UNIT 10 Graphs of Functions

Students' learning outcomes

At the end of the unit, the students will be able to:

- Recall sketch graphs of linear functions (e.g. $y = ax + b$)
- Plot and interpret the graphs of quadratic, cubic, reciprocal and exponential functions.
  - Graph $y = ax^n$ where n is $+ve$ integer, −ve integer, rational number for $x > 0$ and a is any real number.
  - Graph $y = ka^x$, where x is real $a > 1$.
- Discover exponential growth/decay of a practical phenomenon through its graph.
- Determine the gradients of curves by drawing tangents.
- Apply concepts of sketching and interpreting graphs to real-life problems (such as in tax payment, income and salary problems and cost and profit analysis)

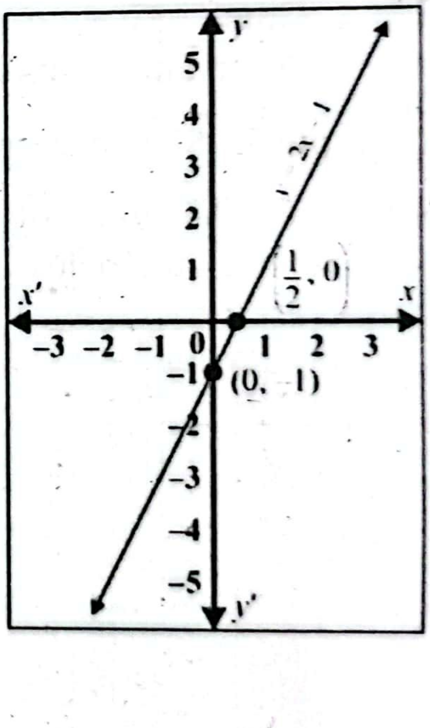
Example 1: Sketch the graph of $y = 2x - 1$.

Solution: To sketch the graph of linear function, we can find its x and y intercepts.

Put $x = 0$, we get $y = 5$. So (0, -1) is the y-intercept.

Put $y = 0$, we get $x = \frac{1}{2}$. So $\left(\frac{1}{2}, 0\right)$ is the x-intercept.

The graph is a straight line that rises to the right because slope is positive.



Graph of Quadratic Functions

A quadratic function is a type of polynomial function that involves x^2 term. Its general form is:

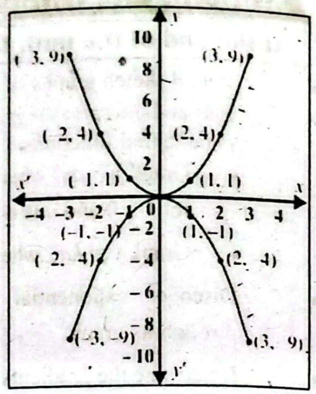
$$y = ax^2 + bx + c$$

Where a, b, c are constants and $a \neq 0$.

Example 2: Plot the graphs of $y = x^2$ and $y = -x^2$ on the same diagram.

Solution: The following table shows several values of x and the given functions are evaluated at those values:

x	$y = x^2$	$y = -x^2$
-3	$(-3)^2 = 9$	9
-2	$(-2)^2 = 4$	-4
1	$(-1)^2 = 1$	-1
0	$(0)^2 = 0$	0
1	$(1)^2 = 1$	1
2	$(2)^2 = 4$	-4
3	$(3)^2 = 9$	-9



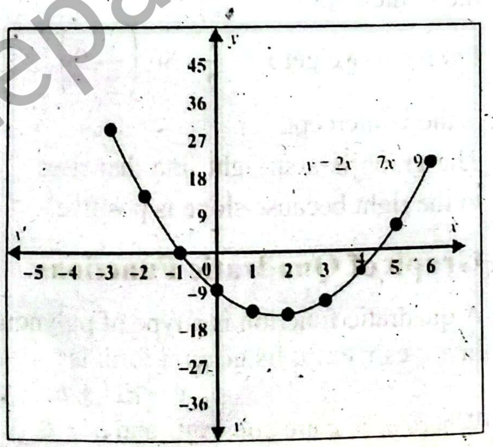
(i) Graph of $y = x^2$ represents parabola, passing through origin and opens upward.

(ii) Graph of $y = -x^2$ represents parabola, passing through origin and opens downward.

Example 3: Sketch the graph of $y = 2x^2 - 7x - 9$ for $-3 \le x \le 6$.

Solution: The values of x and y are given in the table and sketched in figure below:

x	y
3	30
2	13
-1	0
0	-9
1	-14
2	15
3	-12
4	5
5	6
6	21



Graph of $y = 2x^2 - 7x - 9$ represents parabola and opens upward.

It intersects the y-axis at $(0, -9)$ and x-axis at $(-1, 0)$ and $(4.5, 0)$.

Graph of Cubic Functions

A cubic function is a type of polynomial function of degree 3. Its standard form is: $y = ax^3 + bx^2 + cx + d$

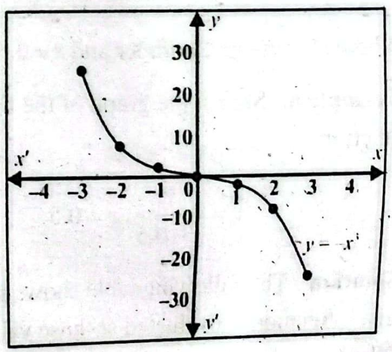
Where a, b, c, d are constants and $a \neq 0$.

> **Remember!**
> - The graph of a cubic function is a curve that can have at most two turning points.
> - It has a general "S-shaped" appearance and depending on the coefficients, the shape may vary.
> - Such functions are much more complicated and show more varied behaviour than linear and quadratic ones.

Example 4: Plot the graph of the following cubic function for $-3 < x < 3$:

Solution: The following table shows several values of x and the given function is evaluated at those values:

x	$y = -x^3$
-3	27
-2	8
-1	1
0	0
1	-1
2	-8
3	-27

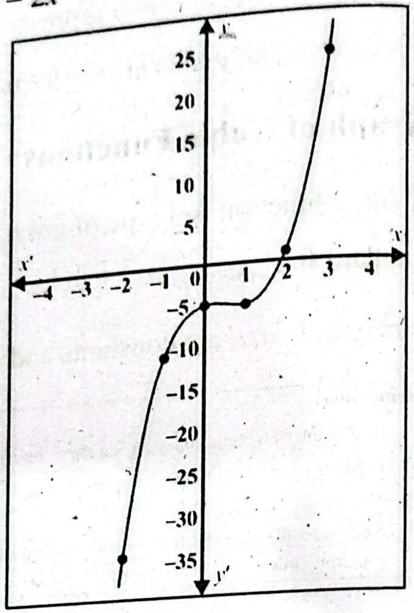


The curve passes through the origin.

Example 5: Plot the graph of $y = 2x^3 - 3x^2 + x - 5$ for $-2 \le x \le 3$.

Solution: The following table shows several values of x and the given function is evaluated at those values:

x	y
−2	−35
−1	−11
0	−5
1	−5
2	1
3	25



The graph tells us that when $x = 0$, the function's value is −5.

Graph of Reciprocal Functions

A reciprocal function is a function of the form:

$$y = \frac{a}{x}$$

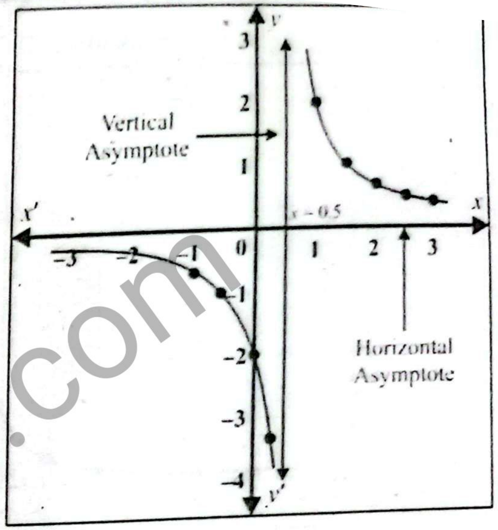
Where a is any real number and $x \neq 0$.

Example 6: Sketch the graph of the following reciprocal function:

$$y = \frac{1}{x - 0.5}, x \neq 0.5$$

Solution: The following table shows several values of x and the given function is evaluated at those values:

x	y
−1	−0.67
−0.5	−1
−0.2	−1.43
0	−2
0.2	3.3
0.5	undefined
1	2
1.2	1.43
1.5	1
2	0.67
2.2	0.59
2.5	0.5
3	0.4



Remember!
An asymptote is a line that a graph approaches but never touches.

Graph of Exponential Functions ($y = ka^x$ where x is real number, $a > 1$)

An exponential function is a mathematical function of the form:

$$y = ka^x$$

Where a, k are constants, x is variable and $a > 1$.

