

Students' learning outcomes

At the end of the unit, the students will be able to:

- Construct a grouped frequency table, histogram (with unequal class interval) and frequency polygon.
- Calculate the mean modal class and median of a grouped frequency distribution.
- Solve real life situations involving mean, weighted mean, median and mode for given data (such as allocation of funds in different projects, forecasting future demographics, marketing, forecasting government budgets).

Example 1: Following are the number of telephone calls made in a week to 30 teachers of a high school.

5 8 11 25 13 16 20 17 15 16 30 21 14 18 19
6 22 26 15 19 35 29 31 23 25 20 10 9 7 26

Construct a frequency distribution with number of classes 7.

Solution: (i) Find range

Greatest value (maximum value) = 35, Smallest value (minimum value) = 5

$$\text{Range} = X_{\max} - X_{\min} = 35 - 5 = 30$$

(ii) Size of class limits = $\frac{\text{Range}}{\text{Number of classes}} = \frac{30}{7} = 4.28 \approx 5$

(iii) Make class limits having size 5. For example, 5 – 9, 10 – 14, 15 – 19 and so on. (see 1st column of table: 1).

(iv) Tally marks are used to count the values, fall in the given class limits. (See 2nd column of table: 1).

(v) Now, count the number of tally marks and write the number as frequency in the third column (see 3rd column of table: 1).

Example 2: The frequency distribution of ages (in years) of 76 members of a locality is available. Draw a histogram for this data.

Class limits	2-4	4-9	9-12	12-17	17-20	20-27	27-30
Frequency (f)	7	10	18	20	10	7	4

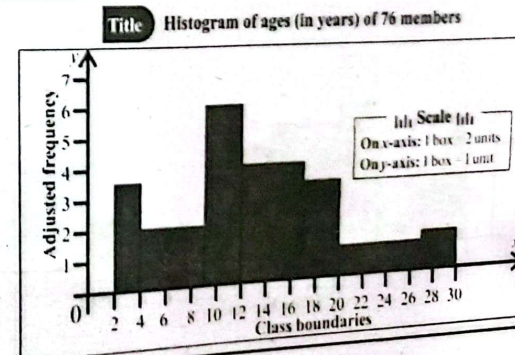
Solution: Look at the table, indicates that the width of the class limits is not equal as first class has width 2, second has 5, the third

has 3, the fourth has 5, the fifth has 3, sixth class has 7, seventh class has width 3. So, there is need to adjust the heights of the rectangles i.e., for the first class we have 2 as width of class and 7 as a frequency, so the height of the first class is $\frac{7}{2} = 3.5$, similarly

for the other $\frac{10}{5} = 2, \frac{18}{3} = 6, \frac{20}{5} = 4, \frac{10}{3} = 3.3, \frac{7}{7} = 1, \frac{4}{3} = 1.3$. These proportional heights are also called adjusted frequencies.

Class limits	Frequency (f)	Width of Class	Height of rectangle (Adjusted frequency)
2-4	7	4-2=2	$\frac{7}{2} = 3.5$
4-9	10	9-4=5	$\frac{10}{5} = 2$
9-12	18	12-9=3	$\frac{18}{3} = 6$
12-17	20	17-12=5	$\frac{20}{5} = 4$
17-20	10	20-17=3	$\frac{10}{3} = 3.3$
20-27	7	27-20=7	$\frac{7}{7} = 1$
27-30	4	30-27=3	$\frac{4}{3} = 1.3$

Taking class boundaries along x - axis and corresponding adjusted frequencies along y - axis, rectangles are drawn and the histogram is given below.

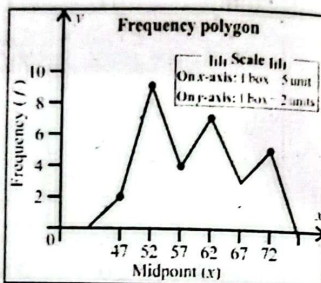


Example 3: The following are the marks obtained by 30 students out of 100 in the subject of Mathematics at their final examination. Construct frequency polygon for the following frequency table.

Marks	45-49	50-54	55-59	60-64	65-69	70-74
Frequency	2	9	4	7	3	5

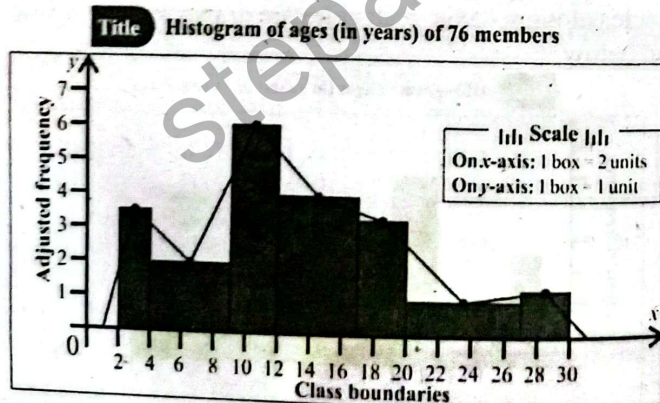
Solution:

Marks	f	Midpoints
45-49	2	$\frac{45+49}{2} = 47$
50-54	9	$\frac{50+54}{2} = 52$
55-59	4	$\frac{55+59}{2} = 57$
60-64	7	$\frac{60+64}{2} = 62$
65-69	3	$\frac{65+69}{2} = 67$
70-74	5	$\frac{70+74}{2} = 72$



Remember!

Frequency polygon on histogram: In histogram, we mark the midpoints on the top of rectangles and join all the points. To touch the base of x-axis, we extend the line at both ends to the next midpoints. The resulting graph is a frequency polygon.



EXERCISE 12.1

1. The following distribution represents the scores achieved by a group of chemistry students in the chemistry laboratory.

Scores	24-28	29-33	34-38	39-43	44-48	49-53	Total
No. of students	3	6	12	23	15	6	65

Answer the following questions.

- What is the upper limit of the last class?
- What is the lower limit of the class 39-43?
- What is the midpoint of the class (34-38)?
- What are the class frequencies of the classes 29-33 and 44-48?
- What is the size of the class limits in the above frequency distribution?
- In which class or group does minimum number of students fall?
- What is the lower limit of the class having 15 as its class frequency?
- What is the number of students having scores between 24 and 43?

Solutions:

- Upper limit of last class = 53.
- Lower limit of class (39-43) is 39.
- Mid point of class (34-38) is $\frac{34+38}{2} = 36$.
- Class frequency of class (29-33) and (44-48) are '6' and 15 respectively.
- Size of class limits = 5.

(iv) The group (24 - 28) contains minimum number of students.

(vii) Lower limit of class of frequency 15 is 44.

(viii) Number of students having scores between 24 and 43 are
 $3 + 6 + 12 + 23 = 44$.

2. For a school staff, the following expenditures (rupees in hundred) are required for the repair of chairs.

145, 152, 153, 156, 158, 160, 146, 152,
 155, 159, 161, 163, 165, 147, 148, 151,
 154, 156, 158, 160, 144, 167, 151, 150,
 152, 149, 145, 153, 152, 155

Prepare a frequency distribution by tally bar method using 3 as the size of class limits and also write down what are the frequencies of the last three classes?

Sol: 145 152 153 156 158 160 146 152 155 159
 161 163 165 147 148 151 154 156 158 160
 144 167 151 150 152 149 145 153 152 155

Class limits	Tally marks	f
144 - 146		4
147 - 149		3
150 - 152		7
153 - 155		5
156 - 158		4
159 - 161		4
162 - 164		1
165 - 167		2
Total		$\Sigma f = 30$

No. of observation = $n = 30$
 Largest value = 167
 Smallest value = 144
 Size of class intervals = 3

The frequencies of last 3 classes are 4, 1 and 2 respectively.

3. Given below are the weights in kg of 30 students of a high school.

30, 33, 24, 21, 15, 39, 37, 44,
 42, 33, 33, 28, 29, 32, 31, 28,
 26, 32, 34, 35, 18, 36, 41, 30,
 35, 41, 23, 26, 18, 34

Taking 5 as the size of the class limit, prepare a frequency table and construct a frequency polygon.

Sol: 30 33 24 21 15 39 37 44 42 33
 33 28 29 32 31 28 26 32 34 35
 38 36 41 30 35 41 23 26 18 34

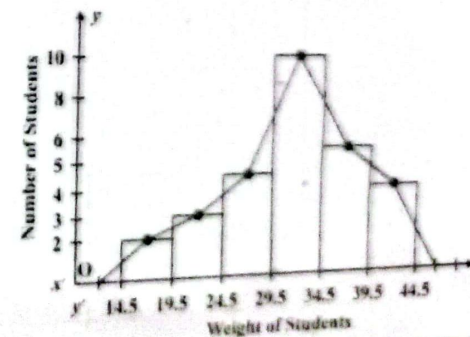
No. of observation = $n = 30$

Largest value = 44

Smallest value = 15

Size of class limits = 5

Class limits	Tally marks	f	C.B
15 - 19		2	14.5 - 19.5
20 - 24		3	19.5 - 24.5
25 - 29		5	24.5 - 29.5
30 - 34		10	29.5 - 34.5
35 - 39		6	34.5 - 39.5
40 - 44		4	39.5 - 44.5
Total			$\Sigma f = 30$



4. A group of Grade -10 students obtained the following marks out of 100 marks in English test.

58, 59, 58, 33, 40, 58, 45, 46,
43, 45, 45, 50, 52, 49, 50, 57,
52, 55, 49, 50, 62, 49, 48, 44,
42, 47, 46, 47, 46, 53, 40, 44

Classify the data into a frequency distribution by (direct method) taking 6 as the size of class limit. Also find the class limit with least class frequency and construct histogram for the data.

