UNIT 1 Real Numbers

Students' learning outcomes

At the end of the unit, the students will be able to:

- Explain, with examples, that civilizations throughout history have systematically studied living things [e.g., the history of numbers from Sumerians and its development to the present Arabic system
- Describe the set of real numbers as combination of rational and irrational numbers
- Demonstrate and verify the properties of equality and inequality of real numbers
- Apply laws of indices to simplify radical expressions
- Apply concepts of real numbers to real word problems(such as temperature, banking, measures of gain and loss, sources of income and expenditure)

Example 1: Identify the following decimal numbers as rational or irrational numbers:

(i) 0.35 (ii) 0.444... (iii) $3.\bar{5}$

(iv) 3.36788542... (v) 1.709975947...

Solution: (i) 0.35 is a terminating decimal number, therefore it is a rational number.

(ii) 0.444... is a recurring decimal number, therefore it is a rational number.

(iii) $3.\bar{5}$ = 3.5555... is a recurring decimal number, therefore it is a rational number.

(iv) 3.3678542... is a non-terminating and non-recurring decimal number. Therefore, it represents an irrational number.

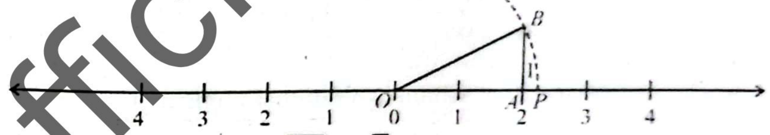
(v) 1.709975947... is a non-terminating and non-recurring decimal number, it is an irrational number.

Example 2: Represent $\sqrt{5}$ on a number line.

Solution: $\sqrt{5}$ can be located on the real line by geometric construction. As $\sqrt{5} = 2.236...$ which is near to 2. Mark a line of $m\overline{AB} = 1$ unit at A, where $m\overline{OA} = 2$ units, and we have a right-angle triangle OAB. By using Pythagoras theorem

$$(m\overline{OB})^2 = (m\overline{OA})^2 + (m\overline{AB})^2$$

$$= (2)^2 + (1)^2 = 4 + 1 = 5 \quad \Rightarrow \quad m\overline{OB} = \sqrt{5}$$



Draw an arc of radius $m\overline{OB} = \sqrt{5}$ taking O as centre, we got point "P" representing $\sqrt{5}$ on the number line

So, $|OP| = \sqrt{5}$

Example 3: Express the following recurring decimals as the rational number $\frac{p}{q}$, where p and q are integers.

(i) $0.\bar{5}$ (ii) $0.\overline{93}$

Solution: (i) $0.\bar{5}$

Let $x = 0.\bar{5}$

$x = 0.55555$ ...(i)

Multiply both sides by 10

$10x = 10(0.5555...)$

$10x = 5.55555$ ...(ii)

Subtracting eq. (i) from eq. (ii)

$10x - x = (5.55555...) - (0.55555...)$

$9x = 5$

$\Rightarrow x = \frac{5}{9}$

Which shows the rational number in the form of $\frac{p}{q}$.

(ii) Let $x = 0.\overline{93}$

$x = 0.939393$ ...(i)

Multiply by 100 on both sides

$100x = 100\ (0.939393...)$

$100x = 93.939393...$ ...(ii)

Subtracting (i) from (ii)

$100x - x = 93.939393... - 0.939393...$

$99x = 93$

$x = \frac{93}{99}$ which is a rational number.

Example 4 : Insert two rational numbers between 2 and 3.

Solution: There are infinite rational numbers between 2 and 3. We find any two of them

For this, find the average of 2 and 3 as $\frac{2+3}{2} = \frac{5}{2}$

So, $\frac{5}{2}$ is a rational number between 2 and 3,to find another rational number between 2and 3 we will again find average of $\frac{5}{2}$ and 3.

i.e., $\frac{\frac{5}{2}+3}{2} = \frac{\frac{5+6}{2}}{2} = \frac{\frac{11}{2}}{2} = \frac{11}{4}$

Hence two rational numbers between 2 and 3 are $\frac{5}{2}$ and $\frac{11}{4}$

Example 5: If $a = \frac{2}{3}, b = \frac{3}{2}, c = \frac{5}{3}$ then verify the distributive property over addition.

Solution: (i) Left distributive property

$a(b+c) = ab + ac$

LHS $= a(b+c)$	RHS $= ab + ac$
$= \frac{2}{3}\left(\frac{3}{2}+\frac{5}{3}\right)$	$= \left(\frac{2}{3}\right)\left(\frac{3}{2}\right) + \left(\frac{2}{3}\right)\left(\frac{5}{3}\right)$
$= \frac{2}{3}\left(\frac{9+10}{6}\right)$	$= 1 + \frac{10}{9}$
$= \frac{2}{3}\left(\frac{19}{6}\right)$	$= \frac{9+10}{9}$
$= \frac{19}{9}$	$= \frac{19}{9}$

Hence proved

(ii) Right distributive property

$(a+b)c = ac + bc$

LHS $= (a+b)c$	RHS $= ac + bc$
$= \left(\frac{2}{3}+\frac{3}{2}\right)\frac{5}{3}$	$= \left(\frac{2}{3}\right)\left(\frac{5}{3}\right) + \left(\frac{3}{2}\right)\left(\frac{5}{3}\right)$
$= \left(\frac{4+9}{6}\right)\frac{5}{3}$	$= \frac{10}{9} + \frac{15}{6}$
$= \left(\frac{13}{6}\right)\left(\frac{5}{3}\right)$	$= \frac{20+45}{18}$
$= \frac{65}{18}$	$= \frac{65}{18}$

Hence, it is verified that $(a+b)c = ac + bc$

Example 6: Identify the property that justifies the statement

(i) If $a > 13$ then $a + 2 > 15$

(ii) If $3 < 9$ and $6 < 12$ then $9 < 21$

(iii) If $7 > 4$ and $5 > 3$ then $35 > 12$

(iv) If $-5 < -4 \Rightarrow 20 > 16$

Solution:

(i) $a > 13$

Add 2 on both sides

$a+2>13+2$

$a+2>15$

(ii) As $3<9$ and $6<12$

$\Rightarrow 3+6<9+12$

$9<21$

(iii) $7<4$ and $5>3$

$\Rightarrow 7\times5>4\times3$

$\Rightarrow 35>12$

(iv) As $-5<-4$

Multiply on both sides by -4

$-5\times-4>-4\times-4$

$\Rightarrow \quad 20>16$

EXERCISE 1.1

1. Identify each of the following as a rational or irrational numbers:

(i)	2.353535	(ii)	$0.\bar{6}$	(iii)	2.236067...
(iv)	$\sqrt{7}$	(v)	e	(vi)	π
(vii)	$5+\sqrt{11}$	(viii)	$\sqrt{3}+\sqrt{13}$		
(ix)	$\frac{15}{4}$	(x)	$(2-\sqrt{2})(2+\sqrt{2})$		

Let's go through each number and determine whether it is rational or irrational.

(i) 2.353535

Solution: This is a repeating decimal $(2.\overline{35})$, which can be written as a fraction. Therefore, it is a **rational number**.

(ii) $0.\bar{6}$

Solution: This is a repeating decimal $(0.\bar{6})$, which can be written as a fraction. Therefore, it is a **rational number**.

(iii) 2.236067...

Solution: This is the decimal expansion of $\sqrt{5}$, which is non-repeating and non-terminating. Therefore, it is an **irrational number**.

(iv) $\sqrt{7}$

Solution: The square root of 7 is a non-repeating and non-terminating decimal. Therefore, it is an **irrational number**

(v) e

Solution: $e = 2.71828182845904...$

The mathematical constant e is a non-repeating and non-terminating decimal. Therefore, it is an **irrational number**.

(vi) π

Solution: $\pi = 3.1415926535897932...$

The mathematical constant π is a non-repeating and non-terminating decimal. Therefore, it is an **irrational number**.

(vii) $5+\sqrt{11}$

Solution: Since $\sqrt{11}$ is irrational and adding a rational number (5) to it results in an irrational number, this is an **irrational number**.

(viii) $\sqrt{3}+\sqrt{13}$

Solution: Both $\sqrt{3}$ and $\sqrt{13}$ are irrational, and their sum is also irrational. Therefore, it is an **irrational number**.

(ix) $\frac{15}{4}$

Solution: This is a fraction, which is a rational number. Therefore, it is a **rational number**.

(x) $(2-\sqrt{2})(2+\sqrt{2})$

Solution: This expression simplifies to $2^2-(\sqrt{2})^2 = 4-2 = 2$, which is a rational number. Therefore, it is a **rational number**.

2. Represent the following numbers on number line:

(i)	$\sqrt{2}$	(ii)	$\sqrt{3}$	(iii)	$4\frac{1}{3}$
(iv)	$-2\frac{1}{7}$	(v)	$\frac{5}{8}$	(vi)	$2\frac{3}{4}$

(i) $\sqrt{2}$

Solution: To represent $\sqrt{2}$ on a number line, we can follow a similar approach to the one outlined in your example:

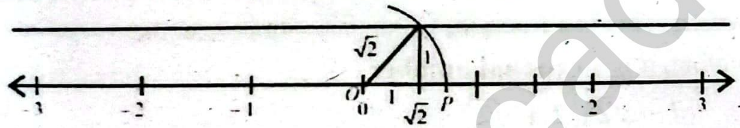
- **Approximate Value:** $\sqrt{2} \approx 1.414$, which is a value slightly greater than 1.
- **Constructing the Number Line:**
    - o Draw a line and mark points for 0 and 1.
    - o Measure the distance from 0 to 1 as 1 unit.
    - o Construct a right-angle triangle with one leg as 1 unit (the segment from 0 to 1) and the other leg also 1 unit.
    - o Use the Pythagorean theorem to find the hypotenuse:

$$(mOB)^2 = (mOA)^2 + (mAB)^2 = 1^2 + 1^2 = 2 \Rightarrow mOB = \sqrt{2}.$$

- o Draw the arc with radius $\sqrt{2}$ from the point O (which is the origin) on the number line.

- **Locating $\sqrt{2}$ on the Number Line**: The point P where the arc intersects the line represents the value $\sqrt{2}$.