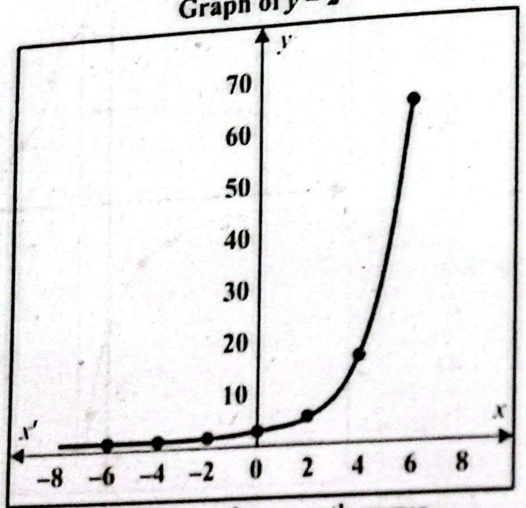
Example 7: Plot the graph of the exponential function $y = 2^x$ for $-6 \le x \le 6$.

Solution: The function $y = 2^x$ has base 2 and variable exponent x. Values of (x, y) are given in the table below:

x	−6	−4	−2	0	2	4	6
$y = 2x$	0.02	0.06	0.25	1	4	16	64

Graph of the above points is given in the figure below:

Graph of $y = 2^x$



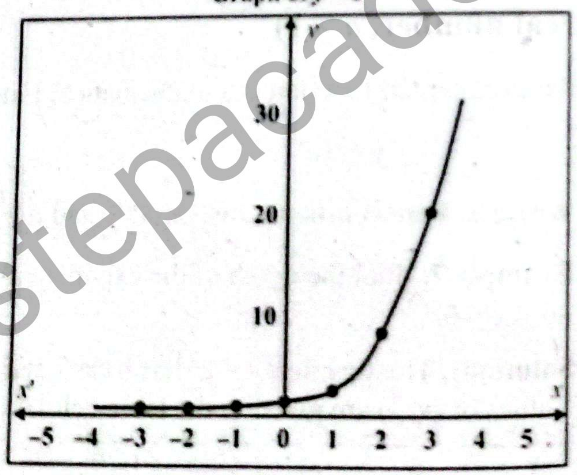
...he graph of $y = 2^x$ represents the growth curve.

Example 8: Plot the graph of the exponential function, $y = e^x$.

Solution: The function $y = e^x$ has base e and variable power x. We know $e = 2.7182818$, correct to two decimal places $e = 2.72$. Table of x and y values is given below.

x	$y = e^x$
-3	0.05
-2	0.14
-1	0.37
0	1
1	2.72
2	7.40
3	20.09

Graph of $y = e^x$



Graphs of $y = ax^n$ (where n is +ve integer, −ve integer or rational number for $x > 0$ and a is any real number)

The graph of the function $y = ax^n$, where n is a positive integer, negative integer or rational number for $x > 0$ and a is any real number, exhibits distinct behaviours depending on the value of n. Following are the examples of these cases:.

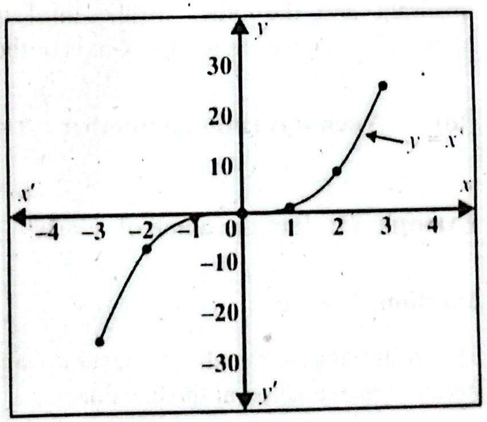
(i) When n is positive integer ($n = 3$)

Example 9: Plot the graph of $y = x^3$ for $-3 \leq x \leq 3$.

Solution: The table shows several values of x and the given function is evaluated at those values:

The curve passes through the origin.

x	$y = x^3$
−3	−27
−2	−8
−1	−1
0	0
1	1
2	8
3	27



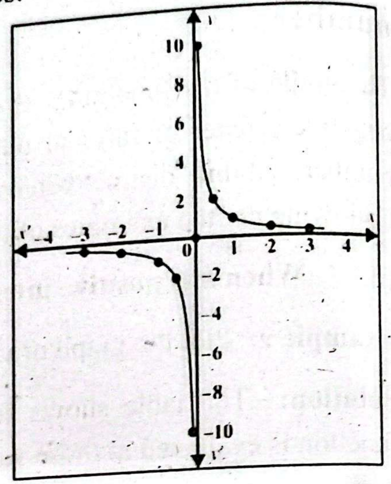
(ii) When n is negative integer ($n = -1$)

Example 10: Plot the graph of $y = x^{-1}$.

Solution: $y = x^{-1} = \dfrac{1}{x}$

The following table shows several values of x and the given function is evaluated at those values:

x	$y=\frac{1}{x}$
−3	−0.3
−2	−0.5
−1	−1
−0.5	−2
−0.1	−10
0.1	10
0.5	2
1	1
2	0.5
3	0.3



The above graph consists of two branches, one in the first quadrant and the other in the third quadrant. Both branches approach but never touch the x-axis or the y-axis.

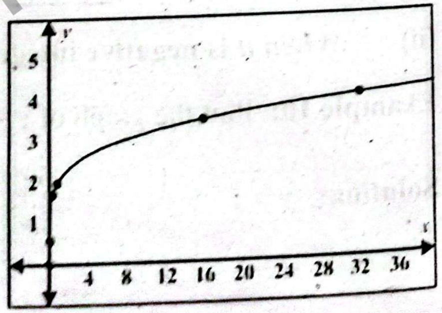
(iii) When n is rational number $\left(n=\frac{1}{5}\right)$

Example 11: Plot the graph of $y=2x^{\frac{1}{5}}$

Solution: $y=2x^{\frac{1}{5}}$

The following table shows several values of x and the given function is evaluated at those values:

x	y
0	0
0.01	0.80
0.5	1.74
1	2
16	3.48
32	4



EXERCISE 10.1

1. Sketch the graph of the following linear functions:

(i) $y=3x-5$

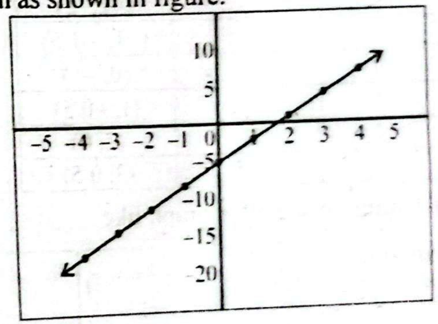
Solution: $y=3x-5$

Step-I: Substitute the values of x in the given equation and find the corresponding values of y, as given below:

x	y	(x, y)
−1	−8	(−1, −8)
0	−5	(0, −5)
1	−2	(1, −2)
2	1	(2, 1)

Step-II: Select a suitable scale for graph like on x-axis, 1 division = 1 unit on y-axis, 1 division = 2 units.

Step-III: Plot the points on the grid and join them to get the required graph as shown in figure.



(ii) $y=-2x+8$

Solution: $y=-2x+8$

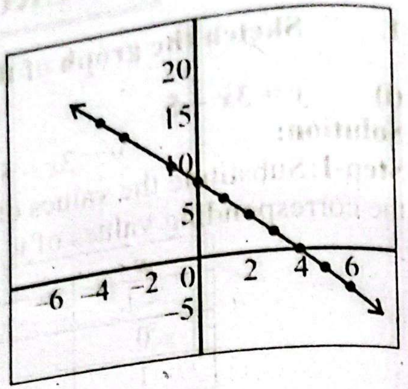
Step-I: Substitute the value of x in the given equation and find the corresponding values of y, as given below:

x	y	(x, y)
−1	10	(−1, 10)
0	8	(0, 8)
1	6	(1, 6)
2	4	(2, 4)
4	0	(4, 0)

Step-II: Select a suitable scale for graph like

On x, axis, 1 division = 1 unit
On y, axis, 1 division = 2 units

Step-III: Plot the points on the grid and join them to get the required graph as shown in figure.