Sol: 58 59 58 33 40 58 45 46 43 45 45
50 52 49 50 57 52 55 49 50 62 49
48 44 42 47 46 47 46 53 40 44

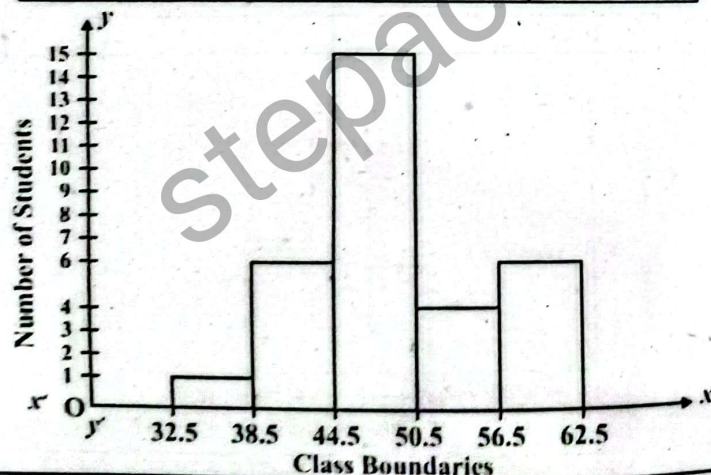
No. of observation = $n = 32$

Largest value = 62

Smallest value = 33

Size of class limits = 6

Class limits	Tally marks	f	C.B.
33 - 38	I	1	32.5 - 38.5
39 - 44		6	38.5 - 44.5
45 - 50		15	44.5 - 50.5
51 - 56		4	50.5 - 56.5
57 - 62		6	56.5 - 62.5
Total		$\Sigma f = 30$	



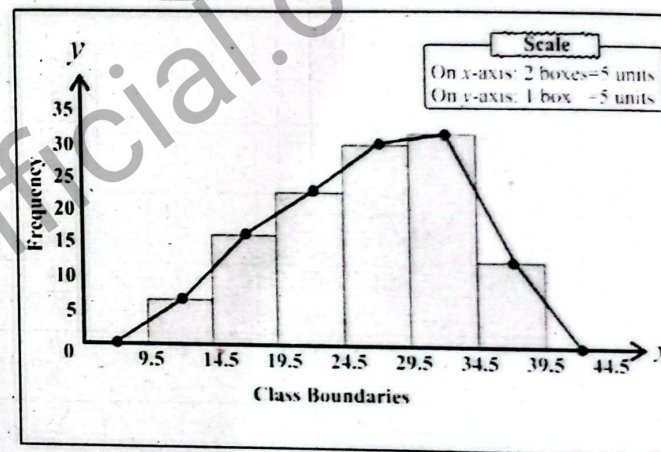
5. From the table given below. Draw a frequency polygon on histogram for the given frequency distribution.

Weight (kg)	10-14	15-19	20-24	25-29	30-34	35-39
Frequency (f)	06	17	23	30	22	13

Sol:

Weight (kg)	10 - 14	15 - 19	20 - 24	25 - 29	30 - 34	35 - 39
Frequency (f)	06	17	23	30	22	13
C.B	9.5 - 14.5	14.5 - 19.5	19.5 - 24.5	24.5 - 29.5	29.5 - 34.5	34.5 - 39.5

Title Frequency polygon on histogram



6. The following data shows the number of heads in an experiment of 50 sets of tossing a coin 5 times. Make a discrete frequency distribution from the information.

3, 3, 4, 0, 5, 4, 3, 3, 1, 2, 4, 5, 0, 3, 2, 4, 4, 0, 0, 0, 5, 5, 3, 2, 1
3, 2, 5, 3, 2, 1, 3, 5, 4, 3, 2, 1, 3, 2, 1, 3, 1, 3, 1, 4, 3, 2, 2, 4

Sol: 3, 3, 4, 0, 5, 4, 3, 3, 1, 2, 4, 5, 0, 3, 2, 4, 4, 0, 0, 0, 5, 5, 3, 2, 1.
4, 3, 2, 5, 3, 2, 1, 3, 5, 4, 3, 2, 1, 3, 2, 1, 3, 1, 3, 1, 4, 3, 2, 2, 4.

Number of Heads	Tally marks	Frequency
0		5
1		7
2		9
3		14
4		9
5		6
Total		$\Sigma f = 50$

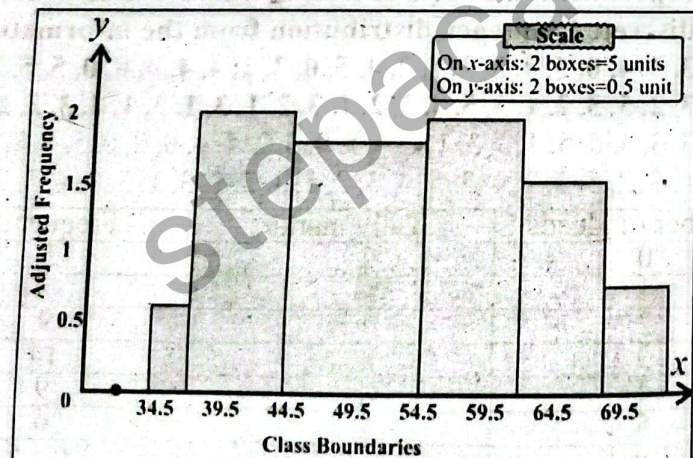
7. The marks obtained by the students of Grade-10 in mathematics test were grouped into the following frequency distribution.

Marks	35-37	38-44	45-54	55-61	62-67	68-72
Frequency	2	12	16	13	9	3

Draw a histogram for the above distribution.

Sol:

Marks	(f)	C.B	h	f/h
35 - 37	2	34.5 - 37.5	3	$\frac{2}{3} = 0.67$
38 - 44	12	37.5 - 44.5	7	$\frac{12}{7} = 1.7$
45 - 54	16	44.5 - 54.5	10	$\frac{16}{10} = 1.6$
55 - 61	13	54.5 - 61.5	7	$\frac{13}{7} = 1.86$
62 - 67	9	61.5 - 67.5	6	$\frac{9}{6} = 1.5$
68 - 72	3	67.5 - 72.5	5	$\frac{3}{5} = 0.6$



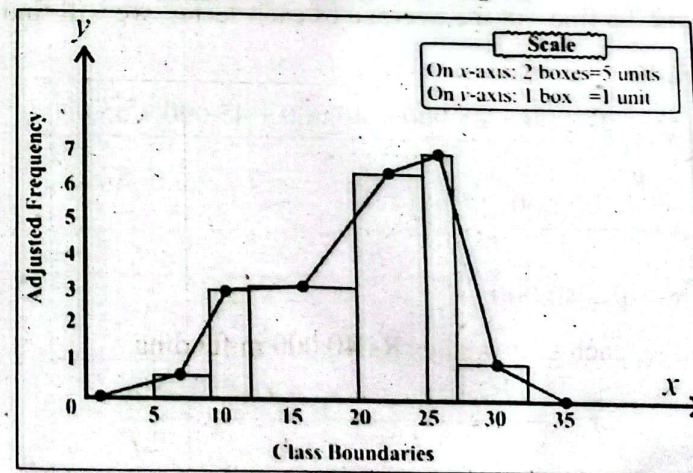
8. Make a frequency polygon on histogram for the following grouped data:

Class limits	5-8	8-12	12-20	20-25	25-27	27-32
Frequency (f)	2	12	25	32	14	5

Sol:

Class limits	(f)	Width of class (h)	f/h
5 - 8	2	$8 - 5 = 3$	$\frac{2}{3} = 0.67$
8 - 12	12	$12 - 8 = 4$	$\frac{12}{4} = 3$
12 - 20	25	$20 - 12 = 8$	$\frac{25}{8} = 3.1$
20 - 25	32	$25 - 20 = 5$	$\frac{32}{5} = 6.4$
25 - 27	14	$27 - 25 = 2$	$\frac{14}{2} = 7$
27 - 32	5	$32 - 27 = 5$	$\frac{5}{5} = 1$

Title Frequency polygon on histogram



Example 4: The marks of a student in five examinations were 64, 75, 81, 87, 90. Find the arithmetic mean of the marks.

