UNIT 2

Logarithms

Students' learning outcomes

At the end of the unit, the students will be able to:

- Express a number in scientific notations and vice versa.
- Describe logarithm of a number
- Differentiate between common and natural logarithm
- Apply concepts of real numbers to real word problems (such as temperature, banking, measures of gain and loss, sources of income and expenditure).

Conversion of Numbers from Ordinary Notation to Scientific Notation

Example 1: Convert 78,000,000 to scientific notation.

Solution:

Step 1: Move the decimal to get a number between 1 and 10:

$$7.8$$

Step 2: Count the number of places you moved the decimal:

7 places

Step 3: Write in scientific notation:

$$78{,}000{,}000 = 7.8 \times 10^7$$

Since we moved the decimal to the **left**, the exponent is **positive**.

Example 2: Convert 0.0000000315 to scientific notation.

Solution:

Step 1: Move the decimal to get a number between 1 and 10:

$$3.15$$

Step 2: Count the number of places you moved the decimal:

8 places

Step 3: Write in scientific notation:

$$0.0000000315 = 3.15 \times 10^{-8}$$

Since we moved the decimal to the **right**, the exponent is **negative**.

Conversion of Numbers from Scientific Notation to Ordinary Notation

Example 3: Convert 3.47×10^6 to ordinary notation.

Solution:

Step 1: Identify the parts:

Coefficient: 3.47

Exponent: 10^6

Step 2: Since the exponent is **positive** 6, move the decimal point 6 places to the right.

$$3.47 \times 10^6 = 3{,}470{,}000$$

Example 4: Convert 6.23×10^{-4} to ordinary notation

Solution:

Step 1: Identify the parts:

Coefficient: 6.23

Exponent: 10^{-4}

Step 2: Since the exponent is **negative** 4, move the decimal point 4 places to the **left.**

$$6.23 \times 10^{-4} = 0.000623$$

EXERCISE 2.1

1. Express the following numbers in scientific notation.

(i)	**2000000**	**(ii)**	**48900**
(iii)	**0.0042**	**(iv)**	**0.0000009**
(v)	$\mathbf{73 \times 10^3}$	**(vi)**	$\mathbf{0.65 \times 10^2}$

Solutions:

(i) 2,000,000

Scientific notation expresses numbers as a product of a number between 1 and 10 and a power of 10. To convert 2,000,000 into scientific notation:

$$2{,}000{,}000 = 2 \times 10^6$$

Thus, the scientific notation for 2,000,000 is 2×10^6.

(ii) 48,900

To express 48,900 in scientific notation, we move the decimal point so that only one non-zero digit remains on the left of the decimal point. For 48,900, move the decimal point 4 places to the left.

$$48{,}900 = 4.89 \times 10^4$$

Thus, the scientific notation for 48,900 is 4.89×10^4.

(iii) 0.0042

For numbers less than 1, we move the decimal point to the right so that the first digit is just to the right of the decimal point. For 0.0042, move the decimal point 3 places to the right.

$$0.0042 = 4.2 \times 10^{-3}$$

Thus, the scientific notation for 0.0042 is 4.2×10^{-3}.

(iv) 0.0000009

Similarly, for very small numbers, we move the decimal point to the right. For 0.0000009, we move the decimal 7 places to the right.

$$0.0000009 = 9 \times 10^{-7}$$

Thus, the scientific notation for 0.0000009 is 9×10^{-7}.

(v) $\mathbf{73 \times 10^3}$

This number is already in a form that is almost in scientific notation, except that the coefficient is not between 1 and 10. We need to adjust the coefficient. To make the coefficient fall between 1 and 10, we can write 73 as 7.3×10^1, and multiply by 10^3:

$$73 \times 10^3 = 7.3 \times 10^1 \times 10^3 = 7.3 \times 10^4$$

Thus, he scientific notation for 73×10^3 is 7.3×10^4.

(vi) $\mathbf{0.65 \times 10^2}$

Here, we have 0.65×10^2, and we want to adjust the coefficient so that it is between 1 and 10. To do this, move the decimal point in 0.65 one place to the right to make it 6.5, and adjust the power of 10 accordingly:

$$0.65 \times 10^2 = 6.5 \times 10^{-1} \times 10^2 = 6.5 \times 10^1$$

Thus, the scientific notation for 0.65×10^2 is 6.5×10^1.

2. **Express the following numbers in ordinary notation.**

(i) 8.04×10^2 (ii) 3×10^5

(iii) 1.5×10^{-2} iv) 1.77×10^7

(v) 5.5×10^{-6} (vi) 4×10^{-5}

Solution:

(i) 8.04×10^2

To convert to ordinary notation, we multiply 8.04 by 10^2 (which is 100). This means we move the decimal point two places to the right:

$$8.04 \times 10^2 = 804$$

Thus, 8.04×10^2 in ordinary notation is 804.

(ii) 3×10^5

To convert to ordinary notation, we multiply 3 by 10^5 (which is 100,000). This means we move the decimal point five places to the right:

$$3 \times 10^5 = 300{,}000$$

Thus, 3×10^5 in ordinary notation is 300,000.