(iii) $y = 0.5x - 1$

Solution: $y = 0.5x - 1$

Step-I: Substitute the values of x in the given equation and find the corresponding values of y, as given below:

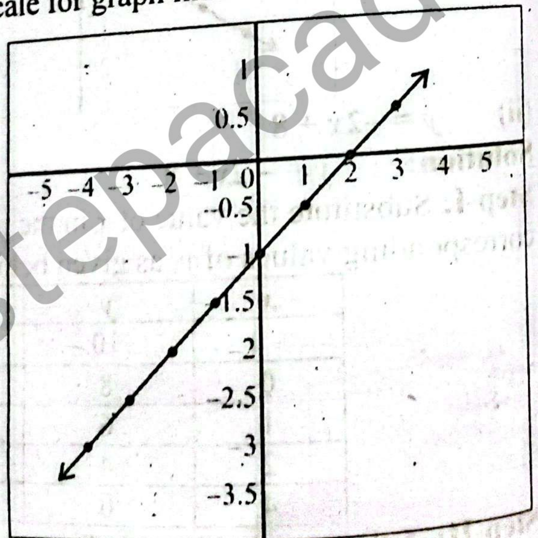
x	y	(x, y)
−2	−2	(−2, −2)
−1	−1.5	(−1, −1.5)
0	−1	(0, −1)
1	−0.5	(1, −0.5)
2	0	(2, 0)
3	0.5	(3, 0.5)

Step-II: Select a suitable scale for graph like

On x-axis, 1 division = 1 unit

On y-axis 1 division = 0.5 unit

Step-III: Plot the points on the grid and join them to get the required graph as shown in figure.



2. Plot the graph of the following quadratic and cubic functions:

(i) $y = x^3 + 2x^2 - 5x - 6; -3.5 \le x \le 2.5$

Solution: $y = x^3 + 2x^2 - 5x - 6; -3.5 \le x \le 2.5$

Step-I: Substitute the values of x from −3.5 to 2.5 in the given equation and find the corresponding values of y, as given below:

x	y	(x, y)
2.5	9.6	(2.5, 9.6)
2	0	(2,0)
1	−8	(1,−8)
0	−6	(0,−6)
−1	0	(−1,0)
−2	4	(−2, 4)
−3	0	(−3, 0)
−3.5	−6.8	(−3.5, −6.8)

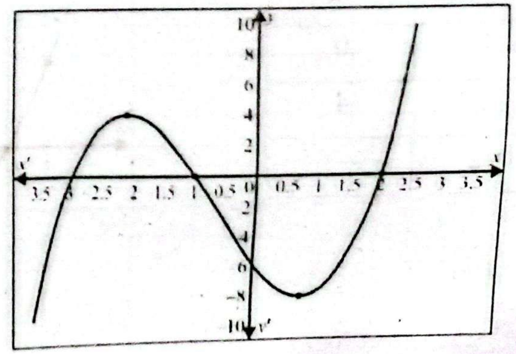
Step-II: Select a suitable scale for graph like

On x-axis, 1 division = 0.5 unit

On y-axis, 1 division = 2 units

Step-III: Plot the points on the grid and join them to get the required graph as shown in figure.

x	y
0	−6
1	−8
−1	0
2	0
−3	0



(ii) $y = x^2 + x - 2$

Solution: $y = x^2 + x - 2$

Step-I: Substitute the values of x in the given equation and find the corresponding values of y, as given below:

x	y	(x, y)
–4	10	(–4,10)
–3	4	(–3,4)
–2	0	(–2,0)
–1	–2	(–1,–2)
0	–2	(0, –2)
1	0	(1, 0)
2	4	(2, 4)
3	10	(3, 10)

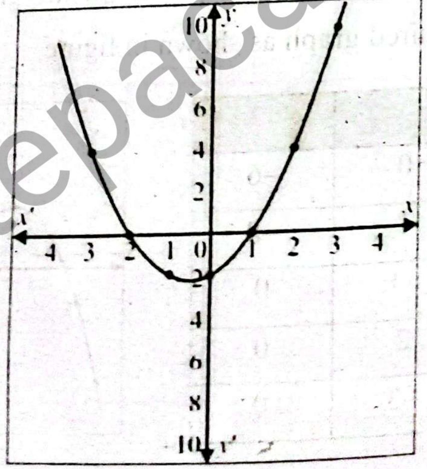
Step-II: Select a suitable scale for graph like

On – x–axis, 1 division = 1 unit

On – y – axis, 1 division = 2 unit

Step-III: Plot the points on the grid and join them to get the required graph as shown in figure.

x	y
0	–2
1	0
2	4
–2	–2



(iii) $y = x^3 + 3x^2 + 2x; -2.5 \le x \le 0.5$

Solution: $y = x^3 + 3x^2 + 2x; -2.5 \le x \le 0.5$

Step-I: Substitute the values of x from –2.5 to 0.5 in the given equation and find the corresponding values of y, as given below:

x	y	(x, y)
–2.5	–1.8	(–2.5, –1.9)
–2	0	(–2, 0)
–1.5	0.4	(–1.5, 0.4)
–1	0	(–1, 0)
–0.5	–0.4	(–0.5, –0.4)
0	0	(0, 0)
0.5	1.8	(0.5, 1.8)

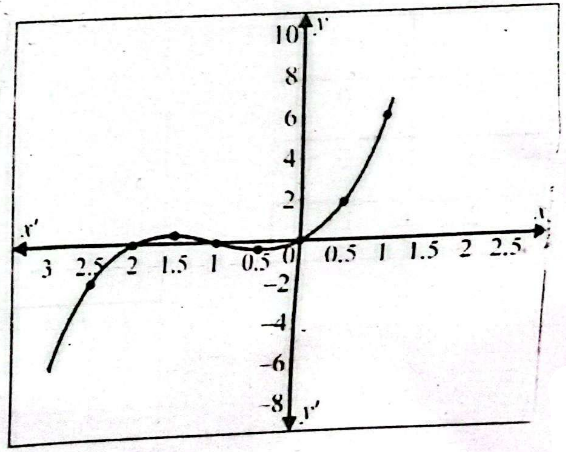
Step-II: Select a suitable scale for graph like

On x-axis, 1 division = 0.5 unit

On y-axis, 1 division = 0.5 unit

Step-III: Plot the points on the grid and join them to get the required graph as shown in figure.

x	y
0	0
1	6
–1	0
–2	0



(iv) $y = 5x^2 - 2x - 3$

Solution: $y = 5x^2 - 2x - 3$

Step-I: Substitute the values of x in the given equation and find the corresponding values of y, as given below:

x	y	(x, y)
−2	21	(−2,21)
−1	4	(−1,4)
0	−3	(0,−3)
0.5	−2.8	(0.5, −2.8)
1	0	(1, 0)
2	13	(2, 13)

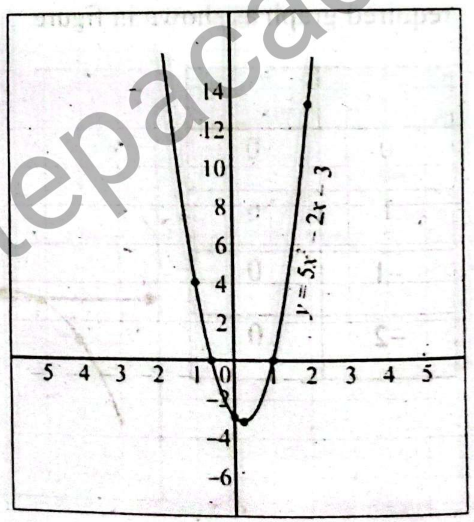
Step-II: Select a suitable scale for graph like

On x-axis, 1 division = 1 unit

On y-axis, 1 division = 2 units

Step-III: Plot the points on the grid and join them to get the required graph as shown in figure.

x	y
0	−3
1	0
−1	3
2	13



3. **Plot the graph of the following functions:**

(i) $y = 4^x$

Solution: $y = 4^x$

Step-I: Substitute the values of x in the given equation and find the corresponding values of y, as given below:

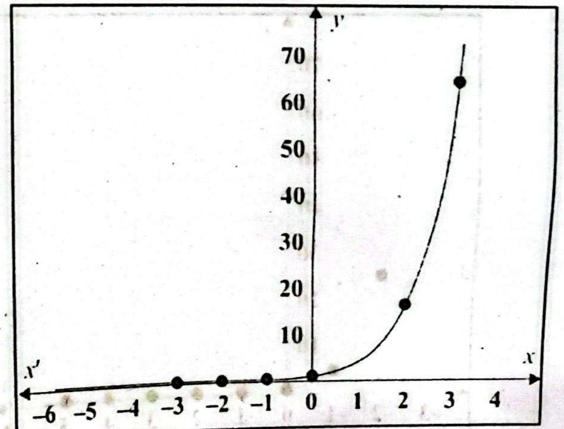
x	y	(x, y)
−2	0.06	(−2, 0.06)
−1	0.25	(−1, 0.25)
0	1	(0, 1)
0.5	2	(0.5, 2)
1	4	(1, 4)
1.5	8	(1.5, 8)
2	16	(2, 16)
2.5	32	(2.5, 32)

Step-II: Select a suitable scale for graph like

On x-axis, 1 division = 0.5 unit

On y-axis, 1 division = 4 unit

Step-III: Plot the points on the grid and join them to get the required graph as shown in figure.