Solution:

$$\text{A.M.} = \bar{X} = \frac{\sum x}{n}$$

$$= \frac{64 + 75 + 81 + 87 + 90}{5}$$

or $\bar{X} = \frac{397}{5} = 79.4$ marks

Example 5: A government allocates funds of Rs.200,000 to five sectors of a school i.e.,

- (i) School Library: Rs. 35,000
- (ii) Sports facilities: Rs. 25,000.
- (iii) Parking area: Rs. 40,000
- (iv) Room renovation: Rs. 45,000
- (v) Furniture: Rs. 55,000

Find the average of fund allocation in each sector of a school.

Solution: To find out the average of each sector, we will find the mean of the given data.

$$\bar{X} = \frac{35,000 + 25,000 + 40,000 + 45,000 + 55,000}{5}$$

$$\bar{X} = \frac{200,000}{5}$$

$$\bar{X} = \text{Rs. } 40,000$$

On average, each sector takes Rs.40,000 in funding.

$$\bar{X} = \frac{f_1x_1 + f_2x_2 + \dots + f_nx_n}{f_1 + f_2 + \dots + f_n} = \frac{\sum fx}{\sum f}$$

Example 6: Given below are the marks out of 100 obtained by 100 students in a examination. Find the average marks of the students.

Marks	30-35	35-40	40-45	45-50	50-55	55-60
No. of students	14	16	18	23	18	11

Marks	Midpoint (x)	Frequency (f)	fx
30-35	32.5	14	455.0
35-40	37.5	16	600.0
40-45	42.5	18	765.0
45-50	47.5	23	1092.5
50-55	52.5	18	945.0
55-60	57.5	11	632.5
Total	—	$\Sigma f = 100$	$\Sigma fx = 4490$

$$\bar{X} = \frac{\Sigma fx}{\Sigma f} = \frac{4490}{100}$$

or $\bar{X} = 44.9$ marks

Hence, the average marks is 44.9 of the students.

$$\bar{X} = A + \frac{\Sigma D}{n} \quad (\text{for ungrouped data}) \quad \dots(i)$$

$$\bar{X} = A + \frac{\Sigma fD}{\Sigma f} \quad (\text{for grouped data}) \quad \dots(ii)$$

Example 7: Find the arithmetic mean using short formula for the runs made by a batsman.

Runs: 40, 45, 50, 52, 50, 60, 56, 70.

Solution: Taking deviations from $A = 52$ (assumed mean)

x	40	45	50	52	50	60	56	70
$D = x - A$	-12	-7	-2	0	-2	8	4	18

Now: $\Sigma D = -23 + 30 = 7$

$$\bar{X} = A + \frac{\Sigma D}{n}$$

So, $\bar{X} = 52 + \frac{7}{8}$

$$= 52 + 0.875 = 52.88 \text{ or } 53 \text{ runs.}$$

Example 8: Deviations from 12.5 of ten different values are 6, -2, 3.5, 9, 8.7, -5.5, 14, 11.3, -6.8, -4.2, find the arithmetic mean.

Solution: Deviations from 12.5 are:

6, -2, 3.5, 9, 8.7, -5.5, 14, 11.3, -6.8, -4.2

Now, $\Sigma D = 34$. Also, $A = 12.5$, using the formula we have,

$$\begin{aligned}\bar{X} &= A + \frac{\Sigma D}{n} \\ &= 12.5 + \frac{34}{10}\end{aligned}$$

$$\text{or } \bar{X} = 12.5 + 3.4 = 15.9$$

Example 9: The heights (in inches) of 200 students are recorded in the following frequency distribution. Find the mean height of the student by short formula.

Height (x) (in inches)	51	52	53	54	55	56	57	58	59	60
Frequency (f)	2	5	8	24	55	45	38	16	6	1

Solution:

Heights (x) (in inches)	Frequency (f)	$A = 55$ $D = x - A$	fD
51	2	-4	-8
52	5	-3	-15
53	8	-2	-16
54	24	-1	-24
$A \leftarrow 55$	55	0	0
56	45	1	45
57	38	2	76
58	16	3	48
59	6	4	24
60	1	5	5
Total	$\Sigma f = 200$		$\Sigma fD = 135$

Now, using the formula (ii), we get

$$\bar{X} = A + \frac{\Sigma fD}{\Sigma f}$$

$$\bar{X} = 55 + \frac{135}{200}$$

$$\text{or } \bar{X} = 55 + 0.675$$

$$\therefore \bar{X} = 55.68 \text{ inches approx.}$$

Hence, the mean height of the students is 55.68 inches.

Example 10: Ten students each from Grade-V section A and B of a well reputed school were taken randomly. Their weights were measured in kg. and recorded as given below:

Weights (kg) Section A	30	28	32	29.5	35	34	31	33	40	37.5
Weights (kg) Section B	35	31.5	34.5	35	32.8	38	29.5	36	36.5	34

- Compute the mean weight for section A and B.
- Conclude which section is better on Average?

Solution: (i) We find arithmetic mean for both the sections by direct method. (Any method can be applied).

As number of observations $n = 10$

$$\text{and } \bar{X}_{(A)} = \frac{\Sigma X_{(A)}}{n}$$

$$\therefore \bar{X}_{(A)} = \frac{330}{10} = 33 \text{ kg}$$

$$\text{and } \bar{X}_{(B)} = \frac{\Sigma X_{(B)}}{n}$$

$$\therefore \bar{X}_{(B)} = \frac{342.8}{10} = 34.28 \text{ kg}$$

$X_{(A)}$	$X_{(B)}$
30	35
28	31.5
32	34.5
29.5	35
35	32.8
34	38
31	29.5
33	36
40	36.5
37.5	34
$\Sigma X_{(A)} = 330$	$\Sigma X_{(B)} = 342.8$

- We have seen from the results that $\bar{X}_{(B)}$ is greater than $\bar{X}_{(A)}$

Therefore, we conclude that section B is better on the average.

Median for ungrouped data

The median of n observations x_1, x_2, \dots, x_n is obtained as:

$$\text{Median } (\tilde{X}) = \left(\frac{n+1}{2} \right)^{\text{th}} \text{ observation} \quad \left(\begin{array}{l} \text{when } n \text{ is} \\ \text{odd number} \end{array} \right)$$

$$\text{Median } (\tilde{X}) = \frac{1}{2} \left[\left(\frac{n}{2} \right)^{\text{th}} \text{ observation} + \left(\frac{n+2}{2} \right)^{\text{th}} \text{ observation} \right] \quad \left(\begin{array}{l} \text{when } n \text{ is} \\ \text{even number} \end{array} \right)$$

Example 11: The following are the scores made by a batsman.

Find the median of the data. 8, 12, 18, 13, 16, 5, 20.

Solution: Writing the scores in an ascending order, we have

5, 8, 12, 13, 16, 18, 20

Since, number of observations is odd i.e., $n = 7$

$$\text{Median } (\tilde{X}) = \left(\frac{n+1}{2} \right)^{\text{th}} \text{ observation}$$

$$= \left(\frac{7+1}{2} \right)^{\text{th}} \text{ observation} = 4^{\text{th}} \text{ observation} = 13$$

Hence, 13 is the median of the given data.

Example 12: Following are the marks out of 100 obtained by 10 students in English.

23, 15, 35, 48, 41, 5, 8, 9, 11, 51. Find the median of the data.

Solution: Arranging the data in an ascending order.

5, 8, 9, 11, 15, 23, 35, 41, 48, 51

Since, number of observation is even, i.e., $n = 10$

$$\therefore \text{Median } (\tilde{X}) = \frac{1}{2} \left[\left(\frac{n}{2} \right)^{\text{th}} \text{ observation} + \left(\frac{n+2}{2} \right)^{\text{th}} \text{ observation} \right]$$

$$\text{As, } \frac{n}{2} = \frac{10}{2} = 5 \text{ and } \frac{n+2}{2} = \frac{12}{2} = 6$$

$$\therefore \text{Median} = \frac{1}{2} [5^{\text{th}} \text{ observation} + 6^{\text{th}} \text{ observation}]$$

$$\text{or Median} = \frac{1}{2} [15 + 23] = \frac{38}{2} = 19$$

Hence, 19 is the median of the data.

Median for Grouped Data

The median for grouped data is obtained by the following formula:

$$\text{Median } (\tilde{X}) = \ell + \frac{h}{f} \left(\frac{n}{2} - c \right)$$

Where, ℓ = Lower class boundary of median class,

h = The size of class limits of median class,

f = Frequency of the median class,

n = Total frequency i.e., Σf ,

and C = Cumulative frequency preceding the median class.

Example 13: The heights of 100 athletes, measured to the nearest (inches) are given in the following table. Find the median.