(iii) 1.5×10^{-2}

To convert to ordinary notation, we multiply 1.5 by 10^{-2} (which is $\frac{1}{100}$). This means we move the decimal point two places to the left:

$$1.5 \times 10^{-2} = 0.015$$

Thus, 1.5×10^{-2} in ordinary notation is 0.015.

(iv) 1.77×10^7

To convert to ordinary notation, we multiply 1.77 by 10^7 (which is 10,000,000). This means we move the decimal point seven places to the right:

$$1.77 \times 10^7 = 17{,}700{,}000$$

Thus, 1.77×10^7 in ordinary notation is 17,700,000.

(v) 5.5×10^{-6}

To convert to ordinary notation, we multiply 5.5 by 10^{-6} (which is $\frac{1}{1{,}000{,}000}$). This means we move the decimal point six places to the left:

$$5.5 \times 10^{-6} = 0.0000055$$

Thus, 5.5×10^{-6} in ordinary notation is 0.0000055.

(vi) 4×10^{-5}

To convert to ordinary notation, we multiply 4 by 10^{-5} (which is $\frac{1}{100{,}000}$). This means we move the decimal point five places to the left:

$$4 \times 10^{-5} = 0.00004$$

Thus, 4×10^{-5} in ordinary notation is 0.00004.

3. **The speed of light is approximately 3×10^8 miles per second. Express it in standard form.**

Solution:

$$30 \times 10^8 \text{ m/s} = 300{,}000{,}000 \times 10^{-8} \times 10^8 \text{ m/s}$$
$$= 300{,}000{,}000 \times 10^{-8+8} \text{ m/s}$$
$$= 300{,}000{,}000 \times 10^0 \text{ m/s}$$
$$= 300{,}000{,}000 \times 1 \text{ m/s}$$
$$= 300{,}000{,}000 \text{ m/s}$$

4. **The circumference of the Earth at the equator is about 40075000 metres. Express this number in scientific notation.**

Solution: To express 40,075,000 metres in scientific notation:

- Place the decimal point after the first non-zero digit: 4.0075.
- Count the number of places the decimal point has moved to get back to the original number. In this case, it moves 7 places to the right.

Thus, the circumference of the Earth in scientific notation is:

4.0075×10^7 metres.

5. **The diameter of Mars is 6.779×10^3 km. Express this number in standard form.**

Solution: The diameter of mass $= 6.779 \times 10^3$ km

$= 6779 \times 10^{-3} \times 10^3$ km

$= 6779 \times 10^{-3+3}$ km

$= 6779 \times 10^0$ km

$= 6779 \times 1$ km

$= 6779$ km

6. **The diameter of Earth is about 1.2756×10^4 km. Express this number in standard form.**

Solution: 1.2756×10^4km $= 12756 \times 10^{-4} \times 10^4$km

$= 12756 \times 10^{-4+4}$ km

$= 12756 \times 10^0$ km

$= 12756 \times 1$km

$= 12756$ km

Example 5: Convert $\log_2 8 = 3$ to exponential form.

Solution: $\log_2 8 = 3$

Its exponential form is: $2^3 = 8$

Example 6: Convert $\log_{10} 100 = 2$ to exponential form.

Solution: $\log_{10} 100 = 2$

Its exponential form is: $10^2 = 100$

Example 7: Find the value of x in each case:

(i) $\log_5 25 = x$ (ii) $\log_2 x = 6$

Solution:

(i) $\log_5 25 = x$

Its exponential form is:

$5^x = 25$

$\Rightarrow 5^x = 5^2$

$\Rightarrow x = 2$

(ii) $\log_2 x = 6$

Its exponential form is:

$2^6 = x$

$\Rightarrow x = 64$

Example 8: Convert the following in logarithmic form:

(i) $3^4 = 81$ (ii) $7^0 = 1$

Solution:

(i) $3^4 = 81$

Its logarithmic form is:

$\log_3 81 = 4$

(ii) $7^0 = 1$

Its logarithmic form is:

$\log_7 1 = 0$

EXERCISE 2.2

1. **Express each of the following in logarithmic form:**

(i)	$10^3 = 1000$	**(ii)**	$2^8 = 256$
(iii)	$3^{-3} = \frac{1}{27}$	**(iv)**	$20^2 = 400$
(v)	$16^{-\frac{1}{4}} = \frac{1}{2}$	**(vi)**	$11^2 = 121$
(vii)	$p = q^r$	**(viii)**	$(32)^{-\frac{1}{5}} = \frac{1}{2}$

Solution: To express the given exponential equations in logarithmic form, we follow the basic rule of logarithms:

$$a^b = c \quad \text{can be rewritten as} \quad \log_a(c) = b$$

Let's solve each part:

(i) $\mathbf{10^3 = 1000}$

We can express this as:

$$\log_{10}(1000) = 3$$

This is the logarithmic form where the base is 10, the result is 1000, and the exponent is 3.

(ii) $\mathbf{2^8 = 256}$

To express this in logarithmic form:

$$\log_2(256) = 8$$

Here, the base is 2, the result is 256, and the exponent is 8.

