selecting a red ball:**

The probability is:

$$P(\text{red ball}) = \frac{\text{Number of red balls}}{\text{Total balls}} = \frac{10}{23}.$$

**(iii) Probability of selecting a blue ball:**

The probability is:

$$P(\text{blue ball}) = \frac{\text{Number of blue balls}}{\text{Total balls}} = \frac{8}{23}.$$

**(iv) Probability of selecting not a red ball:**

The probability of not selecting a red ball is:

$$P(\text{not a red ball}) = 1 - P(\text{red ball}) = 1 - \frac{10}{23} = \frac{13}{23}.$$

**(v) Probability of selecting not a green ball:**

The probability of not selecting a green ball is:

$$P(\text{not a green ball}) = 1 - P(\text{green ball}) = 1 - \frac{5}{23} = \frac{18}{23}.$$

4. **Three coins are tossed together, what is the probability of getting:**
**(i) exactly three heads (ii) at least two tails**
**(iii) not at least two heads**
**(iv) not exactly two heads**

***Sol:*** We will calculate the required probabilities step by step.

Tossing 3 coins results in $2^3 = 8$ possible outcomes:

$$\{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}.$$

**(i) Probability of exactly three heads**

Only one outcome has exactly three heads: $HHH$.

$$P(\text{exactly three heads}) = \frac{\text{Number of favorable outcomes}}{\text{Total outcomes}} = \frac{1}{8}.$$

**(ii) Probability of at least two tails**

At least two tails means the number of tails is 2 or 3. The favorable outcomes are:

$$\{HTT, THT, TTH, TTT\} \quad (4 \text{ outcomes}).$$

$$P(\text{at least two tails}) = \frac{\text{Number of favorable outcomes}}{\text{Total outcomes}} = \frac{4}{8} = \frac{1}{2}.$$

**(iii) Probability of not at least two heads**

At least two heads means the number of heads is 2 or 3. The favorable outcomes for at least two heads are:

$$\{HHT, HTH, THH, HHH\} \quad (4 \text{ outcomes}).$$

The unfavorable outcomes (not at least two heads) are the remaining $8 - 4 = 4$ outcomes:

$$\{HTT, THT, TTH, TTT\}.$$

$$P(\text{not at least two heads}) = \frac{\text{Number of unfavorable outcomes}}{\text{Total outcomes}}$$

$$= \frac{4}{8} = \frac{1}{2}.$$

**(iv) Probability of not exactly two heads**

The favorable outcomes for exactly two heads are:

$$\{HHT, HTH, THH\} \quad (3 \text{ outcomes}).$$

The unfavorable outcomes (not exactly two heads) are the remaining $8 - 3 = 5$ outcomes:

$$\{HHH, HTT, THT, TTH, TTT\}.$$

$$P(\text{not exactly two heads}) = \frac{\text{Number of unfavorable outcomes}}{\text{Total outcomes}}$$

$$= \frac{5}{8}.$$

**5. A card is drawn from a well shuffled pack of 52 playing cards. What will be the probability of getting:**

**(i) king or jack of red colour**

**(ii) not "2" of club and spade**

***Sol:*** **Given:**

A standard pack of 52 playing cards contains:

- 26 red cards (13 hearts + 13 diamonds).
- 26 black cards (13 clubs + 13 spades).
- Each suit has one king, one jack, one "2", etc.

(i) Probability of getting a **king or jack of red color**

- There are 2 red suits: hearts and diamonds.
- Each suit has 1 king and 1 jack, so there are 2 kings and 2 jacks of red color.
- Total favorable outcomes $= 2 + 2 = 4$.
- Total possible outcomes $= 52$.

$$P(\text{king or jack of red color}) = \frac{\text{Favorable outcomes}}{\text{Total outcomes}} = \frac{4}{52} = \frac{1}{13}.$$

(ii) Probability **of not "2" of club and spade**

- The "2 of clubs" and "2 of spades" are specific cards, so there are 2 unfavorable outcomes.
- Total favorable outcomes $= 52 - 2 = 50$.
- Total possible outcomes $= 52$.

$$P(\text{not "2" of club and spade}) = \frac{\text{Favorable outcomes}}{\text{Total outcomes}} = \frac{50}{52} = \frac{25}{26}.$$

**6. Six coins are tossed 600 times. The number of occurrence of tails are recorded and shown in the table given below:**

| No. of tails | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| Frequency | 110 | 90 | 105 | 80 | 76 | 123 | 16 |

**Find the relative frequency of given table.**

***Sol:*** The relative frequency is calculated as:

$$\text{Relative Frequency} = \frac{\text{Frequency of a specific event}}{\text{Total number of trials}}.$$

**Given Data:**

- Total trials = 600
- Frequency for each number of tails:

| No. of tails (x) | Frequency (f) |
|---|---|
| 0 | 110 |
| 1 | 90 |
| 2 | 105 |
| 3 | 80 |
| 4 | 76 |
| 5 | 123 |
| 6 | 16 |

Step-by-step Calculation of Relative Frequencies:

For each $x$, the relative frequency is:

$$\text{Relative Frequency} = \frac{\text{Frequency}}{600}.$$

- For $x = 0$: $\frac{110}{600} = \frac{11}{60}$
- For $x = 1$: $\frac{90}{600} = \frac{3}{20}$
- For $x = 2$: $\frac{105}{600} = \frac{7}{40}$
- For $x = 3$: $\frac{80}{600} = \frac{2}{15}$
- For $x = 4$: $\frac{76}{600} = \frac{19}{150}$
- For $x = 5$: $\frac{123}{600} = \frac{41}{200}$
- For $x = 6$: $\frac{16}{600} = \frac{2}{75}$

**Final Table:**

| No. of tails | 0 | 1 | 2 | 3 | 4 | 5 | 6 | Total |
|---|---|---|---|---|---|---|---|---|
| $f$ | 110 | 90 | 105 | 80 | 76 | 123 | 16 | $\Sigma f = 600$ |
| Relative Frequency | $\frac{11}{60}$ | $\frac{3}{20}$ | $\frac{7}{40}$ | $\frac{2}{15}$ | $\frac{19}{150}$ | $\frac{41}{200}$ | $\frac{2}{75}$ | |

7. **From a lot containing 25 items, 8 items are defective. Find the relative frequency of non-defective items, also find the expected frequency of non-defective items.**

***Sol:*** **Given Data:**

- Total number of items = 25
- Number of defective items = 8
- Number of non-defective items = $25 - 8 = 17$

**(i) Relative Frequency of Non-Defective Items**

The relative frequency is calculated as:

Relative Frequency of non-defective items

$$= \frac{\text{Number of non-defective items}}{\text{Total number of items}} = \frac{17}{25} = 0.68.$$

**(ii) Expected Frequency of Non-Defective Items**

If we know the relative frequency, we can multiply it by the total number of trials (items in this case) to find the expected frequency:

Expected Frequency of non-defective items = Relative Frequency of non-defective items × Total number of items = 0.68 × 25 = 17.