Heights (in inches)	62.5 - 63.5	63.5 - 64.5	64.5 - 65.5	65.5 - 66.5	66.5 - 67.5	67.5 - 68.5	68.5 - 69.5	69.5 - 70.5	70.5 - 71.5
No. of Students	4	6	10	20	30	13	12	3	2

Solution: In the above data, class boundaries have already been given:

Heights (inches)	Frequency (f)	c.f.
62.5 - 63.5	4	4
63.5 - 64.5	6	6 + 4 = 10
64.5 - 65.5	10	10 + 10 = 20
65.5 - 66.5	20	20 + 20 = 40 $\rightarrow c$
66.5 - 67.5	30	30 + 40 = 70 \rightarrow Median class
67.5 - 68.5	13	13 + 70 = 83
68.5 - 69.5	12	12 + 83 = 95
69.5 - 70.5	3	3 + 95 = 98
70.5 - 71.5	2	2 + 98 = 100 $\rightarrow n$
Total	$\Sigma f = 100$	---

Here, $n = 100$

$$\text{so, } \frac{n}{2} = \frac{100}{2} = 50$$

50th item lies in the class boundaries 66.5 - 67.5.

$$\ell = 66.5, \quad h = 1, \quad f = 30, \quad c = 40$$

$$\text{Median} = \ell + \frac{h}{f} \left(\frac{n}{2} - c \right)$$

$$= 66.5 + \frac{1}{30} (50 - 40) \quad (\text{Putting the values})$$

$$= 66.5 + \frac{10}{30}$$

$$= 66.5 + 0.33$$

\therefore Median = 66.83 inches

Example 15: The marks in mathematics of Jamal in eight monthly tests were 75, 76, 80, 80, 82, 82, 82, 85. Find the mode of the marks.

Solution: As 82 is repeated more than any other number so, clearly mode is 82.

Example 16: Ten students were asked about the number of questions they have solved out of 20 questions last week. Records were 13, 14, 15, 11, 16, 10, 19, 20, 18, 17. Find the mode of the data.

Solution: It is obvious that the given data contains no mode. It is ill-defined.

Sometimes data contains several modes. If the data is: 10, 15, 15, 15, 20, 20, 20, 25, 32, then data contains two modes i.e., 15 and 20.

Example 17: A survey was conducted from the 15 students of a school and asked the students about their favourite colour.

The responses are: purple, yellow, purple, yellow, yellow, red, blue, green, yellow, red, blue, yellow, purple, green. Find mode of the data.

Solution: Mode is the most frequent colour.

Mode = yellow

So, the colour "yellow" is the mode of the given data.

Mode for Grouped Data

Mode can be calculated by the following formula:

$$\text{Mode} = \ell + \frac{(f_m - f_1)}{(f_m - f_1) + (f_m - f_2)} \times h$$

Where, ℓ = Lower class boundary of the modal class.

f = Frequency of the modal class.

f_1 = Frequency preceding the modal class.

f_2 = Frequency following the modal class and

h = Size of the modal class.

Example 18: Following are the heights in (inches) of 40 students in Grade - 8.

Heights (inches)	48-50	50-52	52-54	54-56	56-58	58-60
No. of students	5	7	10	9	6	3

Find mode of the above data.

Heights (inches)	Frequency (f)
48 - 50	5
50 - 52	7 $\rightarrow f_1$
52 - 54	10 $\rightarrow f_m$
54 - 56	9 $\rightarrow f_2$
56 - 58	6
58 - 60	3
Total	$\Sigma f = 40$

Activity

Collect data of weights of 50 students. Make a frequency distribution and find mean, median and mode of the data.

Solution: In the above data, class boundaries have already been given. Using the formula for grouped data we find mode as:

$$\ell = 52, h = 2, f_m = 10, f_1 = 7, f_2 = 9$$

$$\text{Mode} = \ell + \frac{(f_m - f_1) \times h}{(f_m - f_1) + (f_m - f_2)}$$

$$\text{or Mode} = 52 + \frac{(10 - 7) \times 2}{(10 - 7) + (10 - 9)}$$

$$\text{or Mode} = 52 + \frac{3 \times 2}{3 + 1} = 52 + \frac{6}{4}$$

$$\text{or Mode} = 52 + 1.5 = 53.5 \text{ (inches)}$$

$$\bar{X}_w = \frac{W_1 X_1 + W_2 X_2 + W_3 X_3 + \dots + W_n X_n}{W_1 + W_2 + W_3 + \dots + W_n} = \frac{\sum_{i=1}^n W_i X_i}{\sum_{i=1}^n W_i} = \frac{\sum W X}{\sum W}$$

Skill practice!

Find the mean, median and mode of the first twenty whole numbers.

Example 19: The following data describes the marks of a student in different subjects and weights assigned to these subjects are also given:

Mark (x)	74	78	74	90
Weights (w)	4	3	5	6

Find its weighted mean.

Solution: Weighted mean $(\bar{X}_w) = \frac{\sum WX}{\sum W}$

$$\bar{X}_w = \frac{4(74) + 3(78) + 5(74) + 6(90)}{4 + 3 + 5 + 6}$$

$$= \frac{296 + 234 + 370 + 540}{18} = \frac{1440}{18}$$

$$\bar{X}_w = 80$$

Example 20: A medicine company started marketing of a sample of medicine in seven different areas of a city. The company distributed the packets of medicine in each area of the city and the weight of each area based on the demand of the medicine. Find the mean and weighted mean of the given data.

Area	Number of packets	Weights (kg)
A	15	5
B	25	4
C	18	3
D	23	4
E	15	2
F	10	1
G	8	2

Solution: Mean = $\frac{\sum X}{n}$

$$= \frac{15 + 25 + 18 + 23 + 15 + 10 + 8}{7}$$

So, the average number of packets of the medicine distributed by the company per area is 16.

Weighted mean = $\frac{\sum [\text{Number of packets} \times \text{Weight}]}{\sum \text{Weights}}$

$$= \frac{15(5) + 25(4) + 18(3) + 23(4) + 15(2) + 10(1) + 8(2)}{5 + 4 + 3 + 4 + 2 + 1 + 2}$$

$$= \frac{377}{21} = 17.95 \approx 18$$

Real Life Situations Involving Mean, Weighted Mean, Median and Mode

Sales and Marketing

Example 21: A toy factory sold toys in a month. Consider the following data:

Class limits	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60
f	15	28	45	29	20

- Calculate mean, median and mode of the number of toys sold by the factory.
- Also tell the modal class of the distribution.

Solution: (i) For mean

Class limits	f	X	fX	c.f.	
10 - 20	15	15	225	15	
20 - 30	28	25	700	28 + 15 = 43	
30 - 40	45	35	1575	45 + 43 = 88	Modal class
40 - 50	29	45	1305	29 + 88 = 117	Median class
50 - 60	20	55	1100	20 + 117 = 137	
Total	$\Sigma f = 137$		4905		

$$\text{Mean } (\bar{X}) = \frac{\Sigma fx}{\Sigma f} = \frac{4905}{137} = 35.8 \approx 36$$

Average sale of the toys is 36.

For median: Here, $n = 137$, so, $\frac{137}{2} = 68.5$; 68.5 lies in 40 - 50.

$$l = 40, h = 10, f = 29, n = 137, c = 48$$

$$\text{Median } (\tilde{X}) = l + \frac{h}{f} \left(\frac{n}{2} - c \right)$$

$$= 40 + \frac{10}{29} \left(\frac{137}{2} - 48 \right)$$

$$= 40 + \frac{10}{29} (68.5 - 48)$$

$$= 40 + \frac{10}{29} (20.5)$$

$$= 40 + 7.07$$

$$\text{Median} = 47.07 \approx 47$$

Thus, median of the sold toys by the factory is 47.07.

For mode: $l = 30, h = 10, f_m = 45, f_1 = 28, f_2 = 29$

$$\begin{aligned}\text{Mode } (\hat{X}) &= l + \frac{(f_m - f_1)}{(f_m - f_1) + (f_m - f_2)} \times h \\ &= 30 + \frac{(45 - 28)}{(45 - 28) + (45 - 29)} \times 10 \\ &= 30 + \frac{17}{17 + 16} \times 10 \\ &= 30 + \frac{17}{33} \times 10 \\ &= 30 + 5.15\end{aligned}$$

$$\text{Mode } (\hat{X}) = 35.15 \approx 35$$

Thus, mode of the sold toys by the factory is 35.

(ii) The modal class of sold toys by the factory is (30 – 40).