(ii) $y = 5^{-x}$

Solution: $y = 5^{x}$

Step-I: Substitute the values of x in the given equation and find the corresponding values of y, as given below:

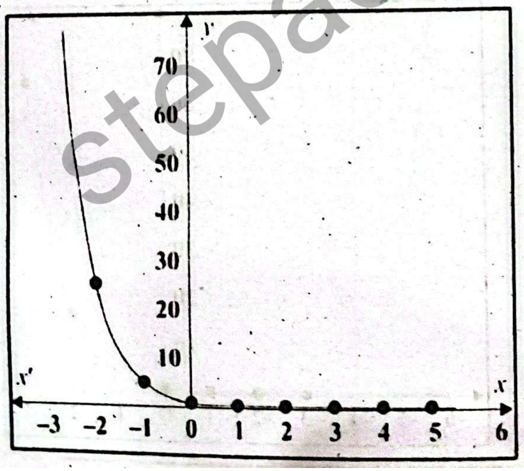
x	$y = 5^x$	(x, y)
–2.5	56	(–2.5, 56)
–2	25	(–2, 25)
–1	5	(–1, 5)
0	–1	(0, –1)
1	0.2	(1, 0.2)
2	0.04	(2, 0.04)
3	0.008	(3, 0.008)

Step-II: Select a suitable scale for graph like

On x-axis, 1 division = 1 unit

On y-axis, 1 division = 10 unit

Step-III: Plot the points on the grid and join them to get the required graph as shown in figure.



(iii) $y = \dfrac{1}{x-3}\, x \neq 3$

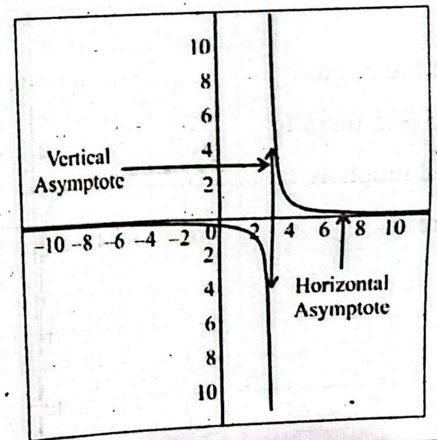
Solution:

Step-I: Substitute the values of x in the given equation and find the corresponding values of y, as given below:

$$y = \frac{1}{x-3}\quad x \neq 3$$

x	y	(x, y)
–3	–0.16	(–3, –0.16)
–2	–0.2	(–2, –0.2)
–1	–0.25	(–1, –0.25)
0	–0.3	(0, –0.3)
1	–0.5	(1, –0.5)
2	–1	(2, –1)
3	∞	(3, ∞)
4	1	(4, 1)
5	0.5	(5, 0.5)
6	0.3	(6, 0.3)

Step-II: Select a suitable scale for graph like
On x-axis, 1 division = 1 unit
On y-axis, 1 division = 0.1 unit
Step-III: Plot the points on the grid and join them to get the required graph as shown in figure.



(iv) $y = \frac{2}{x} + 3, x \neq 0$

Solution: $y = \frac{2}{x} + 3, \ (x \neq 0)$

Step-I: Substitute the values of x in the given equation and find the corresponding values of y, as given below:

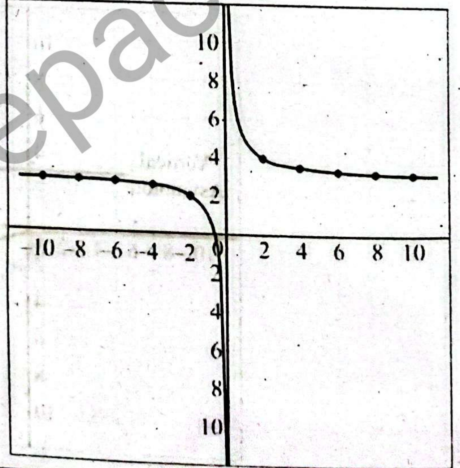
x	y	(x, y)
−10	2.8	(−10, 2.8)
−8	2.75	(−8, 2.75)
−6	2.66	(−6, 2.66)
−4	2.5	(−4, 2.5)
−2	2	(−2, 2)
−1	1	(−1, 1)
0	∞	(0, ∞)
1	5	(1, 5)
2	4	(2, 4)
4	3.5	(4, 3.5)
6	3.3	(6, 3.3)
8	3.25	(8, 3.25)
10	3.2	(10, 3.2)

Step-II: Select a suitable scale for graph like

On x-axis, 1 division = 2 units

On y-axis, 1 division = 1 unit

Step-III: Plot the points on the grid and join them to get the required graph as shown in figure.



(v) $y = x^{\frac{1}{2}}$

Solution: $y = x^{\frac{1}{2}}$

Step-I: Substitute the values of x in the given equation and find the corresponding values of y, as given below:

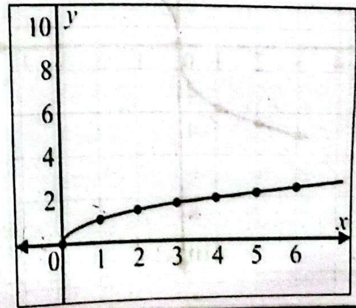
x	$y = x^{\frac{1}{2}} = \sqrt{x}$	(x, y)
0	$y = \sqrt{0} = 0$	(0, 0)
1	$y = \sqrt{1} = 1$	(1, 1)
4	$y = \sqrt{4} = 2$	(4, 2)
9	$y = \sqrt{9} = 3$	(9, 3)
16	$y = \sqrt{16} = 4$	(16, 4)
25	$y = \sqrt{25} = 5$	(25, 5)
36	$y = \sqrt{36} = 6$	(36, 6)

Step-II: Select a suitable scale for graph like

On x-axis, 1 division = 4 units

On y-axis, 1 division = 1 unit

Step-III: Plot the points on the grid and join them to get the required graph as shown in figure.



(vi) $y = 3x^{\frac{1}{3}}$

Solution: $y = 3x^{\frac{1}{3}}$

Step-I: Substitute the values of x in the given equation and find the corresponding values of y, as given below:

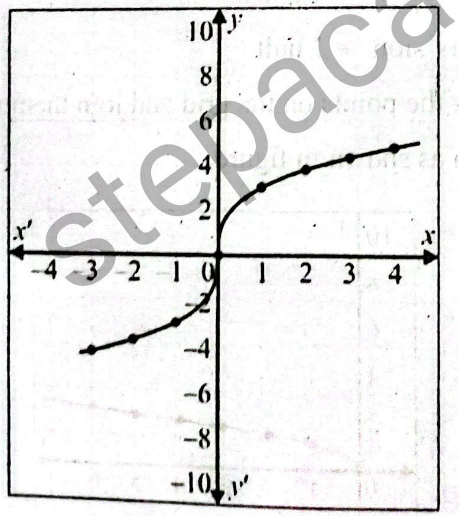
x	y
–4	–4.8
–3	–4.3
–2	–3.8
–1	–3
0	0
1	3
2	3.8
3	4.3
4	4.8

Step-II: Select a suitable scale for graph like

On x-axis, 1 division = 1 unit

On y-axis, 1 division = 1 unit

Step-III: Plot the points on the grid and join them to get the required graph as shown in figure.



(vii) $y = 2x^{-2}$

Solution: $y = 2x^{-2}$

Step-I: Substitute the values of x the given equation and find the corresponding values of y, as given below:

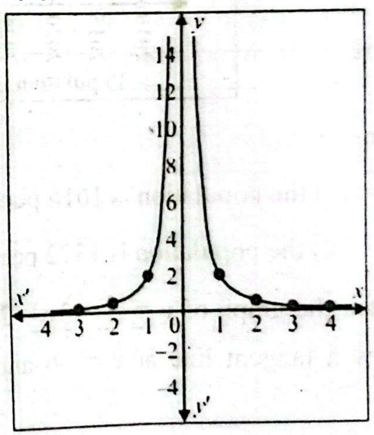
x	y	(x, y)
–4	0.13	(–4, 0.13)
–3	0.22	(4, 0.22)
–2	0.5	(–2, 0.5)
–1	2	(–1, 2)
0	∞	(0, ∞)
1	2	(1, 2)
2	0.5	(2, 0.5)
3	0.22	(3, 0.22)

Step-II: Select a suitable scale for graph like

On x-axis, 1 division = 1 unit

On y-axis, 1 division = 0.5 unit

Step-III: Plot the points on the grid and join them to get the required graph as shown in figure.