Thus, $\sqrt{2}$ is located at approximately 1.414 units from 0, slightly more than 1 but less than 2, on the number line.



(ii) $\sqrt{3}$

Solution: Let LL′ be a number line.

Draw O ⊥ AB on LL′.

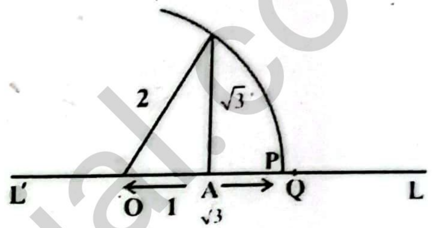
Cut the line segment $\overline{OB}$ equal to 2 units.

OAB is right angled Δ.

$\therefore$ AB = $\sqrt{OB^2 - OA^2} = \sqrt{4-1} = \sqrt{3}$

Cut the number line OP segment equal to $\overline{AB}$.

OP is represents $\sqrt{3}$ on the line segment.

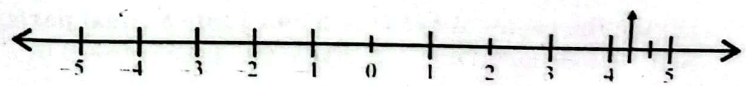


(iii) $4\frac{1}{3}$

Solution: $4\frac{1}{3} = \frac{13}{3}$.

- **Estimate the value:**
  $\frac{13}{3} = 4.333$ ..., which is slightly more than 4 but less than 5.
- **Draw a number line:**
  Mark the points 0, 1, 2, 3, 4, and 5 on the number line.
- **Locate $4\frac{1}{3}$:**
    - o Since $4\frac{1}{3}$ is just slightly more than 4, you will mark a point between 4 and 5.
    - o Divide the segment between 4 and 5 into 3 equal parts, as the denominator is 3.
    - o $4\frac{1}{3}$ will be at the first mark after 4.

Thus, the point representing $4\frac{1}{3}$ will be just past 4 on the number line, about one-third of the way toward 5.

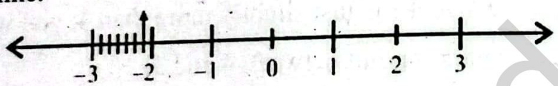


(iv) $-2\frac{1}{7}$

Solution: $-2\frac{1}{7} = -\frac{15}{7}$.

- **Estimate the value:**
  $-\frac{15}{7} \approx -2.142857$, which is slightly more than -2 but less than -3.
- **Draw a number line:**
  Mark the points $-3, -2, -1, 0, 1, 2, 3$ on the number line.
- **Locate $-2\frac{1}{7}$:**
  - Since $-2\frac{1}{7}$ is slightly more than -2, it will be located just past -2 but before -3.
  - Divide the segment between -2 and -3 into 7 equal parts (since the denominator is 7).
  - $-2\frac{1}{7}$ will be at the first mark after -2, which is one-seventh of the way toward -3.

Thus, the point representing $-2\frac{1}{7}$ will be just past -2 on the number line.

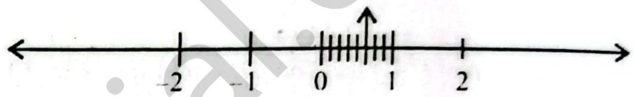


(v) $\frac{5}{8}$

Solution:

1. **Estimate the value:**
   $\frac{5}{8} = 0.625$.
2. **Draw a number line:**
   Mark the points 0, 1 on the number line.
3. **Divide the segment between 0 and 1 into 8 equal parts:**
   Since the denominator is 8, divide the space between 0 and 1 into 8 equal parts.
4. **Locate $\frac{5}{8}$:** Since $\frac{5}{8} = 0.625$, it will be 5 parts out of 8 between 0 and 1.

Thus, the point representing $\frac{5}{8}$ will be located after the 5th division between 0 and 1 on the number line.



(vi) $2\frac{3}{4}$

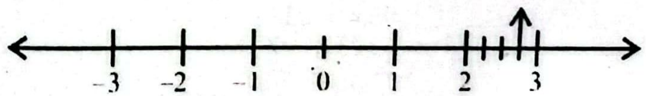
Solution: $2\frac{3}{4}$ can be written as the improper fraction:

$$2\frac{3}{4} = \frac{8}{4} + \frac{3}{4} = \frac{11}{4}.$$

So, $2\frac{3}{4} = 2.75$.

- **Draw a number line:**
  Mark the integers 0, 1, 2, and 3 on the number line.
- **Divide the segment between 2 and 3 into 4 equal parts:**
  Since the denominator is 4, divide the space between 2 and 3 into 4 equal parts.
- **Locate $2\frac{3}{4}$:**
  $2\frac{3}{4} = 2.75$ is 3 parts out of 4 between 2 and 3.

Thus, the point representing $2\frac{3}{4}$ will be located after the 3rd division between 2 and 3 on the number line.



3. **Express the following as a rational number $\frac{p}{q}$ where p and q are integers and $q \neq 0$**

(i) $0.\bar{4}$ (ii) $0.\overline{37}$ (iii) $0.\overline{21}$

Sol: (i) Let: $x = 0.\bar{4}$

This means:

$$x = 0.4444 ...$$

Since the repeating part ("4") has one digit, multiply x by 10:

$$10x = 4.4444 ...$$

$$10x - x = 4.4444 ... - 0.4444 ...$$

$$9x = 4$$

$$x = \frac{4}{9}$$

(ii) **For $0.\overline{37}$:**

Sol: Let $x = 0.\overline{37}$.

This means:

$$x = 0.373737 ...$$

Since the repeating part ("37") has two digits, multiply x by 100:

$$100x = 37.373737 ...$$

$$100x - x = 37.373737 ... - 0.373737 ...$$

$$99x = 37$$

$$x = \frac{37}{99}$$

(iii) **For $0.\overline{21}$:**

Sol: Let $x = 0.\overline{21}$.

This means:

$$x = 0.212121 ...$$

Since the repeating part ("21") has two digits, multiply x by 100:

$$100x = 21.212121 ...$$

$$100x - x = 21.212121 ... - 0.212121 ...$$

$$99x = 21$$

$$x = \frac{21}{99} = \frac{7}{33}$$

4. **Name the property used in the following.**

(i) $(a+4)+b = a+(4+b)$

(ii) $\sqrt{2}+\sqrt{3} = \sqrt{3}+\sqrt{2}$

(iii) $x - x = 0$ (iv) $a(b+c) = ab + ac$

(v) $16 + 0 = 16$ (vi) $100 \times 1 = 100$

(vii) $4 \times (5 \times 8) = (4 \times 5) \times 8$ (viii) $ab = ba$

Here are the properties used in each equation:

(i) $(a+4)+b = a+(4+b)$

Ans. **Property:** *Associative Property of Addition*

(ii) $\sqrt{2}+\sqrt{3} = \sqrt{3}+\sqrt{2}$

Ans. **Property:** *Commutative Property of Addition*

(iii) $x - x = 0$

Ans. **Property:** *Additive Inverse Property*

(iv) $a(b+c) = ab + ac$

Ans. **Property:** *Distributive Property of Multiplication over Addition*

(v) $16 + 0 = 16$

Ans. **Property:** *Additive Identity Property*

(vi) $100 \times 1 = 100$

Ans. **Property:** *Multiplicative Identity Property*

(vii) $4 \times (5 \times 8) = (4 \times 5) \times 8$

Ans. **Property:** *Associative Property of Multiplication*

(viii) $ab = ba$

Ans. **Property:** *Commutative Property of Multiplication*

5. **Name the property used in the following:**

(i) $-3 < -1 \Rightarrow 0 < 2$

(ii) If $a < b$ then $\frac{1}{a} > \frac{1}{b}$

(iii) If $a < b$ Then $a + c < b + c$

(iv) If $ac < bc$ and $c > 0$ then $a < b$

(v) If $ac < bc$ and $c < 0$ then $a > b$

(vi) Either $a > b$ or $a = b$ or $a < b$

(i) $-3 < -1 \Rightarrow < 2$

Property: *Additive Property*

(ii) *If* $a < b$*, then* $\frac{1}{a} > \frac{1}{b}$

Property: *Reciprocal Property*

(iii) *If* $a < b$*, then* $a + c < b + c$

Property: *Additive Property of Inequalities*

(iv) *If* $ac < bc$ *and* $c > 0$*, then* $a < b$

Property: *Multiplicative Property of Inequalities*

(v) *If* $ac < bc$ *and* $c < 0$*, then* $a > b$

Property: *Multiplicative Property of Inequalities*

(vi) *Either* $a > b$*, or* $a = b$*, or* $a < b$

Property: *Trichotomy Property*

6. Insert two rational numbers between

(i) $\frac{1}{3}$ and $\frac{1}{4}$ (ii) 3 and 4

(iii) $\frac{3}{5}$ and $\frac{4}{5}$

Solution:

(i) **Two rational numbers between $\frac{1}{3}$ and $\frac{1}{4}$:**

Average of $\frac{1}{3}$ and $\frac{1}{4}$ $= \left(\frac{1}{3}+\frac{1}{4}\right)\div 2$

$= \left(\frac{4+3}{12}\right)\times\frac{1}{2}$

$= \frac{7}{12}\times\frac{1}{2} = \frac{7}{24}$

Average of $\frac{1}{3}$ and $\frac{7}{24}$ $= \left(\frac{1}{3}+\frac{7}{24}\right)\div 2$

$= \left(\frac{8+7}{24}\right)\times\frac{1}{2}$

$= \frac{15}{24}\times\frac{1}{2} = \frac{15}{48} = \frac{5}{16}$

Thus, $\frac{5}{16}$ and $\frac{7}{24}$ are two rational numbers between $\frac{1}{3}$ and $\frac{1}{4}$.

(ii) **Two rational numbers between 3 and 4.**

Average of 3 and 4 $= \frac{3+4}{2} = \frac{7}{2}$

Average of $\frac{7}{2}$ and 4 $= \left(\frac{7}{2}+4\right)\div 2$

$= \left(\frac{7+8}{2}\right)\times\frac{1}{2}$

$= \frac{15}{2}\times\frac{1}{2} = \frac{15}{4}$

Thus, $\frac{7}{2}$ and $\frac{15}{4}$ are two rational numbers between 3 and 4.