EXERCISE 12.2

1. Find the arithmetic mean in each of the following:

- (i) 4, 6, 10, 12, 15, 20, 25, 28, 30.
- (ii) 12, 18, 19, 0, -19, -18, -12
- (iii) 6.5, 11, 12.3, 9, 8.1, 16, 18, 20.5, 25
- (iv) 8, 10, 12, 14, 16, 20, 22

Sol: (i) Arithmetic Mean of 4, 6, 10, 12, 15, 20, 25, 28, 30

$$\begin{aligned}x &= 9 \\ \Sigma x &= 4 + 6 + 10 + 12 + 15 + 20 + 25 + 28 + 30 \\ &= 150 \\ \bar{x} &= \frac{150}{9} = 16.67\end{aligned}$$

(ii) Arithmetic Mean of 12, 18, 19, 0, -19, -18, -12.

$$\begin{aligned}x &= 7 \\ \Sigma x &= 12 + 18 + 19 + 0 - 19 - 18 - 12 = 0 \\ \bar{x} &= \frac{0}{7} = 0.\end{aligned}$$

(iii) Sum of numbers = $6.5 + 11 + 12.3 + 9 + 8.1 + 16 + 18 + 20.5 + 2.5$

$$\begin{aligned}x &= 9 \\ \Sigma x &= 6.5 + 11 + 12.3 + 9 + 8.1 + 16 + 18 + 20.5 + 2.5 \\ \bar{x} &= \frac{126.4}{9} = 14.04\end{aligned}$$

(iv) Sum of Numbers $8 + 10 + 12 + 14 + 16 + 20 + 22$

$$\begin{aligned}x &= 7 \\ \Sigma x &= 8 + 10 + 12 + 14 + 16 + 20 + 22 = 102 \\ \bar{x} &= \frac{102}{7} = 14.57\end{aligned}$$

2. Following are the heights in (inches) of 12 students.
Find the median height.

55, 53, 54, 58, 60, 61, 62, 56, 57, 52, 51, 63.

Sol: Median height of 12 students (in inches) 55, 53, 54, 58, 60, 61, 62, ... 56, 57, 52, 51, 63

$n - x = 12$ even number. The median is the average of

$\left(\frac{n}{2}\right)^{\text{th}}$ and $\left(\frac{n}{2} + 1\right)^{\text{th}}$ term.

Ascending order

51, 52, 53, 54, 55, 56, 57, 58, 60, 61, 62, 63

Median is $\frac{n}{2}^{\text{th}}$ (6th) and $\left(\frac{n}{2} + 1\right)^{\text{th}}$ (7th) terms.

i.e. Average of 56 and 57 $= \frac{56 + 57}{2} = \frac{113}{2} = 56.5$ inches

3. Following are the earnings (in Rs.) of ten workers:

88, 70, 72, 125, 115, 95, 81, 90, 95, 90. Calculate

(i) Arithmetic Mean (ii) Median

(iii) Mode

Sol: Earnings of 10 workers are: 80, 70, 72, 125, 115, 95, 81, 90, 95, 90.

Ascending order:

70, 72, 81, 88, 90, 90, 95, 95, 115, 125.

Arithmetic Mean

$$n = 10$$

$$\Sigma x = 70 + 72 + 88 + 81 + 90 + 90 + 95 + 95 + 115 + 125$$

$$= 921$$

$$\bar{x} = \frac{921}{10} = 92.1$$

There are 10 terms so median is the Average of 5th $\left(\frac{n}{2} = \frac{10}{2} = 5\right)$ and 6th term.

$$\text{Median} = \frac{90 + 90}{2} = 90$$

$$\text{Mode} = 90, 95.$$

4. The Marks obtained by the students in the subject of English are given below.

Marks	15-19	20-24	25-29	30-34	35-39
Frequency	9	18	35	17	5

Find: (i) Arithmetic mean of their marks by direct and short formula.

- (ii) Median of their marks.

Marks obtained	15-19	20-24	25-29	30-34	35-39	
Frequency (f)	9	18	35	17	5	$\Sigma f = 84$
Class mid points	17	22	27	32	37	
f.x	153	396	945	544	185	$\Sigma fx = 2223$

(i) $\bar{x} = \frac{2223}{84} = 26.46$

(ii) $\text{Median} = \ell + \frac{h}{f} \left(\frac{n}{2} - c \right)$

$$= 24.5 + \frac{5}{35} \left(\frac{84}{2} - 27 \right)$$

$$= 24.5 + \frac{1}{7} (42 - 27)$$

$$= 24.5 + \frac{15}{7}$$

$$= 24.5 + 2.143$$

$$\bar{x} = 26.64$$

5. Given below is a frequency distribution.

Class Interval	5-9	10-14	15-19	20-24	25-29
Frequency	1	8	18	11	2

Find the mode of the frequency distribution.

Sol:

Class Interval	5-9	10-14	15-19	20-24	25-29
C.B	4.5-9.5	9.5-14.5	14.5-19.5	19.5-24.5	24.5-29.5
Frequency	1	8	18	11	2

$$f_m = 18$$

$$f_1 = 8$$

$$f_2 = 11$$

$$h = 5$$

$$l = 14.5$$

$$\text{Mode} = 14.5 + \frac{(18 - 8)5}{(18 - 8) + (18 - 11)}$$

$$= 14.5 + \frac{10 \times 5}{10 + 7}$$

$$= 14.5 + \frac{50}{17}$$

$$= 14.5 + 2.94$$

$$= 17.44 \quad \text{Ans.}$$

6. Ten boys work on a petrol pump station. They get weekly wages as follows: Wages (in Rs.) 4250, 4350, 4400, 4250, 4350, 4410, 4500, 4300, 4500, 4390. Find the arithmetic mean by short formula, median and mode of their wages.

Sol: Wages of 10 boys 4250, 4350, 4400, 4250, 4350, 4410, 4500, 4300, 4500, 4390.

X	D = x - A
4250	4250 - 4350 = -100
4250	4250 - 4350 = -100
4300	4300 - 4350 = -50
4350	4350 - 4350 = 0
4350	4350 - 4350 = 0
4390	4390 - 4350 = 40
4400	4400 - 4350 = 90
4410	4410 - 4350 = 60
4500	4500 - 4350 = 150
4500	4500 - 4350 = 150
	$\Sigma = 200$

Wages in Ascending order

4250, 4250, 4300, 4350, 4350, 4390, 4400, 4410, 4500, 4500

Total number of figures 10

Median is $\frac{10}{2} \cdot \frac{(5^{\text{th}} \text{ term} + 6^{\text{th}} \text{ term})}{2}$

$$= \frac{4350 + 4390}{2} = 4370$$

There are three modes 4250; 4350; 4500

$$\bar{x} = A + \frac{\Sigma D}{n}$$

$$= 4350 + \frac{200}{10}$$

$$= 4350 + 20 = 4370$$

Mode: Since most repeated value of data are 4250, 4350 and 4500, so Mode = 4250, 4350 and 4500.

The arithmetic mean of 45 numbers is 80. Find their sum.

Sol:

$$n = 45$$

$$\bar{X} = 80$$

$$\Sigma x = ?$$

$$\bar{X} = \frac{\Sigma x}{n}$$

$$80 = \frac{\Sigma x}{45}$$

$$\Sigma x = 3600$$

8. Five numbers are 1, 4, 0, 7, 9. Find their mean, median and mode.

Sol: $x = 1, 4, 0, 7, 9$

Ascending order: 0, 1, 4, 7, 9.

$$\Sigma x = 0 + 1 + 4 + 7 + 9 = 21$$

$$n = 5$$

$$\text{A.M.} = \frac{21}{5} = 4.20$$

$$\text{Median} = \frac{n+1}{2}^{\text{th}} = \frac{5+1}{2}^{\text{th}} = \frac{6}{2}^{\text{th}} = 3^{\text{rd}} = 4$$

Data have no mode.

9. A set of data contains the values as 148, 145, 160, 157, 156, 160.

Show that Mode > Median > Mean.

Sol: $x = 148, 145, 160, 157, 156, 160$

Ascending order: 145, 148, 156, 157, 160, 160.

$$\Sigma x = 145 + 148 + 156 + 157 + 160 + 160 = 926$$

$$n = 6$$

$$\text{Arithmetic Mean} = \frac{926}{6} = 154.33$$

$$\text{Mode} = 160$$

$$\text{Median} = \frac{3^{\text{rd}} + 4^{\text{th}}}{2} = \frac{156 + 157}{2} = \frac{313}{2} = 156.5$$

$$\text{Mode} = 160, \text{Median} = 156.5, \text{Mean} = 154.33$$

Clearly Mode > Median > Mean

10. The monthly attendance of 10 students for their lunch in the hostel is recorded as: 21, 15, 16, 18, 14, 17, 15, 12, 13, 11. Find the median and mode of the attendance. Also find the mean if $D = A - 20$.

Sol: Numbers are 21, 15, 16, 18, 14, 17, 15, 12, 13, 11

$$n = 10$$

Ascending order: 11, 12, 13, 14, 15, 15, 16, 17, 18, 21

$$\Rightarrow \text{Mode} = 15$$

$$\Rightarrow \text{Median} = \frac{5^{\text{th}} + 6^{\text{th}}}{2} = \frac{15 + 15}{2} = 15$$

Arithmetic Mean:

X	$D = x - 20$ $A = 20$	
11	$11 - 20 = -9$	
12	$12 - 20 = -8$	
13	$13 - 20 = -7$	
14	$14 - 20 = -6$	
15	$15 - 20 = -5$	
15	$15 - 20 = -5$	
16	$16 - 20 = -4$	
17	$17 - 20 = -3$	
18	$18 - 20 = -2$	
21	$21 - 20 = 1$	$\Sigma D = -48$

$$\begin{aligned} \Rightarrow \bar{x} &= A + \frac{\Sigma D}{n} \\ &= 20 + \left(\frac{-48}{10} \right) \\ &= 20 - 4.8 = 15.2 \end{aligned}$$

Ans.