Example 12: The population of a village was 753 in 2010. If the population grows according to the equation $p = 753e^{0.03t}$, where p is the number of persons in the population at time t,

(a) Graph the population equation for $t = 0$ (in 2010) to $t = 30$

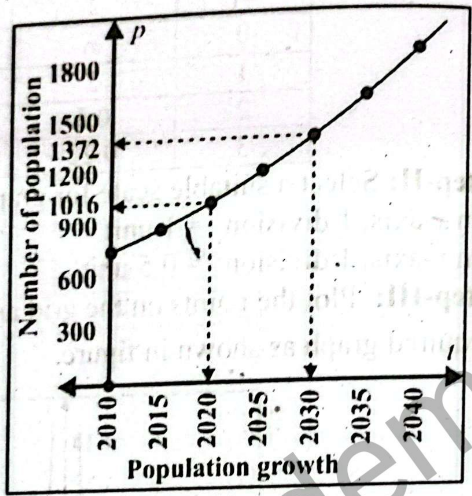
(in 2040).

(b) From the graph, estimate the population (i) in 2020 and (ii) in 2030.

Solution: (a) The general shape of the exponential is known; however, since the graph is being used for estimations, an accurate graph over the required interval, $t = 0$ to $t = 30$, is required.

Calculate a table of values for different time periods and sketched in figure:

t	p
0	753
5	874.9
10	1016.4
15	1180.9
20	1372.1
25	1594.1
30	1852.1



(b) From graph,

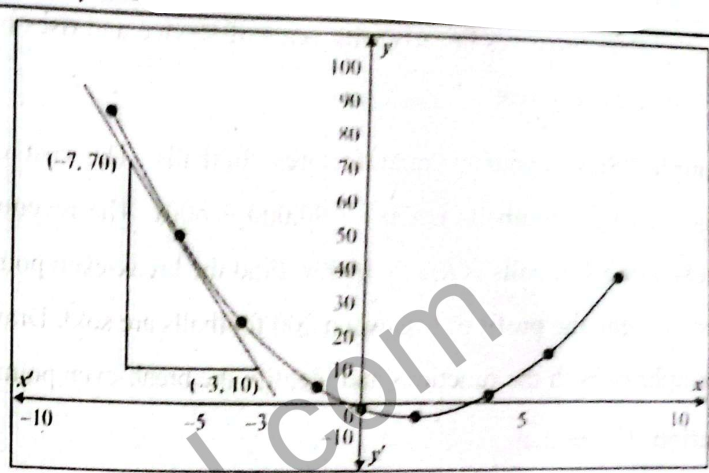
(i) In 2020 ($t = 10$) the population is 1016 persons.

(ii) In 2030 ($t = 20$) the population is 1372 persons.

Example 13: Sketch the graph of $y = x^2 - 3x - 2$ for values of x from -8 to 8, draw a tangent line at $x = -6$ and determine the gradient.

Solution: Calculate the y-values for given values of x. The results are given in the table and sketched in below figure:

x	-8	-6	-4	-2	0	2	4	6	8
y	86	52	26	8	-2	-4	2	16	38



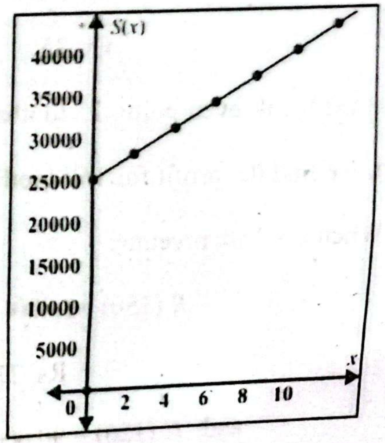
Consider two points $(-3, 10)$ and $(-7, 70)$ on the tangent line.

So, gradient $= \dfrac{70-10}{-7+3} = -15$. Since the gradient is negative, this indicates that the height of the graph decreases as the value of x increases.

Example 14: Majid's salary $S(x)$ in rupees is based on the following formula: $S(x) = 25000 + 1500x$, where x is the number of years he worked. Sketch and interpret the graph of salary function for $0 \leq x \leq 10$.

Solution: Table values and graph are given below:

x	$S(x)$
0	25000
2	28000
4	31000
6	34000
8	37000
10	40000



Majid's salary increases linearly with years of service and rises by Rs. 1500 for every year.

Example 15: A company manufactures footballs. The cost of manufacturing x footballs is $C(x) = 90{,}000 + 600x$. The revenue from selling x footballs is $R(x) = 1{,}800x$. Find the break-even point and determine the profit or loss when 200 footballs are sold. Draw the graphs of both the functions and identify the break-even point.

Solution: Given that

Cost function: $C(x) = 90{,}000 + 600x$

Revenue function: $R(x) = 1{,}800x$

The break-even point occurs when $R(x) = C(x)$

$$1800x = 90000 + 600x$$

$$1200x = 90000$$

$$\Rightarrow \quad x = \frac{90000}{1200}$$

$$x = 75$$

So, at the break-even point, 75 footballs are produced or sold.

Next, we find the profit for 150 footballs

When $x = 150$, revenue:

$$R(150) = 1{,}800(150)$$

$$= \text{Rs. } 270{,}000$$

and $C(150) = 90{,}000 + 600(150)$

$$= \text{Rs. } 180{,}000$$

Now profit: $P(x) = R(x) - C(x)$

Substitute $x = 150$

$$P(150) = R(150) - C(150)$$

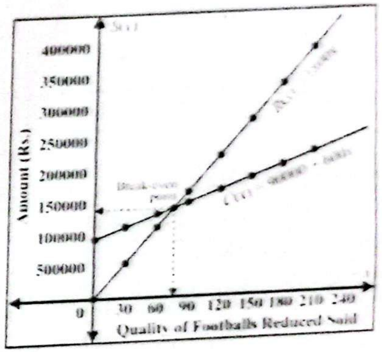
$$= \text{Rs. } 270{,}000 - \text{Rs. } 180{,}000$$

$$= \text{Rs. } 90{,}000$$

Thus, a company earns a profit of Rs. 90,000 when selling 150 footballs.

Table values and graph are given below:

x	$C(x)$	$R(x)$
0	90000	0
30	108000	54000
60	126000	108000
90	144000	162000
120	162000	216000
150	180000	27000
180	198000	324000
210	216000	378000



EXERCISE 10.2

1. **Plot the graph of $y = 2x^2 - 4x + 3$ from -1 to 3. Draw tangent at (2, 3) and find the gradient.**

Solution: $y = 2x^2 - 4x + 3$

Step-I: Substitute the values of x in the given equation and find the corresponding values of y, as given below:

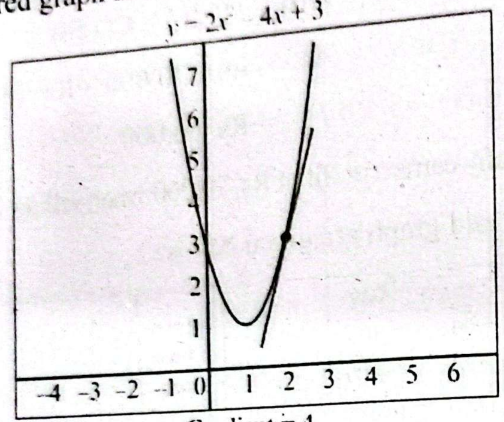
x	y	(x, y)
-1	9	$(-1, 9)$
0	3	$(0, 3)$
1	1	$(1, 1)$
2	3	$(2, 3)$
3	9	$(3, 9)$

Step-II: Select a suitable scale for graph like

On x-axis, 1 division = 1 unit

On y-axis, 1 division = 1 unit

Step-III: Plot the points on the grid and join them to get the required graph as shown in figure.



Gradient = 4

Gradient: Draw a tangent line at point (2, 3). Take any two points on the tangent line. Let two points on the tangent line are: A(2, 3) and B(3, 7)

We know that

Gradient $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ $\qquad m = \dfrac{7-3}{3-2} = \dfrac{4}{1} = 4$

Thus, gradient of tangent line is 4.

2. Plot the graph of $y = 3x^2 + x + 1$ and draw tangent at (1, 5). Also find gradient of the tangent line at this point.

Solution: $y = 3x^2 + x + 1$

Step-I: Substitute the values of x in the given equation and find the corresponding values of y, as given below:

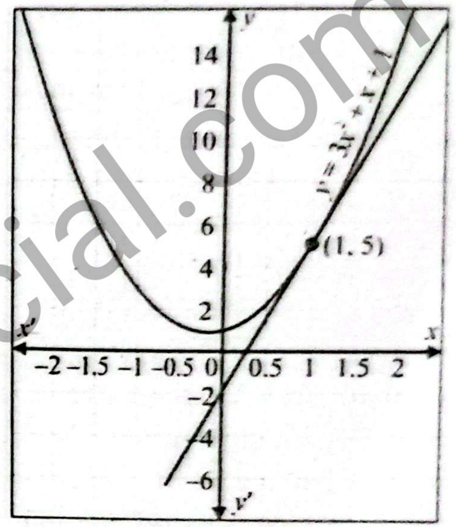
x	y	(x, y)
−2	11	(−2, 11)
−1	3	(−1, 3)
0	1	(0, 1)
1	5	(1, 5)
2	15	(2, 15)

Step-II: Select a suitable scale for graph like

On x-axis, 1 division = 0.5 unit

On y-axis, 1 division = 2 units

Step-III: Plot the points on the grid and join them to get the required graph as shown in figure.