(iii) **Two rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$.**

Average of $\frac{3}{5}$ and $\frac{4}{5}$ $= \left(\frac{3}{5}+\frac{4}{5}\right)\div 2$

$= \left(\frac{3+4}{5}\right)\times\frac{1}{2}$

$= \frac{7}{5}\times\frac{1}{2} = \frac{7}{10}$

Average of $\frac{7}{10}$ and $\frac{4}{5}$ $= \left(\frac{7}{10}+\frac{4}{5}\right)\div 2$

$= \left(\frac{7+8}{10}\right)\times\frac{1}{2}$

$$= \frac{15}{10} \times \frac{1}{2}$$

$$= \frac{15}{20} = \frac{3}{4}$$

Thus, $\frac{7}{10}$ and $\frac{3}{4}$ are two rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$.

Laws of Radicals and Indices

Laws of Radical

(i) $\sqrt[n]{ab} = \sqrt[n]{a}, \sqrt[n]{b}$

(ii) $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

(iii) $\sqrt[n]{a^m} = (\sqrt[n]{a})^m$

(iv) $\left(\sqrt[n]{a}\right)^n = (a^{\frac{1}{n}})^n = a$

Laws of Indices

(i) $a^m . a^n = a^{m+n}$

(ii) $(a^m)^n = a^{mn}$

(iii) $(ab)^n = a^n b^n$

(iv) $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

(v) $\frac{a^m}{a^n} = a^{m-n}$

(vi) $a^0 = 1$

Example 7: Simplify the following:

(i) $\sqrt[4]{16x^4y^8}$ (ii) $\sqrt[3]{27x^6y^9z^3}$

(iii) $(64)^{-\frac{4}{3}}$

Solution: (i) $\sqrt[4]{16x^4y^3}$

$= (16x^4y^3)^{\frac{1}{4}}$ $\quad \sqrt[n]{a} = a^{\frac{1}{n}})$

$= (16)^{\frac{1}{4}}(x^4)^{\frac{1}{4}}(y^3)^{\frac{1}{4}}$ $\quad \because (ab)^m = a^m b^m$

$= 2^{4\cdot\frac{1}{4}} \times x^{4\cdot\frac{1}{4}} \times y^{8\cdot\frac{1}{4}}$ $\quad \because (a^m)^n = a^{mn}$

$= 2xy^2$

(ii) $\sqrt[3]{27x^6y^9z^3}$

$= \left(27x^6y^9z^3\right)^{\frac{1}{3}}$ $\quad \because \sqrt[n]{a} = a^{\frac{1}{n}}$

$= (27)^{\frac{1}{3}}\left(x^6\right)^{\frac{1}{3}}\left(y^9\right)^{\frac{1}{3}}\left(z^3\right)^{\frac{1}{3}}$ $\quad \because (ab)^m = a^m b^m$

$= \left(3^3\right)^{\frac{1}{3}}\left(x^6\right)^{\frac{1}{3}}\left(y^9\right)^{\frac{1}{3}}\left(z^3\right)^{\frac{1}{3}}$ $\quad \because (ab)^m = a^m b^m$

$= 3^{3\times\frac{1}{3}} \cdot x^{6\times\frac{1}{3}} \cdot y^{9\times\frac{1}{3}} . z^{3\times\frac{1}{3}}$

$= 3x^2y^3z$

(iii) $(64)^{-\frac{4}{3}}$

$= \frac{1}{(64)^{\frac{4}{3}}} = \frac{1}{(4)^{\frac{4}{3}}} = \frac{1}{4^{3\times\frac{4}{3}}}$

$= \frac{1}{4^4} = \frac{1}{256}$

Example 8: Rationalize the denominator of (i) $\frac{3}{\sqrt{5}+\sqrt{2}}$

(ii) $\frac{3}{\sqrt{5}-\sqrt{3}}$

Solution (i):

$$\frac{3}{\sqrt{5}+\sqrt{2}} = \frac{3}{\sqrt{5}+\sqrt{2}} \times \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}$$

$$= \frac{3(\sqrt{5}-\sqrt{2})}{(\sqrt{5})^2-(\sqrt{2})^2} = \frac{3\sqrt{5}-\sqrt{2}}{5-2}$$

$$= \frac{3(\sqrt{5}-\sqrt{2})}{3} = \sqrt{5}-\sqrt{2}$$

(ii) $$\frac{3}{\sqrt{5}-\sqrt{3}}=\frac{3}{\sqrt{5}-\sqrt{3}}\times\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$

$$=\frac{3(\sqrt{5}+\sqrt{3})}{(\sqrt{5})^2-(\sqrt{3})^2}=\frac{3(\sqrt{5}+\sqrt{3})}{5-3}$$

$$=\frac{3(\sqrt{5}+\sqrt{3})}{2}$$

EXERCISE 1.2

1. Rationalize the denominator of following:

(i) $\frac{13}{4+\sqrt{3}}$ (ii) $\frac{\sqrt{2}+\sqrt{5}}{\sqrt{3}}$

(iii) $\frac{\sqrt{2}-1}{\sqrt{5}}$ (iv) $\frac{6-4\sqrt{2}}{6+4\sqrt{2}}$

(v) $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ (vi) $\frac{4\sqrt{3}}{\sqrt{7}+\sqrt{5}}$

Solutions:

(i) $\frac{13}{4+\sqrt{3}}$

$$\frac{13}{4+\sqrt{3}}\times\frac{4-\sqrt{3}}{4-\sqrt{3}}=\frac{13(4-\sqrt{3})}{(4+\sqrt{3})(4-\sqrt{3})}$$

First, simplify the denominator using the difference of squares formula:

$$(4+\sqrt{3})(4-\sqrt{3})=4^2-(\sqrt{3})^2=16-3=13$$

Now simplify the numerator:

$$13(4-\sqrt{3})=52-13\sqrt{3}$$

So the rationalized expression is:

$$\frac{52-13\sqrt{3}}{13}$$

Simplifying: $=\frac{52}{13}-\frac{13\sqrt{3}}{13}=4-\sqrt{3}$

(ii) $\frac{\sqrt{2}+\sqrt{5}}{\sqrt{3}}$

$$\frac{\sqrt{2}+\sqrt{5}}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}=\frac{(\sqrt{2}+\sqrt{5})\cdot\sqrt{3}}{\sqrt{3}\cdot\sqrt{3}}$$

Simplify the denominator:

$$\sqrt{3}\cdot\sqrt{3}=3$$

Now simplify the numerator:

$$(\sqrt{2}+\sqrt{5})\cdot\sqrt{3}=\sqrt{6}+\sqrt{15}$$

Thus, the rationalized expression is:

$$\frac{\sqrt{6}+\sqrt{15}}{3}$$

(iii) $\frac{\sqrt{2}-1}{\sqrt{5}}$

$$\frac{\sqrt{2}-1}{\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}}=\frac{(\sqrt{2}-1)\cdot\sqrt{5}}{\sqrt{5}\cdot\sqrt{5}}$$

Simplify the denominator:

$$\sqrt{5}\cdot\sqrt{5}=5$$

Now simplify the numerator:

$$(\sqrt{2}-1)\cdot\sqrt{5}=\sqrt{10}-\sqrt{5}$$

Thus, the rationalized expression is:

$$\frac{\sqrt{10}-\sqrt{5}}{5}$$

(iv) $\frac{6-4\sqrt{2}}{6+4\sqrt{2}}$

$$\frac{6-4\sqrt{2}}{6+4\sqrt{2}}\times\frac{6-4\sqrt{2}}{6-4\sqrt{2}}=\frac{(6-4\sqrt{2})^2}{(6+4\sqrt{2})(6-4\sqrt{2})}$$

First, simplify the denominator using the difference of squares formula:

$$(6+4\sqrt{2})(6-4\sqrt{2})=6^2-(4\sqrt{2})^2=36-32=4$$

Now simplify the numerator:

$$(6-4\sqrt{2})^2 = 6^2 - 2\cdot 6\cdot 4\sqrt{2} + (4\sqrt{2})^2 = 36 - 48\sqrt{2} + 32$$
$$= 68 - 48\sqrt{2}$$

Thus, the rationalized expression is:

$$\frac{68-48\sqrt{2}}{4} = \frac{68}{4} - \frac{48\sqrt{2}}{4} = 17 - 12\sqrt{2}$$

(v) $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

$$\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \frac{(\sqrt{3}-\sqrt{2})^2}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})}$$

First, simplify the denominator using the difference of squares formula:

$$(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2}) = (\sqrt{3})^2 - (\sqrt{2})^2 = 3 - 2 = 1$$

Now simplify the numerator:

$$(\sqrt{3}-\sqrt{2})^2 = (\sqrt{3})^2 - 2\cdot\sqrt{3}\cdot\sqrt{2} + (\sqrt{2})^2 = 3 - 2\sqrt{6} + 2$$
$$= 5 - 2\sqrt{6}$$

Thus, the rationalized expression is:

$$5 - 2\sqrt{6}$$

(vi) $\frac{4\sqrt{3}}{\sqrt{7}+\sqrt{5}}$

$$\frac{4\sqrt{3}}{\sqrt{7}+\sqrt{5}} \times \frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}-\sqrt{5}} = \frac{4\sqrt{3}(\sqrt{7}-\sqrt{5})}{(\sqrt{7}+\sqrt{5})(\sqrt{7}-\sqrt{5})}$$

The denominator is now in the form of $(a+b)(a-b) = a^2 - b^2$. So:

$$(\sqrt{7}+\sqrt{5})(\sqrt{7}-\sqrt{5}) = 7 - 5 = 2$$

Now, distribute $4\sqrt{3}$ across $(\sqrt{7}-\sqrt{5})$:

$$4\sqrt{3}(\sqrt{7}-\sqrt{5})$$

Now, the expression becomes:

$$\frac{4\sqrt{3}(\sqrt{7}-\sqrt{5})}{2} = 2\sqrt{3}(\sqrt{7}-\sqrt{5})$$

2. **Simplify the following**

(i) $\left(\frac{81}{16}\right)^{-\frac{3}{4}}$ (ii) $\left(\frac{3}{4}\right)^{-2} \div \left(\frac{4}{9}\right)^{3} \times \frac{16}{27}$