Mean = 15.2

Median = 15

Mode = 15

11. On a prize distribution day, 50 students brought pocket money as under:

Rupees	5-10	10-15	15-20	20-25	25-30
Frequency (f)	12	9	18	7	4

- (i) Find the median and mode of the above data.
(ii) Find the arithmetic mean of the data given above using coding method.

Sol. (i) Median and Mode:

Rupees	(f)	C.F
5 - 10	12	12
10 - 15	$f_1 \Rightarrow 9$	$12 + 9 = 21$
15 - 20	$f_m \Rightarrow 18$	$21 + 18 = 39$ (Median Group, Modal class)
20 - 25	$f_2 \Rightarrow 7$	$39 + 7 = 46$
25 - 30	4	$46 + 4 = 50$
	$\Sigma f = 50$	

(a) Median = ?

We know that:

$$\begin{aligned} \text{Median}(\bar{x}) &= l + \frac{\frac{n}{2} - c}{f} \left(\frac{n}{2} - c \right) \\ &= 15 + \frac{5}{18} \left(\frac{50}{2} - 21 \right) \\ &= 15 + \frac{5}{18} (25 - 21) \\ &= 15 + \frac{5}{18} (4) \\ &= 15 + \frac{20}{18} \end{aligned}$$

$$\bar{x} = 16.11$$

- (b) Mode = ?
We know that:

$$\begin{aligned}\text{Mode } (\hat{X}) &= l + \frac{(f_m - f_1) \times h}{(f_m - f_1) + (f_m - f_2)} \\ &= 15 + \frac{(18 - 9) \times 5}{(18 - 9) + (18 - 7)} \\ &= 15 + \frac{9 \times 5}{9 + 11} \\ &= 15 + \frac{45}{20} \\ &= 15 + 2.25 \quad \hat{X} = 17.25\end{aligned}$$

(ii) Arithmetic mean by using coding:

Rupees	(f)	(x)	$u = \frac{D}{h} = \frac{x - A}{h}$	fu
5 - 10	12	7.5	-2	-24
10 - 15	9	12.5	-1	-9
15 - 20	18	17.5	0	0
20 - 25	7	22.5	1	7
25 - 30	4	27.5	2	8
	$\Sigma f = 50$			-18

We know that:

$$\begin{aligned}\bar{X} &= A + \frac{\Sigma fu}{\Sigma f} \times h \\ &= 17.5 + \left(\frac{-18}{50} \right) \times 5 \\ &= 17.5 + \left(\frac{-90}{50} \right) \\ &= 17.5 + (-1.8) \\ &= 17.5 - 1.8\end{aligned}$$

$$\boxed{\bar{X} = 15.7}$$

12. The arithmetic mean of the ages of 20 boys is 13 years, 4 months and 5 days.

Find the sum of their ages. If one of the boys is of age exactly 15 years. What is the average age of the remaining boys?

Sol: Average age of 20 students = 13 years, 4 months and 5 days

$$\begin{aligned}&= 13 + \frac{4}{12} + \frac{5}{365} \\ &= 13 + 0.333 + 0.0136 \\ &= 13.3466 \text{ years}\end{aligned}$$

$$\text{Total age} = 13.3466 \times 20 = 266.932 \text{ years}$$

The students gone = 15 years

$$\begin{aligned}\text{Remaining sum} &= 266 \text{ years } 11 \text{ months } 6 \text{ days} \\ &\quad - 15 \text{ years} \\ &= 251 \text{ years } 11 \text{ months } 6 \text{ days}\end{aligned}$$

$$\text{Balance total} = 251.932 \text{ years}$$

$$\text{Now, remaining boys} = 20 - 1 = 19$$

$$\begin{aligned}\text{Average} &= \frac{251.932}{19} \\ &= 13.259 \text{ years} \\ &= 13 \text{ years} + 0.259 \times 12 \text{ months} \\ &= 13 \text{ years } 3.108 \text{ months} \\ &= 13 \text{ years} + 3 \text{ months} + 0.108 \times 30 \text{ days} \\ &= 13 \text{ years, } 3 \text{ months and } 4 \text{ days (approx.)}\end{aligned}$$

13. Calculate the arithmetic mean from the following information:

(i) If $D = X - 140$, $\Sigma D = 500$ and $n = 10$

(ii) If $U = \frac{x-130}{6}$, $\Sigma U = -150$ and $n = 15$

(iii) If $D = x - 25$, $\Sigma fD = 300$ and $\Sigma f = 20$

(vi) If $U = \frac{x-120}{5}$, $\Sigma fU = 60$ and $\Sigma f = 100$

Sol: (i) If $D = x - 140$, $\Sigma D = 500$ and $n = 10$

$$D = X - A \quad \text{So} \quad A = 140$$

We know that:

$$\begin{aligned} \bar{X} &= A + \frac{\Sigma D}{n} \\ &= 140 + \frac{500}{10} \\ &= 140 + 50 \end{aligned}$$

$$\boxed{\bar{X} = 190}$$

(ii) If $u = \frac{x-130}{6}$, $\Sigma u = -150$ and $n = 15$.

We know that:

$$u = \frac{x-A}{h}$$

$$u = \frac{x-130}{6}$$

So, $A = 130$, $h = 6$

We also know that:

$$\bar{X} = A + \frac{\Sigma u}{n} \times h$$

$$\begin{aligned} \bar{X} &= 130 + \frac{-150}{15} \times 6 \\ &= 130 - 10 \times 6 \\ &= 130 - 60 \end{aligned}$$

$$\boxed{\bar{X} = 70}$$

(iii) If $D = x - 25$, $\Sigma fD = 300$ and $\Sigma f = 20$

$$D = x - 25$$

We get: $A = 25$

We know that:

$$\bar{X} = A + \frac{\Sigma fD}{\Sigma f}$$

$$= 25 + \frac{300}{20}$$

$$= 25 + 15$$

$$\boxed{\bar{X} = 40}$$

(iv) If $u = \frac{x-120}{5}$, $\Sigma fu = 60$ and $\Sigma f = 100$

We know that:

$$u = \frac{x-A}{h}$$

$$u = \frac{x-120}{5}$$

So, $A = 120$, $h = 5$

We also know that:

$$\bar{X} = A + \frac{\Sigma fu}{\Sigma f} \times h$$

$$= 120 + \frac{60}{100} \times 5$$

$$= 120 + \frac{300}{100}$$

$$= 120 + 3$$

$$\boxed{\bar{X} = 123}$$

14. The three children Haris, Maham and Minal made the following scores in a game conducted by a group of teachers in the school.

Haris scores	50	55	70	85	90
Maham	75	60	60	45	53
Minal scores	80	77	66	42	48

It is decided that the candidate who gets the highest average score will be awarded rupees 1000. Who will get the awarded amount?

Sol: Average of Mr. Haris = $\frac{50 + 55 + 70 + 85 + 90}{5} = 70$

Average of Maham = $\frac{75 + 60 + 60 + 45 + 53}{5} = 58.6$

Average of Minal = $\frac{80 + 77 + 66 + 42 + 48}{5}$
 $= \frac{313}{5} = 62.6$

Award goes to Haris.

15. Given below is a frequency distribution derived by making a substitution as $D = X - 20$. Calculate the arithmetic mean.

D	-6	-4	-2	0	2	4	6
f	1	3	6	20	26	12	2

Sol:

D	-6	-4	-2	0	2	4	6	
f	1	3	6	20	26	12	2	$\Sigma f = 70$
fD	-6	-12	-12	0	52	48	12	$\Sigma fD = 82$

$$D = X - A$$

$$D = X - 20 \Rightarrow A = 20$$

$$\bar{X} = A + \frac{\Sigma fD}{\Sigma f} = 20 + \frac{82}{70} = 20 + 1.17 = 21.17$$

16. Being partners Hafsa and Fatima took part in a quiz programme. They made the following number of points 45, 51, 58, 61, 74, 48, 46 and 50. Compute the average number of points using deviation $D = x - 58$.

Solution: $D = X - 58 \Rightarrow A = 58$

Interval (Points X)	Deviation $D = x - 58$
45	-13
46	-12
48	-10
50	-8
51	-7
58	0
61	3
74	16
ΣD	-31

No. of points = 8

If \bar{X} is the average

then, $\bar{X} = A + \frac{\Sigma D}{f}$
 $= 58 + \frac{-31}{8}$
 $= 58 - 3.875$
 $= 54.125$
 $\bar{X} = 54.13$

17. A person purchased the following food items:

Food item	Quantity (in Kg)	Cost per Kg (in Rs.)
Rice	10	96
Flour	12	48
Ghee	4	190
Sugar	3	49
Mutton	2	650

What is the weighted mean of cost of food items per kg?