Gradient = 7

Gradient: Draw a tangent line at point (1, 5). Take any two points on the tangent line. Let two points on the tangent line are: A(1,5) and B(2, 12)

We know that

Gradient $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$m = \dfrac{12-5}{2-1} = \dfrac{7}{1} = 7$

Thus, gradient of tangent line is 7.

3. The strength of students in a school was 1000 in 2016. If the strength decay according to the equation $S = 1000\, e^{-t}$, where S is the number of students at time t.

(a) Graph the given equation for $t = 0$ (in 2016) to $t = 9$ (*in* 2025).

(b) From the graph, estimate the student's strength in 2019 and in 2023.

Solution: (a) $S = 1000\ e^{-t}$ Or $S = \dfrac{1000}{e^{t}}$

Step-I: Substitute the values of "t" in the given equation and find the corresponding values of S, as given below:

2016, (t = 0)	1000
2017, (t = 1)	367.9 ≈ 368
2018, (t = 2)	135.3 ≈ 135
2019, (t = 3)	49.8 ≈ 50
2020, (t = 4)	18.3 ≈ 18
2021, (t = 5)	6.7 ≈ 7
2022, (t = 6)	2.48 ≈ 3
2023, (t = 7)	0.91 ≈ 1
2024, (t = 8)	0.33 ≈ 0
2025, (t = 9)	0.12 ≈ 0

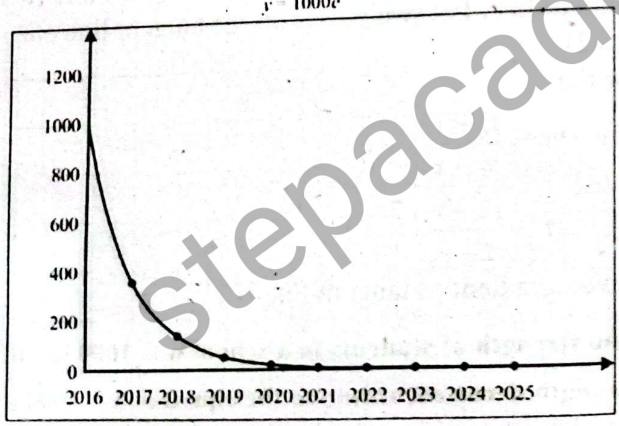
Step-II: Select a suitable scale for graph like

On x-axis, 1 division = t = 1 unit (year)

On y-axis, 1 division = 200 units

Step-III: Plot the points on the grid and join them to get the required graph as shown in figure.

$y = 1000e^{-t}$



Solution: (b)

The estimated student's strength in 2019 is 50.

The estimated student's strength in 2023 is 1.

4. **The demand and supply functions for a product are given by the equations $P_d = 400 - 5Q$, $P_s = 3Q + 24$:**

Plot the graph of each function over the interval

$Q = 0$ to $Q = 300$.

Solution: $Pd = 400 - 5Q$ and $P(s) = 3Q + 24$

The interval ($Q = 0$ to $Q = 300$)

Step-I: Substitute the values of "Q" in the given equation and find the corresponding values of Pd and Ps, as given below:

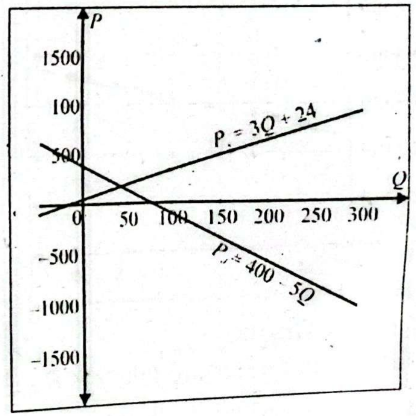
Q	$Pd = 400 - 5Q$	$Ps = 3Q + 24$
0	400	24
50	150	174
100	– 100	324
150	– 350	474
200	– 600	624
250	– 850	774
300	– 1100	924

Step-II: Select a suitable scale for graph like

On x-axis, 1 division = 50 units

On y-axis, 1 division = 200 units

Step-III: Plot the points on the grid and join them to get the required graph as shown in figure.



5. **Shahid's salary S(x) in rupees is based on the following formula:**

$$S(x) = 45000 + 4500x,$$

where x is the number of years he has been with the company. Sketch and interpret the graph of salary function for $0 \le x \le 5$.

Solution: $S(x) = 45000 + 4500x$ and $0 \le x \le 5$

Step-I: Substitute the values of "x" in the given function $S(x)$ and find the corresponding values of $S(x)$, as given below:

x	$S(x)$
0	45000
1	49500
2	54000
3	58500
4	63000
5	67500

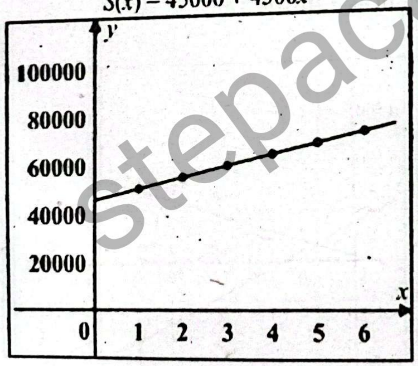
Step-II: Select a suitable scale for graph like

On x-axis, 1 division = 1 unit

On y-axis, 1 division = 10000 units

Step-III: Plot the points on the grid and join them to get the required graph as shown in figure.

$S(x) = 45000 + 4500x$



Interpretation of Salary Graph:

The graph shows that Shahid's starting salary is Rs. 45,000. After 1 year Shahid's salary increases by Rs. 4500.

6. **A company manufactures school bags. The cost function of producing x bags is $C(x) = 1200 + 20x$ and the revenue from selling x bags is $R(x) = 50x$.**

(a) Find the break-even point.

(b) Determine the profit or loss when 250 bags are sold.

(c) Plot the graphs of both the functions and identify the break-even point.

Solution: (a) The break-even point is no profit or loss. It happens when total cost and revenue are equal.

$$R(x) = C(x)$$
$$50x = 1200 + 20x$$
$$50x - 20x = 1200$$
$$30x = 1200$$
$$x = \frac{1200}{30} = 40$$

The company reaches at breakeven point at the production of 40 bags.

Solution: (b)

Since break-even is at 40 bags. Surely it will be profit at 250 bags.

Profit $= R(x) - C(x)$

Profit $= 50x - (1200 + 20x)$

$= 50x - 1200 - 2x$

$= 30x - 1200$

Put $x = 250$ in profit function

Profit $= 30(250) - 1200$

$= 7500 - 1200 = 6300$

Solution: (c) $R(x) = 50x$ and $C(x) = 1200 + 20x$

Step-I: Substitute the values of "x" in given function $S(x)$ and find the corresponding values of $S(x)$, as given below:

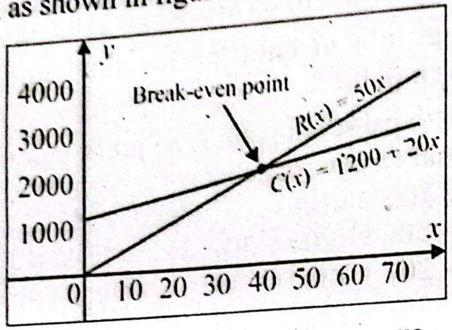
x	$R(x) = 50x$	$C(x) = 1200 + 20x$
0	0	1200
10	500	1400
20	1000	1600
30	1500	1800
40	2000	2000
50	2500	2200
60	3000	2400

Step-II: Select a suitable scale for graph like

On x-axis, 1 division = 10 units

On y-axis, 1 division = 200 units

Step-III: Plot the points on the grid and join them to get the required graph as shown in figure.



7. **A newspaper agency fixed cost of Rs. 70 per edition and marginal printing and distribution costs of Rs. 40 per copy. Profit function is $p(x) = 10x - 70$, where x is the number of newspapers. Plot the graph and find profit for 500 newspapers.**

Solution: $p(x) = 10x - 70$

Step-I: Substitute the values of "x" in the given function $S(x)$ and find the corresponding values of $S(x)$, as given below:

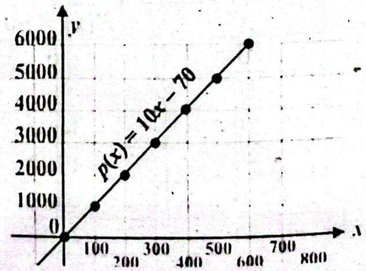
x	p(x)
0	– 70
100	930
200	1930
300	2930
400	3930
500	4930
600	5930

Step-II: Select a suitable scale for graph like

On x-axis, 1 division = 100 units

On y-axis, 1 division = 1000 units

Step-III: Plot the points on the grid and join them to get the required graph as shown in figure.