(iii) $(0.027)^{-\frac{1}{3}}$ (iv) $\sqrt[7]{\frac{x^{14} \times y^{21} \times z^{35}}{y^{14} z^{7}}}$

(v) $\frac{5\cdot(25)^{n+1} - 25\cdot(5)^{2n}}{5\cdot(25)^{2n+3} - (25)^{n+1}}$ (vi) $\frac{(16)^{x+1} + 20(4^{2x})}{2^{x-3} \times 8^{x+2}}$

(vii) $(64)^{-\frac{2}{3}} \div (9)^{-\frac{3}{2}}$ (viii) $\frac{3^{n} \times 9^{n+1}}{3^{n-1} \times 9^{n-1}}$

(ix) $\frac{5^{n+3} - 6.5^{n+1}}{9 \times 5^{n} - 2^{n} \times 5^{n}}$

(i) $\left(\frac{81}{16}\right)^{-\frac{3}{4}}$

$$\left(\frac{81}{16}\right)^{-\frac{3}{4}} = \frac{1}{\left(\frac{81}{16}\right)^{\frac{3}{4}}}$$

Next, simplify the expression $\left(\frac{81}{16}\right)^{\frac{3}{4}}$:

$$\left(\frac{81}{16}\right)^{\frac{3}{4}} = \frac{81^{\frac{3}{4}}}{16^{\frac{3}{4}}}$$

Now calculate the powers:

$$81 = 3^4 \quad \Rightarrow \quad 81^{\frac{3}{4}} = (3^4)^{\frac{3}{4}} = 3^3 = 27$$

$$16 = 2^4 \quad \Rightarrow \quad 16^{\frac{3}{4}} = (2^4)^{\frac{3}{4}} = 2^3 = 8$$

Thus:

$$\frac{81^{\frac{3}{4}}}{16^{\frac{3}{4}}} = \frac{27}{8}$$

Therefore, the simplified expression is:

$$\frac{1}{\frac{27}{8}} = \frac{8}{27}$$

(ii) $\left(\frac{3}{4}\right)^{-2} \div \left(\frac{4}{9}\right)^{3} \times \frac{16}{27}$

First, simplify each part step by step.

$$\left(\frac{3}{4}\right)^{-2} = \frac{1}{\left(\frac{3}{4}\right)^{2}} = \frac{1}{\frac{9}{16}} = \frac{16}{9}$$

$$\left(\frac{4}{9}\right)^{3} = \frac{4^3}{9^3} = \frac{64}{729}$$

Now combine all the terms:

$$\frac{16}{9} \div \frac{64}{729} \times \frac{16}{27}$$

To divide by a fraction, multiply by its reciprocal:

$$\frac{16}{9} \times \frac{729}{64} \times \frac{16}{27}$$

Multiply the numerators and denominators:

$$\frac{16 \times 729 \times 16}{9 \times 64 \times 27} = \frac{16^2 \times 729}{9 \times 64 \times 27}$$

Simplify the expression step by step:

$$16^2 = 256 \quad \text{and} \quad 64 \times 27 = 1728$$

Now simplify:

$$\frac{256 \times 729}{9 \times 1728}$$

Factor the denominator:

$$9 \times 1728 = 15552$$

Thus, the simplified result is: $\frac{256 \times 729}{15552} = \frac{186624}{15552} = 12$

(iii) $(0.027)^{-\frac{1}{3}}$

Simplify $(0.027)^{-\frac{1}{3}}$

Expression $(0.027)^{-\frac{1}{3}} = (27)^{-\frac{1}{3}} \times (10^{-3})^{-\frac{1}{3}}$

$$= (3^3)^{-\frac{1}{3}} \times (10^{-3})^{-\frac{1}{3}}$$

$$= 3^{3 \times -\frac{1}{3}} \times 10^{-3 \times \frac{-1}{3}}$$

$$= 3^{-1} \times 10^{1}$$

$$= \frac{10}{3}$$

(iv) $\sqrt[7]{\frac{x^{14}y^{21}z^{35}}{y^{14}z^{7}}}$

First, simplify the expression inside the radical:

$$\frac{x^{14}y^{21}z^{35}}{y^{14}z^{7}} = x^{14} \times y^{21-14} \times z^{35-7} = x^{14} \times y^{7} \times z^{28}$$

Now take the 7th root of each term:

$$\sqrt[7]{x^{14}} = x^2, \quad \sqrt[7]{y^7} = y, \quad \sqrt[7]{z^{28}} = z^4$$

Thus, the simplified result is:

$$x^2yz^4$$

(v) $\frac{5\cdot(25)^{n+1} - 25\cdot(5)^{2n}}{5\cdot(5)^{2n+3} - (25)^{n+1}}$

Sol: $\frac{5\cdot(25)^{n+1} - 25\cdot(5)^{2n}}{5\cdot(5)^{2n+3} - (25)^{n+1}}$

Expression $= \frac{5(25)^{n+1} 25(5)^{2n}}{5(5)^{2n+3} - (25)^{2n+1}} = \frac{5(5^2)^{n+a} - 5^2(5)^{2n}}{5(5)^{2n+3} - (5^2)^{n+1}}$

$$= \frac{5^{2n+3} - 5^{2n+2}}{5^{2n+4} - 5^{2n+2}} = \frac{5^{2n+2}(5-1)}{5^{2n+2}(5^2-1)} = \frac{4}{24} = \frac{1}{6}$$

(vi) $\frac{(16)^{x+1}+20(4^{2x})}{2^{x-3}\times 8^{x+2}}$

Simplify the powers of 16 and 4 in terms of powers of 2:

$$16 = 2^4, \quad 4 = 2^2, \quad 8 = 2^3$$

So:

$$\frac{(2^4)^{x+1} + 20(2^2)^{2x}}{2^{x-3} \times (2^3)^{x+2}}$$

Simplify the powers:

$$= \frac{2^{4(x+1)} + 20 \cdot 2^{4x}}{2^{x-3} \times 2^{3(x+2)}}$$

Simplify:

$$= \frac{2^{4x+4} + 20 \cdot 2^{4x}}{2^{x-3} \times 2^{3x+6}}$$

Now factor out 2^{4x} from the numerator:

$$= \frac{2^{4x}(2^4 + 20)}{2^{x-3} \times 2^{3x+6}}$$

Simplify further:

$$= \frac{2^{4x} \times 36}{2^{x-3} \times 2^{3x+6}} = \frac{36 \times 2^{4x}}{2^{4x+3}}$$

$$= \frac{36}{2^3}$$

Cancel out the 2^{4x}:

$$= \frac{36}{8} = \frac{9}{2}$$

(vii) $(64)^{-\frac{2}{3}} \div (9)^{-\frac{3}{2}}$

We begin by rewriting 64 and 9 as powers of 2 and 3, respectively:

$$64 = 2^6 \quad \text{and} \quad 9 = 3^2$$

Now apply the negative exponents:

$$(64)^{-\frac{2}{3}} = (2^6)^{-\frac{2}{3}} = 2^{-4}$$

$$(9)^{-\frac{3}{2}} = (3^2)^{-\frac{3}{2}} = 3^{-3}$$

Now simplify the division:

$$2^{-4} \div 3^{-3} = 2^{-4} \times 3^3 = \frac{3^3}{2^4}$$

Simplifying powers:

$$3^3 = 27 \quad \text{and} \quad 2^4 = 16$$

Thus, the simplified result is:

$$\frac{27}{16}$$

(viii) $\frac{3^n \times 9^{n+1}}{3^{n-1} \times 9^{n-1}}$

First, rewrite 9 as 3^2:

$$9^{n+1} = (3^2)^{n+1} = 3^{2(n+1)} = 3^{2n+2}$$

$$9^{n-1} = (3^2)^{n-1} = 3^{2(n-1)} = 3^{2n-2}$$

Now simplify the expression:

$$\frac{3^n \times 3^{2n+2}}{3^{n-1} \times 3^{2n-2}}$$

Use the property of exponents $a^m.a^n = a^{m+n}$

$$= \frac{3^{n+2n+2}}{3^{n-1+2n-2}} = \frac{3^{3n+2}}{3^{3n-3}}$$

Now subtract the exponents:

$$= 3^{(3n+2)-(3n-3)} = 3^5$$

Thus, the simplified result is:

$$3^5 = 243$$

(ix) $\frac{5^{n+3} - 6.5^{n+1}}{9 \times 5^n - 2^2 \times 5^n}$

First, factor the denominator:

$$9 \times 5^n - 2^2 \times 5^n = 5^n(9 - 2^2)$$

Thus, the expression becomes:

$$\frac{5^{n+3} - 6.5^{n+1}}{5^n(9 - 2^2)}$$

Now factor out 5^n from the numerator:

$$5^{n+3} = 5^n \times 5^3 = 5^n \times 125$$

So, the expression becomes:

$$= \frac{5^n(125 - 6.5)}{5^n(9 - 2^2)}$$

$$= \frac{125 - 30}{9 - 2^2}$$

$$= \frac{95}{9-2^n} = \frac{\overset{19}{\cancel{95}}}{\underset{1}{\cancel{5}}} = 19$$

3. If $x = 3 + \sqrt{8}$ then find the value of

(i) $x + \frac{1}{x}$ (ii) $x - \frac{1}{x}$

(iii) $x^2 + \frac{1}{x^2}$ (iv) $x^2 - \frac{1}{x^2}$

(v) $x^4 + \frac{1}{x^4}$ (vi) $\left(x - \frac{1}{x}\right)^2$

Given $x = 3 + \sqrt{8}$

(i) $x + \frac{1}{x}$

$$x = 3 + \sqrt{8}$$

$$\frac{1}{x} = \frac{1}{3+\sqrt{8}} \times \frac{3-\sqrt{8}}{3-\sqrt{8}} = \frac{3-\sqrt{8}}{(3+\sqrt{8})(3-\sqrt{8})}$$

Now, simplify the denominator using the difference of squares formula $(a + b)(a - b) = a^2 - b^2$:

$$(3+\sqrt{8})(3-\sqrt{8}) = 3^2 - (\sqrt{8})^2 = 9 - 8 = 1$$

So,

$$\frac{1}{x} = 3 - \sqrt{8}$$

Now, calculate $x + \frac{1}{x}$:

$$x + \frac{1}{x} = (3+\sqrt{8}) + (3-\sqrt{8}) = 3 + 3 = 6$$

Thus, the value of $x + \frac{1}{x}$ is:

$\boxed{6}$

(ii) $x - \frac{1}{x}$

$$x = 3 + \sqrt{8} \quad \text{and} \quad \frac{1}{x} = 3 - \sqrt{8}$$

Now, calculate $x - \frac{1}{x}$:

$$x - \frac{1}{x} = (3+\sqrt{8}) - (3-\sqrt{8}) = 3 + \sqrt{8} - 3 + \sqrt{8} = 2\sqrt{8}$$

Since $\sqrt{8} = 2\sqrt{2}$, we have:

$$x - \frac{1}{x} = 2 \times 2\sqrt{2} = 4\sqrt{2}$$

Thus, the value of $x - \frac{1}{x}$ is:

$\boxed{4\sqrt{2}}$

(iii) $x^2 + \frac{1}{x^2}$

$$x = 3 + \sqrt{8}$$

$$\frac{1}{x} = \frac{1}{3+\sqrt{8}} \times \frac{3-\sqrt{8}}{3-\sqrt{8}}$$

$$= \frac{3-\sqrt{8}}{(3)^2 - (\sqrt{8})^2}$$

$$= \frac{3-\sqrt{8}}{9-8} = \frac{3-\sqrt{8}}{1}$$

$$\frac{1}{x} = 3 - \sqrt{8}$$

Now, $x + \frac{1}{x} = (3+\sqrt{8}) + (3-\sqrt{8})$

$= 3 + 3 = 6$

$$x + \frac{1}{x} = 6$$

$$\left(x + \frac{1}{x}\right)^2 = (6)^2$$

$$x^2 + \frac{1}{x^2} + 2 = 36$$

$$x^2 + \frac{1}{x^2} + 2 = 36 - 2$$

$$x^2 + \frac{1}{x^2} + 2 = 34$$

(iv) $x^2 - \frac{1}{x^2}$

First, we square $x = 3 + \sqrt{8}$:

$$x^2 = \left(3 + \sqrt{8}\right)^2$$

Expanding:

$$x^2 = 3^2 + 2(3)(\sqrt{8}) + (\sqrt{8})^2$$
$$x^2 = 9 + 6\sqrt{8} + 8$$
$$x^2 = 17 + 6\sqrt{8}$$

Now,

$$\frac{1}{x} = \frac{1}{3 + \sqrt{8}} \times \frac{3 - \sqrt{8}}{3 - \sqrt{8}}$$

Now, we simplify the denominator:

$$(3 + \sqrt{8})(3 - \sqrt{8}) = 3^2 - (\sqrt{8})^2 = 9 - 8 = 1$$

So, $\frac{1}{x} = 3 - \sqrt{8}$.

Next, square $\frac{1}{x}$:

$$\frac{1}{x^2} = (3 - \sqrt{8})^2$$

Expanding:

$$\frac{1}{x^2} = 3^2 - 2(3)(\sqrt{8}) + (\sqrt{8})^2$$
$$\frac{1}{x^2} = 9 - 6\sqrt{8} + 8$$
$$\frac{1}{x^2} = 17 - 6\sqrt{8}$$

Now, subtract $\frac{1}{x^2}$ from x^2:

$$x^2 - \frac{1}{x^2} = (17 + 6\sqrt{8}) - (17 - 6\sqrt{8})$$
$$x^2 - \frac{1}{x^2} = 17 + 6\sqrt{8} - 17 + 6\sqrt{8}$$
$$x^2 - \frac{1}{x^2} = 12\sqrt{8}$$
$$x^2 - \frac{1}{x^2} = 12 \times 2\sqrt{2} = 24\sqrt{2}$$

(v) $x^4 + \frac{1}{x^4}$

First, we square $x = 3 + \sqrt{8}$:

$$x^2 = \left(3 + \sqrt{8}\right)^2$$

Expanding:

$$x^2 = 3^2 + 2(3)(\sqrt{8}) + (\sqrt{8})^2$$
$$x^2 = 9 + 6\sqrt{8} + 8$$
$$x^2 = 17 + 6\sqrt{8}$$

Now, $(x^2)^2 = \left(17 + 6\sqrt{8}\right)^2$

$$x^4 = 17^2 + 2(17)\left(6\sqrt{8}\right) + \left(6\sqrt{8}\right)^2$$
$$= 289 + 204\sqrt{8} + 288$$
$$= 577 + 204\sqrt{8}$$

Now, $\frac{1}{x} = \frac{1}{3+\sqrt{8}} \times \frac{3-\sqrt{8}}{3-\sqrt{8}}$

Now, we simplify the denominator:

$$(3 + \sqrt{8})(3 - \sqrt{8}) = 3^2 - (\sqrt{8})^2 = 9 - 8 = 1$$

So, $\frac{1}{x} = 3 - \sqrt{8}$.

Next, square $\frac{1}{x}$:

$$\frac{1}{x^2} = (3-\sqrt{8})^2$$

Expanding:

$$\frac{1}{x^2} = 3^2 - 2(3)(\sqrt{8}) + (\sqrt{8})^2$$

$$\frac{1}{x^2} = 9 - 6\sqrt{8} + 8$$

$$\frac{1}{x^2} = 17 - 6\sqrt{8}$$

Now;

$$\left(\frac{1}{x^2}\right)^2 = \left(17-6\sqrt{8}\right)^2$$

$$\frac{1}{x^4} = (17)^2 - 2(17)\left(6\sqrt{8}\right) + \left(6\sqrt{8}\right)^2$$

$$= 289 - 204\sqrt{8} + 288$$

$$= 577 - 204\sqrt{8}$$

Now, add x^4 and $\frac{1}{x^4}$:

$$x^4 + \frac{1}{x^4} = (577 + 204\sqrt{8}) + (577 - 204\sqrt{8})$$

$$x^4 + \frac{1}{x^4} = 577 + 577$$

$$x^4 + \frac{1}{x^4} = 1154$$

(vi) $\left(x - \frac{1}{x}\right)^2$

$$x = 3 + \sqrt{8}$$

$$\frac{1}{x} = \frac{1}{3+\sqrt{8}} \times \frac{3-\sqrt{8}}{3-\sqrt{8}}$$

$$= \frac{3-\sqrt{8}}{(3)^2 - \left(\sqrt{8}\right)^2}$$

$$= \frac{3-\sqrt{8}}{9-8}$$

$$= \frac{3-\sqrt{8}}{1}$$

$$\frac{1}{x} = 3 - \sqrt{8}$$

Now, subtract $\frac{1}{x}$ from x:

$$x - \frac{1}{x} = (3+\sqrt{8}) - (3-\sqrt{8})$$

$$x - \frac{1}{x} = 3 + \sqrt{8} - 3 + \sqrt{8}$$

$$x - \frac{1}{x} = 2\sqrt{8} = 4\sqrt{2}$$

Now, square $x - \frac{1}{x}$:

$$\left(x - \frac{1}{x}\right)^2 = (4\sqrt{2})^2$$

$$\left(x - \frac{1}{x}\right)^2 = 16 \times 2 = 32$$

4. Find the rational numbers p and q such that $\frac{8-3\sqrt{2}}{4+3\sqrt{2}} = p + q\sqrt{2}$

Multiply the expression by $\frac{4-3\sqrt{2}}{4-3\sqrt{2}}$:

$$\frac{8-3\sqrt{2}}{4+3\sqrt{2}} \times \frac{4-3\sqrt{2}}{4-3\sqrt{2}} = \frac{(8-3\sqrt{2})(4-3\sqrt{2})}{(4+3\sqrt{2})(4-3\sqrt{2})}$$

The denominator is a difference of squares:

$$(4+3\sqrt{2})(4-3\sqrt{2}) = 4^2 - (3\sqrt{2})^2 = 16 - 18 = -2$$

Now, expand the numerator $(8-3\sqrt{2})(4-3\sqrt{2})$:

$$(8-3\sqrt{2})(4-3\sqrt{2}) = 8 \cdot 4 + 8 \cdot (-3\sqrt{2}) + (-3\sqrt{2}) \cdot 4 + (-3\sqrt{2}) \cdot (-3\sqrt{2})$$

$$= 32 - 24\sqrt{2} - 12\sqrt{2} + 18$$
$$= 32 + 18 - 36\sqrt{2}$$
$$= 50 - 36\sqrt{2}$$

We now have:

$$\frac{50 - 36\sqrt{2}}{-2}$$

$$= \frac{50}{-2} - \frac{36\sqrt{2}}{-2} = -25 + 18\sqrt{2}$$

Now we have the expression:

$$-25 + 18\sqrt{2}$$

This is in the form $p + q\sqrt{2}$, where $p = -25$ and $q = 18$.

5. **Simplify the following:**

(i) $\frac{(25)^{\frac{3}{2}} \times (243)^{\frac{3}{5}}}{(16)^{\frac{5}{4}} \times (8)^{\frac{4}{3}}}$ (ii) $\frac{54 \times \sqrt[3]{(27)^{2x}}}{9^{x+1} + 216(3^{2x-1})}$

(iii) $\sqrt{\frac{(216)^{\frac{2}{3}} \times (25)^{\frac{1}{2}}}{(0.04)^{\frac{-3}{2}}}}$

(iv) $\left(a^{\frac{1}{3}} + b^{\frac{2}{3}}\right) \times \left(a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{2}{3}} + b^{\frac{4}{3}}\right)$

(i) $\frac{(25)^{\frac{3}{2}} \times (243)^{\frac{3}{5}}}{(16)^{\frac{5}{4}} \times (8)^{\frac{4}{3}}}$