Sol.

Food item	Quantity (kg W)	Cost per kg (X)	Price paid (WX)
Rice	10	96	960
Flour	12	48	576
Ghee	4	190	760
Sugar	3	49	147
Mutton	2	650	1300
Total price/weight	31		3743

$$\bar{X}_w = \text{Rs. } 120.74 \text{ per kg}$$

18. For the following data, find the weighted mean.

Item	Quantity	Cost of item (in thousands)
Washing Machine	5	35
Heater	3	5
Stove	2	13
Dispenser	6	18

Sol:

Item	Quantity (W)	Cost of item in thousands	Total cost (WX)
Washing machine	5	35	175
Heater	3	5	15
Stove	2	13	26
Dispenser	6	18	108
	$\Sigma W = 16$		$\Sigma WX = 324$

Average cost per

$$\text{item} = \frac{\Sigma WX}{\Sigma W}$$

$$= \frac{324}{16}$$

$$= 20.25 (\text{in thousands})$$

19. A company is planning its next year marketing budget across five years: yearly budgets (in million) are: 5, 7, 8, 6, 7. Find the average budget for the next year.

Sol: Total budget for 5 years = $5 + 7 + 8 + 6 + 7$
 $= 33$ millions

$$\text{Average per year} = \frac{33}{5} = 6.6 \text{ million per year}$$

20. Ahmad obtained the following marks in a certain examination. Find the weighted mean if weights 5, 4, 2, 3, 2, 4 respectively are allotted to the subjects.

Urdu	English	Science	Math	Islamiyat	Computer
78	65	80	90	85	72

Solution:

Paper	Urdu	English	Science	Math	Islamiyat	Computer
X	78	65	80	90	85	72
W	5	4	2	3	2	4
WX	390	260	160	270	170	288

$$\Sigma WX = 1538$$

$$\Sigma W = 5 + 4 + 2 + 3 + 2 + 4 = 20$$

$$\bar{X}_w = \frac{\Sigma WX}{\Sigma W} = \frac{1538}{20} = 76.9 \text{ Marks}$$

REVIEW EXERCISE

12

1. Four options are given against each statement. Encircle the correct option.

- Which data takes only some specific values?
 (a) continuous data (b) discrete data
 (c) grouped data (d) ungrouped data
- The number of times a value occurs in a data is called:
 (a) frequency (b) relative frequency
 (b) class limit (d) class boundaries.
- Midpoint is also known as:
 (a) mean (b) median
 (c) class limit (d) class mark
- Frequency polygon is also drawn / constructed by using:
 (a) histogram (b) bar graph
 (c) class boundaries (d) class limit
- The difference between the greatest value and the smallest value is called:
 (a) class limits (b) midpoint
 (c) relative frequency (d) range
- Measure of central tendency is used to find out the _____ of a data set.
 (a) class boundaries (b) cumulative frequency
 (c) middle or centre value (d) frequency
- If the mean of 5, 7, 8, 9 and x is 7.5, what will be the value of x?
 (a) 10 (b) 8 (c) 8.5 (d) 5.8

- Find the mode of the given data: 2, 5, 8, 9, 0, 1, 3, 7 and 10.
 (a) 5 (b) 7 (c) 0 (d) no mode
- (ix) In a data the values (observations) which appears or occurs most often is called:
 (a) mean (b) mode
 (c) median (d) weighted mean
- (x) Find the median of the given data: 110, 125, 122, 130, 124, 127 and 120
 (a) 124 (b) 120 (c) 125 (d) 127

Answers:

(i)	(b)	(ii)	(a)	(iii)	(d)	(iv)	(a)	(v)	(d)
(vi)	(c)	(vii)	(c)	(viii)	(d)	(ix)	(b)	(x)	(a)

2. Define the following:

- (i) frequency distribution
 (ii) histogram (unequal class limits)
 (iii) mean (iv) median

Ans. (i) Frequency Distribution

A frequency distribution is a table or chart that organizes a dataset into groups or intervals (known as classes) and shows the number of occurrences (frequency) of each group or interval. It provides a summary of the data, making it easier to see patterns, trends, and distributions within the dataset.

(ii) Histogram (Unequal Class Limits)

A histogram is a graphical representation of a frequency distribution where data is grouped into intervals (classes) and represented as adjacent rectangles (bars). When the class intervals are of unequal width, the height of each bar is adjusted to represent the **frequency density**, which is calculated as:

$$\text{Frequency Density} = \frac{\text{Frequency}}{\text{Class Width}}$$

This ensures that the area of each bar is proportional to the frequency of the interval.

(iii) Mean

The mean is a measure of central tendency that represents the average value of a dataset. It is calculated by dividing the sum of all data points by the number of data points:

$$\text{Mean}(\bar{x}) = \frac{\sum x_i}{n}$$

Where:

- x_i represents each data point.
- n is the total number of data points.

(iv) Median

The median is the middle value of a dataset when the data is arranged in ascending or descending order. If the dataset contains an odd number of observations, the median is the middle value. If the dataset contains an even number of observations, the median is the average of the two middle values.

For grouped data, the median can be calculated using the formula:

$$\text{Median} = l + \frac{h}{f} \left(\frac{n}{2} - c \right)$$

Where:

- l = Lower boundary of the median class.
- n = Total number of frequencies.
- c = Cumulative frequency of the class before the median class.
- f = Frequency of the median class.
- h = Width of the median class.

3. Following are the weights of 40 students recorded to the nearest (lbs).

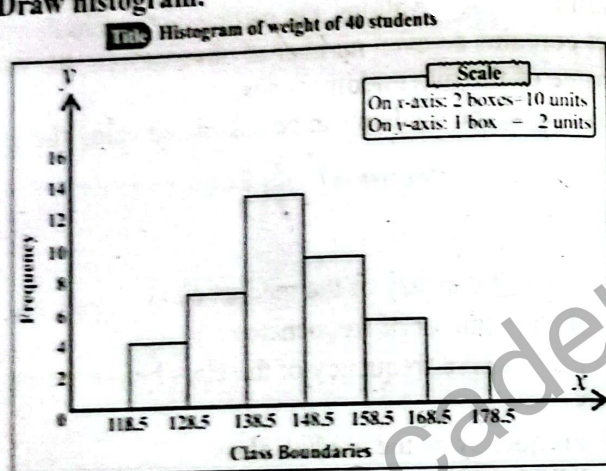
138, 164, 150, 132, 144, 125, 149, 157, 146, 158, 140, 147, 136, 148, 152, 144, 168, 126, 138, 176, 163, 119, 154, 165, 146, 173, 142, 147, 135, 153, 140, 135, 161, 145, 135, 142, 150, 156, 145, 128.

- (a) Make a frequency table taking size of class limits as 10.
 (b) Draw histogram,
 (c) Draw a frequency polygon of the given data.

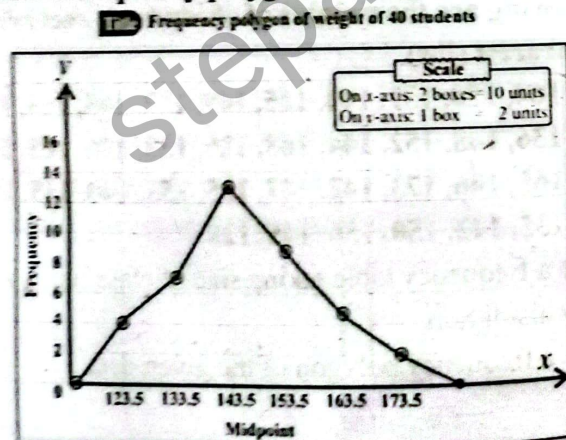
Sol: (a) Make a frequency table taking size of class limits as 10.

Class limits	Tally marks	f	C.B	Midpoints
119 - 128		4	118.5 - 128.5	123.5
129 - 138		7	128.5 - 138.5	133.5
139 - 148		13	138.5 - 148.5	143.5
149 - 158		9	148.5 - 158.5	153.5
159 - 168		5	158.5 - 168.5	163.5
169 - 178		2	168.5 - 178.5	173.5

(b) Draw histogram.



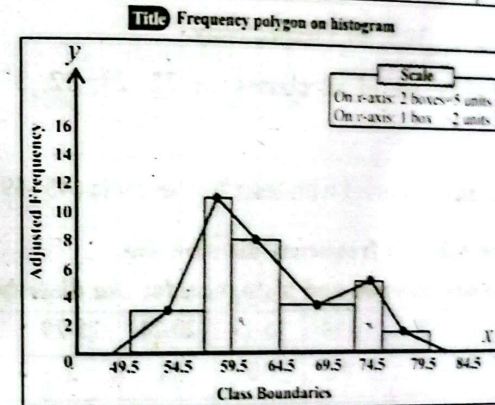
(c) Draw a frequency polygon of the given data.