Finding the profit:

Profit function is $p(x) = 10x - 70$,

Put $x = 500$ in $p(x)$

$p(500) = 10(500) - 70$

$p(500) = 5000 - 70$

$p(500) = 4{,}930$

Thus, profit for 500 newspapers is Rs. 4930.

8. **Ali manufactures expensive shirts for sale to a school. Its cost (in rupees) for x shirts is $C(x) = 1500 + 10x + 0.2x^2$, $0 \le x \le 150$. Plot the graph and find the cost of 200 shirts.**

Solution: Plotting the Graph:

Step-I: Substitute the values of "x" in the given function $C(x)$ and find the corresponding values of $C(x)$, as given below:

$C(x) = 1500 + 10x + 2.0x^2$ and $0 \le x \le 1500$

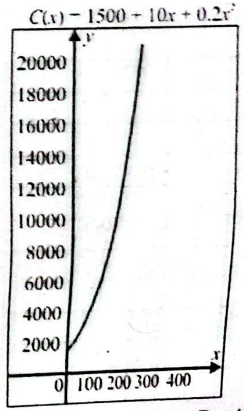
x	C(x)
0	1500
25	1875
50	2500
75	3375
100	4500
125	5875
150	7500

Step-II: Select a suitable scale for graph like

On x-axis, 1 division = 25 units

On y-axis, 1 division = 1500 units

Step-III: Plot the points on the grid and join them to get the required graph as shown in figure.



Cost of 200 shirts = Rs. 11500

Finding the Cost:

$C(x) = 1500 + 10x + 0.2x^2$

Put $x = 200$

$C(200) = 1500 + 10(200) + 2.0(200)^2$
$= 1500 + 2000 + 8000$
$= 11,500$

Thus, cost for 200 shirts is Rs. 11,500.

REVIEW EXERCISE 10

1. Four options are given against each statement. Encircle the correct option.

(i) $x = 5$ represents:
(a) x-axis (b) y-axis
(c) line || to x-axis (d) line || to y-axis

(ii) Slope of the line $y = 5x + 3$ is:
(a) 3 (b) −3 (c) 5 (d) -5

(iii) The y– intercepts of $y = -2x - 1$ is:
(a) −2 (b) 2 (c) −1 (d) 1

(iv) The graph of $y = x^3$, cuts the x-axis at:
(a) $x = 0$ (b) $x = 1$ (c) $x = -1$ (d) $x = 2$

(v) The graph of 3^x represents:
(a) growth (b) decay
(c) both (a) and (b) (d) a line

(vi) The graph of $y = -x^2 + 5$ opens:
(a) upward (b) downward
(c) left side (d) right side

(vii) The graph of $y = x^2 - 9$ opens:
(a) upward (b) downward
(c) left side (d) right side

(viii) $y = 5^x$ is ________ function.
(a) linear (b) quadratic
(c) cubic (d) exponential

(ix) Reciprocal function is:
(a) $y = 7^x$ (b) $y = \frac{2}{x}$
(c) $y = 2x^2$ (d) $y = 5x^3$

(x) $y = -3x^3 + 7$ is ________ function.
(a) exponential (b) cubic
(c) linear (d) reciprocal

Answers:

(i)	(d)	(ii)	(c)	(iii)	(c)	(iv)	(a)	(v)	(a)
(vi)	(b)	(vii)	(a)	(viii)	(d)	(ix)	(b)	(x)	(b)

2. Plot the graph of the following functions:

(i) $y = 3^{-x}$ for x from −2 to 4

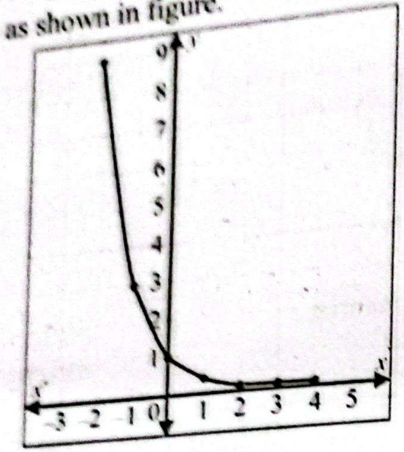
Solution: $y = 3^{-x} = \frac{1}{3^x}$

Step-I: Substitute the values of "x" in the given function and find the corresponding values of y, as given below:

x	$y = 3^{-x}$	(x, y)
−2	9	(−2, 9)
−1	3	(−1, 3)
0	1	(0, 1)
1	$\frac{1}{3} = 0.33$	(1, 0.33)
2	$\frac{1}{9} = 0.11$	(2, 0.11)
3	$\frac{1}{27} = 0.03$	(3, 0.03)
4	$\frac{1}{81} = 0.01$	(4, 0.01)

Step-II: Select a suitable scale for graph like
On x-axis, 1 division = 1 unit
On y-axis, 1 division = 1 unit

Step-III: Plot the points on the grid and join them to get the required graph as shown in figure.



(ii) $y = \frac{2}{x}, x \neq 0$

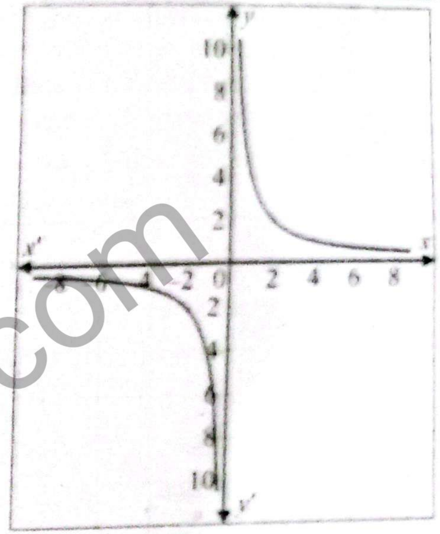
Solution: $y = \frac{2}{x}, x \neq -0$

Step-I: Substitute the values of "x" in the given function and find the corresponding values of y, as given below:

x	$y = \frac{2}{x}$
–8	–0.25
–6	–0.33
–4	–0.5
–2	–1
–1	–2
0	undefined
1	2
2	1
4	0.5
6	0.33
8	0.25

Step-II: Select a suitable scale for graph like
On x-axis, 1 division = 2 units
On y-axis, 1 division = 0.5 unit

Step-III: Plot the points on the grid and join them to get the required graph as shown in figure.



3. Sales for a new magazine are expected to grow according to the equation: $S = 200000\,(1 - e^{-0.05t})$, where t is given in weeks.

(a) Plot graph of sales for the first 50 weeks.

Solution: (a) Graph for First 50 weeks:

Step-I: Substitute the values of "t" in the given function and find the corresponding values of S, as given below:

$S = 200000\,(1 - 2^{-0.05t})$ (t is time in weeks)

We know that $.e \approx 2.718 \approx 2.72$

t	$S = 200000\,(1 - e^{-0.05t})$
0	0
5	44239
10	78694
15	105527
20	126424
25	142699
30	155374
$t = 35$	165245
$t = 40$	172932
$t = 45$	178920
$t = 50$	183583

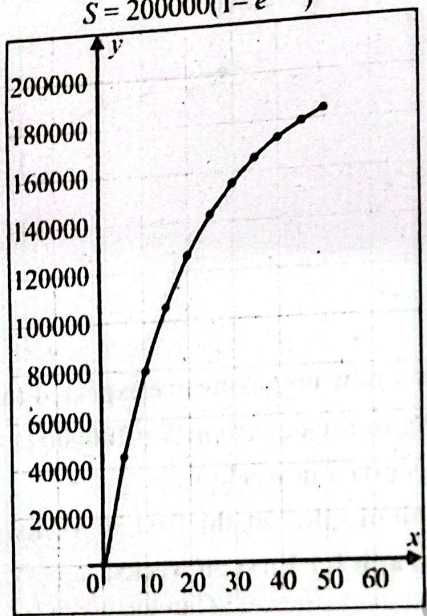
Step-II: Select a suitable scale for graph like

On x-axis, 1 division = 10 units

On y-axis, 1 division = 20000 units

Step-III: Plot the points on the grid and join them to get the required graph as shown in figure.

$S = 200000(1 - e^{0.05t})$



(b) Calculate the number of magazines sold, when $t = 5$ and $t = 35$.

Solution: (b)

Given that $S = 20000\,(1 - e^{-0.05t})$ ... (i)

Put $t = 5$ in function.

$$S = 200,000\,(1 - e^{-0.05t}) = 200,000\,(1 - e^{-0.05 \times 5})$$
$$= 44,239.84 \approx 44,240$$

Thus, 44, 240 magazines are sold when $t = 5$.