Sol: $\frac{(25)^{\frac{3}{2}} \times (243)^{\frac{3}{5}}}{(16)^{\frac{5}{4}} \times (8)^{\frac{4}{3}}}$

$$= \frac{(5^2)^{\frac{3}{2}} \times (3^5)^{\frac{3}{5}}}{(2^4)^{\frac{5}{4}} \times (2^3)^{\frac{4}{3}}}$$

$$= \frac{5^3 \times 3^3}{2^5 \times 2^4}$$

$$= \frac{125 \times 27}{32 \times 16}$$

$$= \frac{3375}{512}$$

(ii) $\frac{54 \times \sqrt[3]{(27)^{2x}}}{9^{x+1} + 216(3^{2x-1})}$

Sol: $\frac{54 \times \sqrt[3]{(27)^{2x}}}{9^{x+1} + 216(3^{2x-1})}$

$$= \frac{54 \times \sqrt[3]{(3^3)^{2x}}}{(3^2)^{x+1} + 3^3 \times 2^3 (3^{2x-1})}$$

$$= \frac{54 \times 3^{2x}}{3^{2x+2} + 2^3 \times 3^{2x+2}}$$

$$= \frac{54 \times 3^{2x}}{3^{2x+2}(1+8)}$$

$$= \frac{54 \times 3^{2x}}{9 \times 3^{2x+2}} = \frac{54}{9} \times \frac{1}{3^2} = 6 \times \frac{1}{9} = \frac{6}{9} = \frac{2}{3}$$

(iii) $\sqrt{\frac{(216)^{\frac{2}{3}} \times (25)^{\frac{1}{2}}}{(0.04)^{\frac{-3}{2}}}}$

$$= \sqrt{\frac{(6^3)^{\frac{2}{3}} \times (5^2)^{\frac{1}{2}}}{\left(\frac{1}{25}\right)^{\frac{-3}{2}}}}$$

$$= \sqrt{\frac{6^2 \times 5}{25^{\frac{3}{2}}}}$$

$$\sqrt{\frac{36\times5}{125}}=\sqrt{\frac{180}{125}}=\sqrt{\frac{36}{25}}=\frac{6}{5}$$

(iv) $\left(a^{\frac{1}{3}}+b^{\frac{2}{3}}\right)\times\left(a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{2}{3}}+b^{\frac{4}{3}}\right)$

Sol: $\left(a^{\frac{1}{3}}+b^{\frac{2}{3}}\right)\times\left(a^{\frac{2}{3}}-a^{\frac{1}{3}}b^{\frac{2}{3}}+b^{\frac{4}{3}}\right)$

$$=\left(a^{\frac{1}{3}}+b^{\frac{2}{3}}\right)\times\left[\left(a^{\frac{1}{3}}\right)^2-\left(a^{\frac{1}{3}}\right)\left(b^{\frac{2}{3}}\right)+\left(b^{\frac{2}{3}}\right)^2\right]$$

$$\because (x+y)(x^2-xy+y^2)=x^3+y^3$$

$$\left(a^{\frac{1}{3}}\right)^3+\left(b^{\frac{2}{3}}\right)^3=a^{\frac{1}{3}\times3}+b^{\frac{2}{3}\times3}=a+b^2$$

Example 9: The sum of two real numbers is 8, and their difference is 2. Find the numbers.

Solution: Let a and b be two real numbers then

$a+b=8$ ...(i)

$a-b=2$ ...(ii)

Add eq. (i)and eq. (ii)

$2a=10\Rightarrow a=5$ put in eq. (i)

$\Rightarrow 5-b=2\Rightarrow -b=2-5\Rightarrow -b=-3\Rightarrow b=3$

So, 5 and 3 are required real numbers

Example 10: Normal human body temperature is 98.6 F. Convert it into Celsius and kelvin scale.

Solution: Given that F = 98.6

So convert it into Celsius scale, we use

$$C=\frac{5}{9}(F-32)$$

$$C=\frac{5}{9}(98.6-32)$$

$$=\frac{5}{9}(66.6)$$

$$=(0.55)(66.6)$$

$$C=37^\circ$$

Hence, normal human body temperature at Celsius scale is 37°

(i) Profit = selling Price – cost price

P = SP – CP

$$\text{Profit \%}=\left(\frac{profit}{CP}\times100\right)\%$$

(ii) Loss = cost price – selling price

Loss = CP – SP

$$\text{Loss \%}=\left(\frac{loss}{CP}\times100\right)\%$$

Example 11: Hamail purchased a bicycle for Rs. 6590 and sold it for 6850. Find the profit percentage.

Solution:

Cost Price = CP = Rs. 6590

Selling Price = SP = Rs. 6850

Profit = SP – CP

= 6850 – 6590

= Rs 260

Now, we find the profit percentage.

$$\text{Profit \%}=\left(\frac{profit}{CP}\times100\right)\%$$

$$=\left(\frac{260\times100}{6590}\right)\%$$

$$=3.94\%$$

$$\approx 4\%$$

Example 12: Umair bought a book for Rs. 850 and sold it for Rs.720. What was his loss percentage?

Solution:

Cost price of book = CP = Rs. 850

Selling price of book = SP = Rs. 720

Loss = CP – SP

$= 850 - 720$

$= \text{Rs. } 130$

Loss percentage $= \left(\frac{Loss}{CP} \times 100\right)\%$

$= \left(\frac{130}{850} \times 100\right)\%$

$= 15.29\%$

Example 13: Saleem, Nadeem, and Tanveer earned a profit of Rs. 4,50,000 from a business. If their investments in the business are the ratio 4: 7: 14, find each person's profit.

Solution: Profit earned = Rs. 4,50,000

Given ratio = 4: 7: 14

Sum of ratio = 4 + 7 + 1 = 25

Saleem earned profit $= \frac{4}{25} \times 4,50,000$

= Rs. 72,000

Nadeem earned profit $= \frac{7}{25} \times 4,50,000$

= Rs. 126,000

Tanveer earned profit $= \frac{14}{25} \times 4,50,000$

= Rs. 252,000

Example 14: If the simple profit on Rs. 6400 for 12 years is Rs. 3840. Find the rate of profit.

Solution: Principal = Rs. 6400

Simple profit = Rs. 3840

Time = 12 years

To find the rate we use the following formula

Rate $= \frac{\text{amount of profit} \times 100}{\text{time} \times \text{principal}}$

$= \frac{3840 \times 100}{12 \times 6400}$

$= 5\%$ Thus, rate of profit is 5%

EXERCISE 1.3

1. The sum of three consecutive integers is forty-two, find three integers.

Solution: Let the three consecutive integers be:

$$x, x+1, x+2$$

The sum of these integers is 42:

$$x + (x+1) + (x+2) = 42$$

Simplifying the equation:

$$3x + 3 = 42$$

Now, subtract 3 from both sides:

$$3x = 39$$

Now, divide by 3:

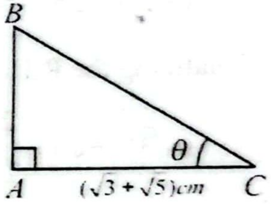
$$x = 13$$

So, the three consecutive integers are:

13,14,15

Thus, the three integers are **13, 14, and 15.**

2. The diagram shows right angled ΔABC in which the length of AC is $(\sqrt{3}+\sqrt{5})$cm. The area of ΔABC is $(1+\sqrt{15})\text{cm}^2$, Find the length AB in the form $(a\sqrt{3}+b\sqrt{5})$cm where a and b are integers.



Solution: Given:

- Triangle ΔABC, where $AC = \sqrt{3} + \sqrt{5}$ cm.
- Area of $\Delta ABC = 1 + \sqrt{15}$ cm².
- AB is to be expressed as $AB = a\sqrt{3} + b\sqrt{5}$ cm, where a and b are integers.

Step-by-Step Solution:

1. Formula for the area of the triangle:

The area of a triangle can be written as:

$$\frac{1}{2} AC.AB$$

Angle $A = 90o$

Angle $A = \theta$

$$\overline{AC} = \sqrt{3}+\sqrt{5}$$

$$AB = a\sqrt{3}+b\sqrt{5}$$

$$\text{Area} = 1+\sqrt{15}$$

a and b are to determined.

$$\text{Area of } \Delta ABC = \frac{1}{2} AC \times AB$$

$$\frac{1}{2} AC \times AB = 1+\sqrt{15}$$

$$\frac{1}{2}\left(\sqrt{3}+\sqrt{5}\right)a\sqrt{3}+b\sqrt{5} = 1+\sqrt{15}$$

$$a\sqrt{3}+b\sqrt{5} = \frac{2\left(1+\sqrt{15}\right)}{\sqrt{3}+\sqrt{5}}$$

Rationalizing the R.H.S

$$a\sqrt{3}+b\sqrt{5} = \frac{2\left(1+\sqrt{15}\right)\left(\sqrt{3}-\sqrt{5}\right)}{\left(\sqrt{3}+\sqrt{5}\right)\left(\sqrt{3}-\sqrt{5}\right)}$$

$$= \frac{2\left(1+\sqrt{15}\right)\left(\sqrt{3}-\sqrt{5}\right)}{3-5}$$

$$= \frac{2\left(1+\sqrt{15}\right)\left(\sqrt{3}-\sqrt{5}\right)}{-2}$$

$$a\sqrt{3}+b\sqrt{5} = -\left(1+\sqrt{15}\right)\left(\sqrt{3}-\sqrt{15}\right)$$

$$a\sqrt{3}+b\sqrt{5} = -\left(1+\sqrt{15}\right)\left(\sqrt{3}-\sqrt{5}\right)$$

$$= -\left[\sqrt{3}-\sqrt{5}+3\sqrt{5}-5\sqrt{3}\right]$$

$$= -\left[2\sqrt{5}-4\sqrt{3}\right]$$

$$= -2\sqrt{5}+4\sqrt{3}$$

$$a\sqrt{3}+b\sqrt{5} = 4\sqrt{3}-2\sqrt{5}$$

Comparing the coefficients of $\sqrt{3}$ and $\sqrt{5}$ we get $a = 4$; $b = -2$

3. A rectangle has sides of length $(2+\sqrt{18})m$ and $(5-\frac{4}{\sqrt{2}})m$.

Expressthe area of the rectangle in the form $a+b\sqrt{2}$ where $a$and$b$ are integers.

Sol: **Problem Statement:**

We are given a rectangle with sides of length:

- $(2+\sqrt{18})$ meters and
- $\left(5-\frac{4}{\sqrt{2}}\right)$ meters.

We are asked to find the area of the rectangle in the form $a + b\sqrt{2}$, where a and b are integers.