4. From the table given below. Draw a frequency polygon on histogram for the given frequency distribution.

Weight (kg)	50-56	57-59	60-64	65-72	73-75	76-80
Frequency	25	32	40	30	15	8

Weight (kg)	(f)	Size of class (h)	Height of rectangle adjusted frequency f/h	C.B.
50 - 56	25	7	$25 \div 7 = 3.6$	49.5 - 56.5
57 - 59	32	3	$32 \div 3 = 10.7$	56.5 - 59.5
60 - 64	40	5	$40 \div 5 = 8$	59.5 - 64.5
65 - 72	30	8	$30 \div 8 = 3.8$	64.5 - 72.5
73 - 75	15	3	$15 \div 3 = 5$	72.5 - 75.5
76 - 80	8	5	$8 \div 5 = 1.6$	75.5 - 80.5



5. Given below are marks obtained by 45 students in the monthly test of Biology:

Marks	20-24	25-29	30-34	35-39	40-44	45-49
No. of students	05	08	12	15	03	02

With reference to the above table find the following:

- upper class boundary of the 5th class.
- lower class boundaries of all the classes.
- midpoint of all the classes.
- the class interval with the least frequency.

Sol:

Marks	No. of students (f)	Midpoints (x)	C.B.
20 - 24	5	22	19.5 - 24.5
25 - 29	8	27	24.5 - 29.5
30 - 34	12	32	29.5 - 34.5
35 - 39	15	37	34.5 - 39.5
40 - 44	3	42	39.5 - 44.5
45 - 49	2	47	44.5 - 49.5

- (i) The upper class boundary of 5th class is 44.5.
- (ii) The lower class boundaries of all classes are 19.5, 24.5, 29.5, 34.5, 39.5, 44.5 respectively.
- (iii) The midpoints of all classes are 22, 27, 32, 37, 42, 47 respectively.
- (iv) The classes interval with least frequency is (45-49).

6. Given below is frequency distribution.

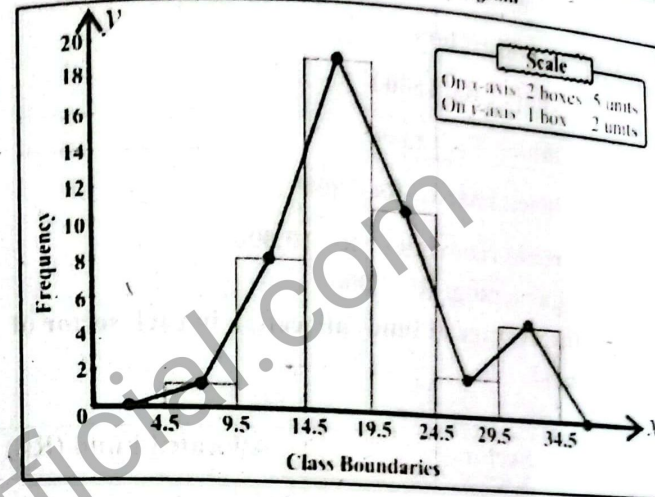
Draw frequency polygon and histogram for the distribution.

Class limits	5-9	10-14	15-19	20-24	25-29	30-34
Frequency	1	8	18	11	2	5

Solution:

Class limits	(f)	Midpoints (x)	C.B.
5 - 9	1	7	4.5 - 9.5
10 - 14	8	12	9.5 - 14.5
15 - 19	18	17	14.5 - 19.5
20 - 24	11	22	19.5 - 24.5
25 - 29	2	27	24.5 - 29.5
30 - 34	5	32	29.5 - 34.5

Title Frequency polygon on histogram



7. For the following data, find the weighted mean.

Item	Quantity	Cost of item (Rs.)
Chair	20	500
Table	20	400
Black board	10	750
Tube light	25	230
Cupboard	09	950

Sol:

Item	Quantity (W)	Cost of item (Rs.)	WX
Chair	20	500	10,000
Table	20	400	8,000
Black board	10	750	7,500
Tube light	25	230	5,750
Cupboard	09	950	8,550
	$\Sigma W = 84$		$\Sigma WX = 39,800$

$$\bar{X}_w = \frac{\Sigma WX}{\Sigma W} = \frac{39,800}{84} = 473.81$$

Average price of each item is Rs. 473.81.

8. A principal of a school allocates funds of Rs.50, 000 to five different sectors:

- (i) chairs: Rs. 15000
- (ii) tables: Rs. 12,000
- (iii) black boards: Rs.6,000
- (iv) room renovation: Rs. 10,000
- (v) gardening: Rs. 7,000

Find the average of funds allocation in each sector of the school.

Sol:

Sectors	Allocated Fund (Rs.)
Chairs	15,000
Tables	12,000
Black boards	6,000
Room renovation	10,000
Gardening	7,000
Total	Rs. 50,000

Number of sectors = 5

Total amount = Rs. 50,000

$$\bar{X} = \frac{\Sigma W}{n} = \frac{50,000}{5} = 10,000$$

Thus, the average of funds allocation in each sector is Rs.10,000.

9. The marks of a student Saad in six tests were 84, 91, 72, 68, 87, 78. Find the arithmetic mean of his marks.

Sol:

$$\bar{X} = \frac{\Sigma x}{n} = \frac{84+91+72+68+87+78}{6}$$

$$= \frac{480}{6} = 80 \text{ marks}$$

10. Adjoining distribution showed maximum load (in kg) supported by certain ropes. Find the mean load using short method.

Max-Load kg	93-97	98 - 102	103 -107	108-112	113-117	118-122
No. of ropes	2	5	8	12	6	2

Sol:

Max. load kg	No. of ropes(f)	Midpoints (x)	D = x - A	fD
93 - 97	2	95	95 - 100 = -5	-10
98 - 102	5	100	100 - 100 = 0	0
103 - 107	8	105	105 - 100 = 5	40
108 - 112	12	110	110 - 100 = 10	120
113 - 117	6	115	115 - 100 = 15	90
118 - 122	2	120	120 - 100 = 20	40
	$\Sigma W = 35$			$\Sigma fD = 280$

$$\bar{X} = A + \frac{\Sigma fD}{\Sigma f} = 100 + \frac{280}{35} = 100 + 8 = 108$$

Thus, the average load support by certain ropes is 108kg.

11. Usman rolled a fair dice eight times. Each time their sum was recorded as 8, 5, 6, 6, 9, 4, 3, 11. Find the median and mode of the sum.

Sol: Ascending order:

3, 4, 5, 6, 6, 8, 9, 11

$$n = 8$$

mode = 6

Median = Average of 4th and 5th term

$$= \frac{6+6}{2} = 6$$

12. Two partners Mr. Aslam and Mrs. Kalsoom run a company. In the following data the weekly wages (in Rs.) of employees who work in the company are given:

Wages	600 - 700	700 - 800	800 - 900	900-1000	1000-1100
Employees	3	5	7	21	11

Sol: Mean

Wages (Rs.)	Employees (f)	Midpoints (x)	fx
600 - 700	3	650	1,950
700 - 800	5	750	3,750
800 - 900	7	850	5,950
900 - 1000	21	950	19,950
1000 - 1100	11	1050	11,550
	$\Sigma f = 47$		$\Sigma fx = 43,150$

$$\bar{X} = \frac{\Sigma fx}{\Sigma f} = \frac{43150}{47} = 918.09$$

Median

Wages (Rs.)	Employees (f)	Cumulative Frequency (C.F)
600 - 700	3	3
700 - 800	5	3 + 5 = 8
800 - 900	7	3 + 5 = 15 $\rightarrow C$
900 - 1000	$f \rightarrow 21$	15 + 21 = 36
1000 - 1100	11	1050

$$\text{Median} = l + \frac{h}{f} \left[\frac{n}{2} - c \right]$$

l = lower limit of Medium class = 900

h = size of the medium class = 100

$n = \Sigma f$ (Total frequency) = 47

c = Commutative frequency proceeding the Medium class
= 3 + 5 + 7 = 15

f = Frequency of the Medium class = 21

$$\therefore \text{Median} = 900 + \frac{100}{21} \left[\frac{47}{2} - 15 \right]$$

$$= 900 + 4.76 (23.5 - 15)$$

$$= 900 + 4.76 (8.5)$$

$$= 900 + 40.46 = 940.46$$

Mode:

Wages (Rs.)	Employees (f)
600 - 700	3
700 - 800	5
800 - 900	$f_1 \rightarrow 7$
$l \rightarrow 900$ - 1000	$f_m \rightarrow 21$
1000 - 1100	$F_2 \rightarrow 11$

Modal class (900 - 1100)

$$l = 900$$

$$h = 100$$

$$fm = 21$$

$$f_1 = 7$$

$$f_2 = 11$$

$$\text{Mode} = l + \frac{(f_m - f_1)h}{(f_m - f_1) + (f_m - f_2)}$$

$$= 900 + \frac{(21 - 7) \times 100}{(21 - 7) + (21 - 11)}$$

$$= 900 + \frac{14 \times 100}{14 + 10} = 900 + \frac{1400}{24}$$

$$\text{Mode} = 900 + 58.33 = 958.33$$