Put $t = 35$ in function.

$$S = 200,000\,(1 - e^{-0.05 \times 35})$$
$$= 200,000\,(1 - e^{-1.75})$$
$$= 165,245.2 \approx 165,245$$

Thus, 165,245 magazines are sold when $t = 35$.

4. Plot the graph of following for x from −5 to 5:

(i) $y = x^2 - 3$

Solution: $y = x^2 - 3$

Step-I: Substitute the values of "x" in the given function and find the corresponding values of y, as given below:

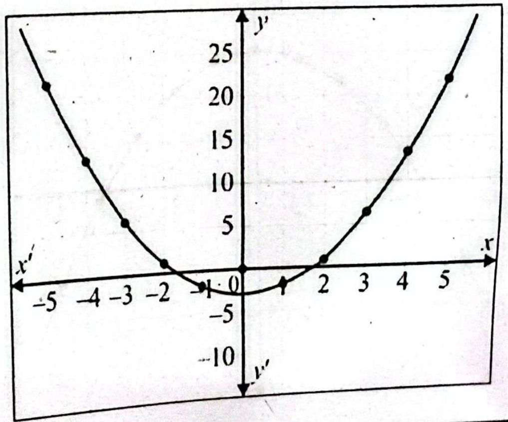
x	$y = x^2 - 3$	(x, y)
−5	22	(−5, 22)
−4	13	(−4, 13)
−3	6	(−3, 6)
−2	1	(−2, 1)
−1	−2	(−1, −2)
0	−3	(0, −3)
1	−2	(1, −2)
2	1	(2, 1)
3	6	(3, 6)
4	13	(4, 13)
5	22	(5, 22)

Step-II: Select a suitable scale for graph like

On x-axis, 1 division = 1 unit

On y-axis, 1 division = 2 units

Step-III: Plot the points on the grid and join them to get the required graph as shown in figure.



(ii) $y = 15 - x^2$

Solution: $y = 15 - x^2$

Step-I: Substitute the values of "x" in the given function and find the corresponding values of y, as given below:

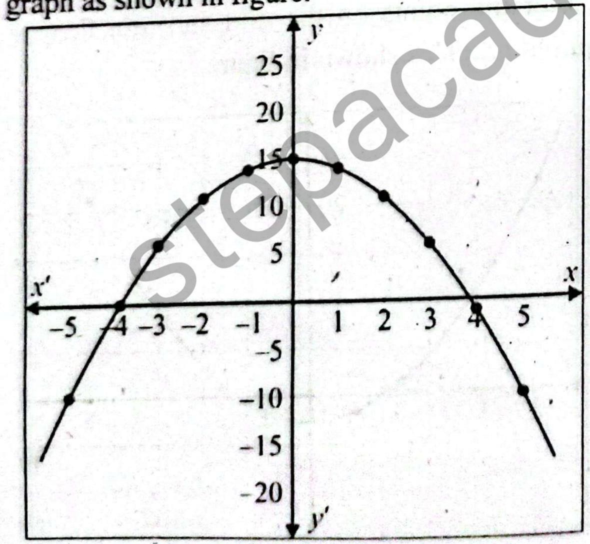
x	$y = 15 - x^2$	(x, y)
–5	–10	(–5, –10)
–4	–1	(–4, –1)
–3	6	(–3, 6)
–2	11	(–2, 11)
–1	14	(–1, 14)
0	–15	(0, –15)
1	14	(1, 14)
2	11	(2, 11)
3	6	(3, 6)
4	–1	(4, –1)
5	–10	(5, –10)

Step-II: Select a suitable scale for graph like

On x-axis, 1 division = 1 unit

On y-axis, 1 division = 2 units

Step-III: Plot the points on the grid and join them to get the required graph as shown in figure.



5. **Plot the graph of $y = \frac{1}{2}(x+4)(x-1)(x-3)$ from –5 to 4.**

Solution: $y = \frac{1}{2}(x+4)(x-1)(x-3)$

Step-I: Substitute the values of "x" in the given function and find the corresponding values of y, as given below:

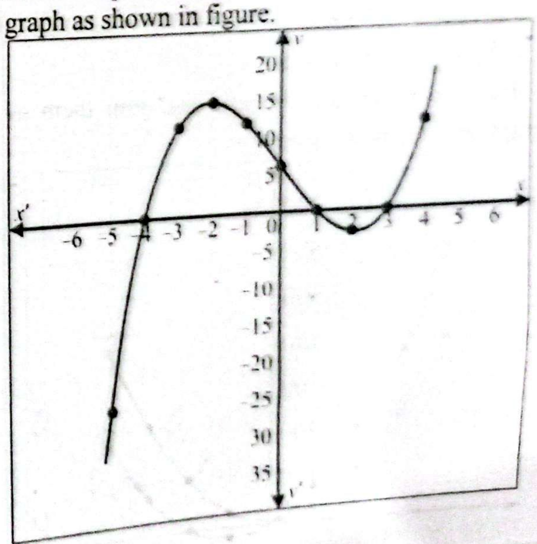
x	y	(x, y)
-4	0	(-4,0)
-3	12	(-3,12)
-2	15	(-2,15)
-1	12	(-1,12)
0	6	(0, 6)
1	0	(1, 0)
2	-3	(2, -3)
3	0	(3, 0)
4	12	(4, 12)

Step-II: Select a suitable scale for graph like

On x-axis, 1 division = 1 unit

On y-axis, 1 division = 3 units

Step-III: Plot the points on the grid and join them to get the required graph as shown in figure.



6. **The supply and demand functions for a particular market are given by the equations:**

(i) **$P_s = Q^2 + 5$ and $P^d = Q^2 - 10Q$, where P represents price and Q represents quantity,**

(ii) **Sketch the graph of each function over the interval $Q = -20$ to $Q = 20$.**

Solution: $P_s = Q^2 + 5$, $Pd = Q^2 - 10Q$

Step-I: Substitute the values of "Q" in the given function and find the corresponding values of P_s and P_s, as given below:

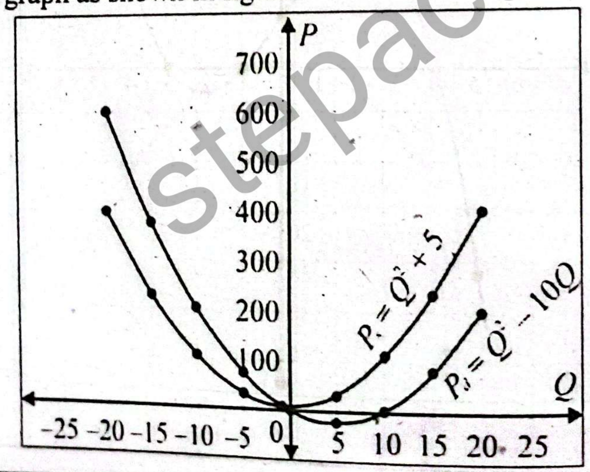
Q	P_s	P_d
– 20	405	600
– 15	230	375
–10	150	200
–5	30	75
0	5	0
5	30	– 25
10	105	0
15	230	75
20	405	200

Step-II: Select a suitable scale for graph like

On x-axis, 1 division = 5 units

On y-axis, 1 division = 100 units

Step-III: Plot the points on the grid and join them to get the required graph as shown in figure.



7. **A television manufacturer company make 40 inches LEDs. The cost of manufacturing x LEDs is $C(x) = 60{,}000 + 250x$ and the revenue from selling x LEDs is $R(x) = 1200x$. Find the break-even point and find the profit or loss when 100 LEDs are sold. Identify the break-even point graphically.**

Solution: **Finding Break-even point:**

The break-even point means no profit or loss. It happens when total cost and revenue are equal.

$$R(x) = C(x)$$
$$1200x = 60{,}000 + 250x$$
$$1200x - 250x = 60{,}000$$
$$950x = 60{,}000$$
$$x = \frac{60000}{950} = 63.16 \approx 63$$

Since break-even is at approximately 64 LEDs. Surely it will be profit on selling 100 LEDs.

$$\text{Profit} = R(x) - C(x)$$
$$\text{Profit} = (1200x) - (60{,}000 + 250x)$$
$$= 1200x - 60{,}000 - 250x$$
$$= 950x - 60{,}000$$

Put the x = 100 in the profit function.

$$\text{Profit} = 950(100) - 60{,}000$$
$$= 95000 - 60{,}000$$
$$= 35{,}000$$

Plotting the graph:

$C(x) = 60,000 + 250x, \quad R(x) = 1200x$

x	$C(x)$	(x, y)
0	60,000	(0, 0)
20	65,000	24,000
40	70,000	48,000
60	75,000	72,000
80	80,000	96,000
100	85,000	
120	90,000	

Step-II: Select a suitable scale for graph like

On x-axis, 1 division = 20 units

On y-axis, 1 division = 20,000 units

Step-III: Plot the points on the grid and join them to get the required graph as shown in figure.