Step 1: Simplify the lengths of the sides

FIRST SIDE: $2+\sqrt{18}$

We can simplify $\sqrt{18}$ as:

$$\sqrt{18} = \sqrt{9 \times 2} = \sqrt{9} \times \sqrt{2} = 3\sqrt{2}$$

So, the first side becomes:

$$2+\sqrt{18} = 2+3\sqrt{2}$$

SECOND SIDE: $5-\frac{4}{\sqrt{2}}$

To simplify $\frac{4}{\sqrt{2}}$, multiply both the numerator and denominator by $\sqrt{2}$:

$$\frac{4}{\sqrt{2}} = \frac{4}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{4\sqrt{2}}{2} = 2\sqrt{2}$$

So, the second side becomes:

$$5-\frac{4}{\sqrt{2}} = 5-2\sqrt{2}$$

Step 2: Find the area of the rectangle

The area A of the rectangle is the product of its two sides:

$$A = (2+3\sqrt{2}) \times (5-2\sqrt{2})$$

We now expand this expression using the distributive property (FOIL method):

$$A = 2 \times 5 + 2 \times (-2\sqrt{2}) + 3\sqrt{2} \times 5 + 3\sqrt{2} \times (-2\sqrt{2})$$

Let's simplify each term:

- $2 \times 5 = 10$
- $2 \times (-2\sqrt{2}) = -4\sqrt{2}$
- $3\sqrt{2} \times 5 = 15\sqrt{2}$
- $3\sqrt{2} \times (-2\sqrt{2}) = -6 \times 2 = -12$

Now combine all the terms:

$$A = 10 - 4\sqrt{2} + 15\sqrt{2} - 12$$

Step 3: Combine like terms

- The constant terms: $10 - 12 = -2$
- The terms with $\sqrt{2}$: $-4\sqrt{2} + 15\sqrt{2} = 11\sqrt{2}$

Thus, the area is:

$$A = -2 + 11\sqrt{2}$$

4. Find two numbers whose sum is 68 and difference is 22.

Sol: Problem Statement:

We need to find two numbers whose sum is 68 and whose difference is 22. Let's denote these two numbers by x and y. We have the following two equations:

- $x + y = 68$ (Equation 1: The sum of the numbers is 68)
- $x - y = 22$ (Equation 2: The difference of the numbers is 22)

Step 1: Add the two equations

To eliminate y, we can add Equation 1 and Equation 2 together. This will help us solve for x.

$$(x + y) + (x - y) = 68 + 22$$

Simplifying both sides:

$$x + x = 90$$

$$2x = 90$$

Now, solve for x:

$$x = \frac{90}{2} = 45$$

Step 2: Substitute $x = 45$ into Equation 1

Now that we have $x = 45$, we can substitute this value into Equation 1 to solve for y.

$$x + y = 68$$

Substituting $x = 45$:

$$45 + y = 68$$

Solve for y:

$$y = 68 - 45 = 23$$

5. The weather in Lahore was unusually warm during the summer of 2024. The TV news reported temperatures as high as 48°C. By using the formula, $(F = \frac{9}{5}C + 32)$ find the temperature as Fahrenheit scale.

Sol: $F = \frac{9}{5}C + 32$

Given:

The temperature in Lahore during the summer of 2024 is 48°C.

Step 1: Plug the given value of $C = 48$ into the formula

$$F = \frac{9}{5}(48) + 32$$

Step 2: Calculate the value of $\frac{9}{5} \times 48$

$$\frac{9}{5} \times 48 = \frac{432}{5} = 86.4$$

Step 3: Add 32 to 86.4

$$F = 86.4 + 32 = 118.4$$

6. **The sum of the ages of the father and son is 72. Six years ago the father's age was 2 times the age of the son. What was Son's age six years ago?**

Sol: Let the present age of the father be F and the present age of the son be S.

Given:

1. The sum of their ages is 72:

$$F + S = 72 \quad \text{(Equation 1)}$$

2. Six years ago, the father's age was twice the son's age:

$$F - 6 = 2(S - 6) \quad \text{(Equation 2)}$$

Step 1: Solve the system of equations

FROM EQUATION 1:

$$F + S = 72$$

Solving for F:

$$F = 72 - S$$

SUBSTITUTE F = 72 – S INTO EQUATION 2:

$$72 - S - 6 = 2(S - 6)$$

Simplifying:

$$66 - S = 2(S - 6)$$
$$66 - S = 2S - 12$$

Now, solve for S:

$$66 + 12 = 2S + S$$
$$78 = 3S$$
$$S = \frac{78}{3} = 26$$

Step 2: Find the Father's current age

Now that we know the son's age, we can substitute $S = 26$ into Equation 1:

$$F + 26 = 72$$
$$F = 72 - 26 = 46$$

Step 3: Find the Son's age 6 years ago

The son's current age is 26, so 6 years ago:

$$S - 6 = 26 - 6 = 20$$

7. **Mirha bought a toy for Rs. 1500 and sold for Rs. 1520. What was her profit percentage?**

Sol: C. P = 1500

S. P = 1520

Profit % = 20

$$\text{Profit \%} = \frac{20 \times 100}{1500} = \frac{20}{15} = \frac{4}{3}$$

$$= 1.33\%$$

8. **The annual income of Tayyab is Rs. 9,60,000, while the exempted amount is Rs. 1,30,000. How much tax would he have to pay at the rate of 0.75%.**

Sol: **Given:**

- Annual income of Tayyab = Rs. 9,60,000
- Exempted amount = Rs. 1,30,000
- Tax rate = 0.75%

Step 1: Calculate the taxable income

Taxable income is calculated by subtracting the exempted amount from the total income.

$$\text{Taxable Income} = \text{Annual Income} - \text{Exempted Amount}$$
$$\text{Taxable Income} = 9{,}60{,}000 - 1{,}30{,}000$$
$$\text{Taxable Income} = \text{Rs. } 8{,}30{,}000$$

Step 2: Calculate the tax to be paid

The tax is calculated at a rate of 0.75% on the taxable income.

$$\text{Tax} = \frac{0.75}{100} \times \text{Taxable Income}$$
$$\text{Tax} = \frac{0.75}{100} \times 8{,}30{,}000$$
$$\text{Tax} = 0.0075 \times 8{,}30{,}000$$
$$\text{Tax} = \text{Rs. } 6{,}225$$

9. **Find the compound markup on Rs. 3,75,000 for one year at the rate of 14% compounded annually.**

Sol: To calculate the compound markup on Rs. 3,75,000 at a rate of 14% compounded annually for one year, we can use the formula for compound interest:

$$A = P\left(1+\frac{r}{100}\right)^n$$

Given:

- $P = 3,75,000$ (principal amount)
- $r = 14\%$ (interest rate)
- $n = 1$ year (since the interest is compounded for one year)

Step 1: Substitute the values into the compound interest formula:

$$A = 3,75,000\left(1+\frac{14}{100}\right)^1$$

$$A = 3,75,000(1+0.14)$$

$$A = 3,75,000 \times 1.14$$

Step 2: Calculate the amount after one year:

$$A = 3,75,000 \times 1.14 = 4,27,500$$

Step 3: Calculate the compound markup:

The compound markup is the difference between the final amount and the principal:

$$\text{Compound Markup} = A - P$$

$$\text{Compound Markup} = 4,27,500 - 3,75,000$$

$$\text{Compound Markup} = 52,500$$

REVIEW EXERCISE 1

1. **Choose the correct option.**

(i) $\sqrt{7}$ is:

(a) Integer (b) Rational number
(c) Irrational number (d) Natural number

(ii) π and e are:

(a) Natural number (b) Integers
(c) Rational number (d) Irrational number

(iii) If n is not a perfect square then $\sqrt{n}$ is:

(a) Rational number (b) Natural number
(c) Integer (d) Irrational number

(iv) $\sqrt{3}+\sqrt{5}$ is:

(a) Whole number (b) Integer
(c) Rational number (d) Irrational number

(v) For all $x \in R$, $x = x$ is called:

(a) Reflexive property (b) Transitive number
(c) Symmetric property (d) Trichotomy property

(vi) Let $a,b,c \in R$ then $a > b$ and $b > c \Rightarrow a > c$ is called ________ property.

(a) Trichotomy (b) Transitive
(c) Additive (d) Multiplicative

(vii) $2^x \times 8^x = 64$ then $x =$

(a) $\frac{3}{2}$ (b) $\frac{3}{4}$
(c) $\frac{5}{6}$ (d) $\frac{2}{3}$

(viii) Let $a,b,\in R$ then $a = b$ and $b = a$ is called ____ property.

(a) Reflexive (b) Symmetric
(c) Transitive (d) Additive

(ix) $\sqrt{75}+\sqrt{27} =$

(a) $\sqrt{102}$ (b) $9\sqrt{3}$

(c) $5\sqrt{3}$ (d) $8\sqrt{3}$

(x) The product of $(3+\sqrt{5})(3-\sqrt{5})$ is:

(a) Prime number (b) odd number

(c) Irrational number (d) Rational number

Answers:

(i)	c	(ii)	d	(iii)	d	(iv)	d	(v)	a
(vi)	b	(vii)	a	(viii)	a	(ix)	d	(x)	d

2. If $a=\frac{3}{2}, b=\frac{5}{3}$ and $c=\frac{7}{5}$, then verify that

(i) $a(b+c)=ab+ac$ (ii) $(a+b)c=ac+bc$

We are given:

$$a=\frac{3}{2},\quad b=\frac{5}{3},\quad c=\frac{7}{5}$$

(i) **Verify $a(b+c)=ab+ac$**

L.H.S $a(b+c)$

First, calculate $b+c$:

$$b+c=\frac{5}{3}+\frac{7}{5}$$

$$b+c=\frac{25+21}{15}$$

$$b+c=\frac{46}{15}$$

Now, multiply by a:

$$a(b+c)=\frac{3}{2}\times\frac{46}{15}$$

$$a(b+c)=\frac{3\times 46}{2\times 15}=\frac{138}{30}=\frac{23}{5}\quad ...(A)$$

R.H.S: $ab+ac$

Now, calculate ab and ac.

First, calculate ab: $ab=\frac{3}{2}\times\frac{5}{3}$

$$ab=\frac{15}{6}=\frac{5}{2}$$

Next, calculate ac:

$$ac=\frac{3}{2}\times\frac{7}{5}$$

$$ac=\frac{21}{10}$$

Now, add ab and ac:

$$ab+ac=\frac{5}{2}+\frac{21}{10}$$

$$=\frac{25+21}{10}=\frac{46}{10}=\frac{23}{5}\ ...(B)$$

Thus, the first equation is verified: $a(b+c)=ab+ac$.

(ii) **Verify $(a+b)c=ac+bc$**

L.H.S: $(a+b)c$

First, calculate $a+b$:

$$a+b=\frac{3}{2}+\frac{5}{3}=\frac{9+10}{6}=\frac{19}{6}$$

Now, multiply by c:

$$(a+b)c=\frac{19}{6}\times\frac{7}{5}$$

$$(a+b)c=\frac{19\times 7}{6\times 5}=\frac{133}{30}$$

R.H.S: $ac+bc$

Now, calculate ac and bc.

First, calculate ac:

$$ac=\frac{3}{2}\times\frac{7}{5}=\frac{21}{10}$$

Next, calculate bc:

$$bc=\frac{5}{3}\times\frac{7}{5}=\frac{35}{15}=\frac{7}{3}$$

Now, add ac and bc:

$$ac+bc=\frac{21}{10}+\frac{7}{3}$$

$$=\frac{63+70}{30}=\frac{133}{30}$$

Thus, the second equation is also verified: $(a+b)c = ac + bc$.

3. If $a=\frac{4}{3}, b=\frac{5}{2}, c=\frac{7}{4}$, Then, verify the Associative property of real numbers. w.r.t addition and multiplication.

Sol: $a=\frac{4}{3}, \quad b=\frac{5}{2}, \quad c=\frac{7}{4}$

(i) Associative Property of Addition:

The associative property of addition states:

$$(a+b)+c = a+(b+c)$$

L.H.S: $(a+b)+c$

First, calculate $a+b$:

$$a+b = \frac{4}{3}+\frac{5}{2}$$
$$= \frac{8+15}{6} = \frac{23}{6}$$

Now, add c to this result:

$$(a+b)+c = \frac{23}{6}+\frac{7}{4}$$
$$= \frac{46+21}{12} = \frac{67}{12}$$

R.H.S: $a+(b+c)$

First, calculate $b+c$:

$$b+c = \frac{5}{2}+\frac{7}{4}$$
$$= \frac{10+7}{4} = \frac{17}{4}$$

Now, add a to this result:

$$a+(b+c) = \frac{4}{3}+\frac{17}{4}$$
$$= \frac{16+51}{12} = \frac{67}{12}$$

Conclusion:

Both sides are equal:

$$(a+b)+c = a+(b+c) = \frac{67}{12}$$

Thus, the associative property of addition is verified.

(ii) Associative Property of Multiplication:

The associative property of multiplication states:

$$(a\times b)\times c = a\times(b\times c)$$

L.H.S: $(a\times b)\times c$

First, calculate $a\times b$:

$$a\times b = \frac{4}{3}\times\frac{5}{2}$$
$$= \frac{20}{6} = \frac{10}{3}$$

Now, multiply this result by c:

$$(a\times b)\times c = \frac{10}{3}\times\frac{7}{4}$$
$$= \frac{70}{12} = \frac{35}{6}$$

R.H.S: $a\times(b\times c)$

First, calculate $b\times c$:

$$b\times c = \frac{5}{2}\times\frac{7}{4} = \frac{35}{8}$$

Now, multiply a by this result:

$$a\times(b\times c) = \frac{4}{3}\times\frac{35}{8} = \frac{140}{24} = \frac{35}{6}$$

Conclusion:

Both sides are equal:

$$(a\times b)\times c = a\times(b\times c) = \frac{35}{6}$$

Thus, the associative property of multiplication is verified.

4. Is 0 a rational number? Explain.

Ans. Yes, 0 is a rational number.

Explanation:

A rational number is any number that can be expressed in the form:

$\frac{p}{q}$, where p and q are integers, and $q \neq 0$.

For 0:

- It can be written as $\frac{0}{q}$, where $q \neq 0$.

For example:

$$0 = \frac{0}{1},\quad 0 = \frac{0}{-5},\quad 0 = \frac{0}{10}, \text{ and so on.}$$

Since 0 satisfies the condition of being expressible as a fraction with an integer numerator (0) and a non-zero integer denominator (q), it is classified as a **rational number**.

5. State trichotomy property of real numbers.

Ans. The **Trichotomy Property** of real numbers states:

For any two real numbers a and b, exactly one of the following three statements is true:

1. $a < b$ (i.e., a is less than b),
2. $a = b$ (i.e., a is equal to b),
3. $a > b$ (i.e., a is greater than b).

This property ensures that any two real numbers can be compared, and there is no overlap or ambiguity in their relationship.

6. Find two rational numbers between 4 and 5.

Solution:

Two rational numbers between 4 and 5.

Average of 4 and 5 $= \frac{4+5}{2} = \frac{9}{2}$

Average of $\frac{9}{2}$ and 5 $= \left(\frac{9}{2}+5\right) \div 2$

$= \left(\frac{9+10}{2}\right) \times \frac{1}{2} = \frac{19}{2} \times \frac{1}{2} = \frac{19}{4}$

Thus, two rational numbers between 4 and 5 are $\frac{9}{2}$ and $\frac{19}{4}$.

7. Simplify the following:

(i) $\sqrt[5]{\frac{x^{15}y^{35}}{z^{20}}}$ (ii) $\sqrt[3]{(27)^{2x}}$

(iii) $\frac{6(3)^{n+2}}{3^{n+1}-3^n}$

Problem (i): Simplify

$$\sqrt[5]{\frac{x^{15}y^{35}}{z^{20}}}$$

SOLUTION:

The fifth root simplifies as:

$$\sqrt[5]{\frac{x^{15}y^{35}}{z^{20}}} = \frac{\sqrt[5]{x^{15}} \cdot \sqrt[5]{y^{35}}}{\sqrt[5]{z^{20}}}.$$

Using the property $\sqrt[n]{a^m} = a^{\frac{m}{n}}$:

$$\sqrt[5]{x^{15}} = x^{\frac{15}{5}} = x^3,\quad \sqrt[5]{y^{35}} = y^{\frac{35}{5}} = y^7,\quad \sqrt[5]{z^{20}} = z^{\frac{20}{5}} = z^4.$$

Thus:

$$\sqrt[5]{\frac{x^{15}y^{35}}{z^{20}}} = \frac{x^3y^7}{z^4}.$$

Answer: $\frac{x^3y^7}{z^4}$.

Problem (ii): Simplify

$$\sqrt[3]{(27)^{2x}}.$$

SOLUTION:

The cube root simplifies as:

$$\sqrt[3]{(27)^{2x}} = (27^{2x})^{\frac{1}{3}} = 27^{\frac{2x}{3}}.$$

Express 27 as 3^3:

$$27^{\frac{2x}{3}} = (3^3)^{\frac{2x}{3}}.$$

Using the property $(a^m)^n = a^{m \cdot n}$:

$$(3^3)^{\frac{2x}{3}} = 3^{3 \cdot \frac{2x}{3}} = 3^{2x}.$$

Answer:

$$3^{2x}.$$

Problem (iii): Simplify

$$\frac{6 \cdot 3^{n+2}}{3^{n+1} - 3^n}.$$

SOLUTION:

$$\frac{6 \cdot 3^{n+2}}{3^{n+1} - 3^n}.$$

$$= \frac{6 \,.\, 3^{n+2}}{3^n \,.\, 3 - 3^n}$$

$$= \frac{6 \,.\, 3^{n+2}}{3^n(3-1)}$$

$$= \frac{6 \,.\, 3^{n+2}}{3^n \,.\, 2}$$

$$= 3 \cdot 3^{(n+2)-n} = 3 \cdot 3^2 = 3 \cdot 9 = 27.$$

Answer: 27.

8. **The sum of three consecutive odd integers is 51. Find the integers.**

Let the three consecutive odd integers be x, $x + 2$, and $x + 4$.
The sum of the three integers is given as 51:

$$x + (x + 2) + (x + 4) = 51.$$
$$x + x + 2 + x + 4 = 51,$$
$$3x + 6 = 51.$$

Subtract 6 from both sides:

$$3x = 45.$$

Divide both sides by 3:

$$x = 15.$$

The three integers are: $x = 15, x + 2 = 17, x + 4 = 19.$

Sum the three integers:

$$15 + 17 + 19 = 51.$$

This is correct.

Final Answer:1

The three consecutive odd integers are:

15,17, and 19.

9. **Abdullah picked up 96 balls and placed them into two buckets. One bucket has twenty-eight more balls than the other bucket. How many balls are in each bucket?**

Let the number of balls in the smaller bucket be x.

Then the number of balls in the larger bucket will be $x + 28$, as it has 28 more balls than the smaller bucket.

The total number of balls in the two buckets is 96:

$$x + (x + 28) = 96.$$

Combine like terms:

$$x + x + 28 = 96,$$
$$2x + 28 = 96.$$

Subtract 28 from both sides:

$$2x = 68.$$

Divide both sides by 2:

$$x = 34.$$

The smaller bucket has $x = 34$ balls.

The larger bucket has $x + 28 = 34 + 28 = 62$ balls.

The smaller bucket has **34 balls**, and the larger bucket has **62 balls.**

Q.10. Salma invested Rs. 3,50,000 in a bank which paid simple profit at a rate $7\frac{1}{4}$ % per annum. After 2 years, the

rate was increased to 8% per annum. Find the amount she has at the end of 7 years.

Sol. P = Rs. 3,50,000

$R = 7\frac{1}{4}\%$ or 7.25%

T = 2 years

We know that

Profit = P × T × R

= 350,000 × 2 × 7.25%

$= 700{,}000 \times \frac{7.25}{100}$

$= 700{,}000 \times \frac{725}{100 \times 100}$

= 70 × 725

= Rs. 50,750

Step (II): Finding the profit for 5 years

P = Rs. 350,000

R = 8%

T = 5years

We know that

Profit = P × T × R

= 350,000 × 5 × 8%

$= 1{,}750{,}0\cancel{0}\cancel{0}\ \frac{8}{1\cancel{00}}$

= Rs. 140,000

Step (III): Finding the total profit:

Total profit = Rs. (50,750 + 140,000)

= Rs. 190,750

Step (IV): Finding the total amount:

Total amount at the end of 7 years

= Rs. 350,000 + 190,750

= Rs. 540,750