(iii) $\mathbf{3^{-3} = \frac{1}{27}}$

This can be expressed as:

$$\log_3\left(\frac{1}{27}\right) = -3$$

The base is 3, the result is $\frac{1}{27}$, and the exponent is -3.

(iv) $20^2 = 400$

In logarithmic form, this is:

$$\log_{20}(400) = 2$$

Here, the base is 20, the result is 400, and the exponent is 2.

(v) $16^{-\frac{1}{4}} = \frac{1}{2}$

To express this in logarithmic form:

$$\log_{16}\left(\frac{1}{2}\right) = -\frac{1}{4}$$

The base is 16, the result is $\frac{1}{2}$, and the exponent is $-\frac{1}{4}$.

(vi) $11^2 = 121$

Step 1: Exponential Form

$$11^2 = 121$$

Step 2: Logarithmic Form

$$\log_{11}(121) = 2$$

Explanation:

- The **base** is 11.
- The **result** is 121.
- The **exponent** is 2.

(vii) $p = q^r$

Expressing this in logarithmic form:

$$\log_q(p) = r$$

The base is q, the result is p, and the exponent is r.

(viii) $(32)^{\frac{-1}{5}} = \frac{1}{2}$

To express this in logarithmic form:

$$\log_{32}\left(\frac{1}{2}\right) = -\frac{1}{5}$$

Here, the base is 32, the result is $\frac{1}{2}$, and the exponent is $-\frac{1}{5}$.

2. Express each of the following in exponential form:

(i) $\log_5 125 = 3$ **(ii)** $\log_2 16 = 4$

(iii) $\log_{23} 1 = 0$ **(iv)** $\log_5 5 = 1$

(v) $\log_2 \frac{1}{8} = -3$ **(vi)** $\frac{1}{2} = \log_9 3$

(vii) $5 = \log_{10} 100000$ **(viii)** $\log_4 \frac{1}{16} = -2$

Solution: To convert the given logarithmic expressions into exponential form, we use the basic rule of logarithms:

$$\log_b(a) = c \quad \text{can be rewritten as} \quad b^c = a$$

Let's solve each part separately:

(i) $\log_5(125) = 3$

We can express this in exponential form as:

$$5^3 = 125$$

This means 5 raised to the power of 3 equals 125.

(ii) $\log_2(16) = 4$

In exponential form, this is:

$$2^4 = 16$$

This means 2 raised to the power of 4 equals 16.

(iii) $\log_{23}(1) = 0$

We express this as:

$$23^0 = 1$$

Any number raised to the power of 0 equals 1.

(iv) $\log_5(5) = 1$

This can be expressed as:

$$5^1 = 5$$

This means 5 raised to the power of 1 equals 5.

(v) $\log_2\left(\frac{1}{8}\right) = -3$

In exponential form:

$$2^{-3} = \frac{1}{8}$$

This means 2 raised to the power of -3 equals $\frac{1}{8}$.

(vi) $\frac{1}{2} = \log_9(3)$

We express this in exponential form as:

$$9^{\frac{1}{2}} = 3$$

This means 9 raised to the power of $\frac{1}{2}$ (the square root of 9) equals 3.

(vii) $5 = \log_{10}(100000)$

In exponential form:

$$10^5 = 100000$$

This means 10 raised to the power of 5 equals 100000.

(viii) $\log_4\left(\frac{1}{16}\right) = -2$

We express this as:

$$4^{-2} = \frac{1}{16}$$

This means 4 raised to the power of -2 equals $\frac{1}{16}$.

3. **Find the value of x in each of the following:**

(i) $\log_x 64 = 3$ (ii) $\log_5 1 = x$

(iii) $\log_x 8 = 1$ (iv) $\log_{10} x = -3$

(v) $\log_4 x = \frac{3}{2}$ (vi) $\log_2 1024 = x$

Solutions:

(i) $\log_x(64) = 3$

The equation $\log_x(64) = 3$ can be rewritten in exponential form as:

$$x^3 = 64$$

Now, to find x, we take the cube root of both sides:

$$x = \sqrt[3]{64} = 4$$

Thus, the value of x is:

$$\boxed{x = 4}$$

(ii) $\log_5(1) = x$

We know that any logarithm of 1 is equal to 0, because:

$$\log_b(1) = 0 \quad \text{for any base } b$$

Thus:

$$x = 0$$

So the value of x is:

$$\boxed{x = 0}$$

(iii) $\log_x(8) = 1$

This equation can be rewritten in exponential form as:

$$x^1 = 8$$

Thus, $x = 8$.

So the value of x is:

$$\boxed{x = 8}$$

(iv) $\log_{10}(x) = -3$

Rewriting the equation in exponential form:

$$10^{-3} = x$$

This gives: $x = \frac{1}{10^3} = \frac{1}{1000}$

(v) $\log_4(x) = \frac{3}{2}$

Rewriting this in exponential form:

$$x = 4^{\frac{3}{2}}$$

We can simplify $4^{\frac{3}{2}}$ as follows:

$$x = 4^{\frac{3}{2}} = (2^2)^{\frac{3}{2}} = 2^3 = 8$$

Thus:

$$x = 8$$

So the value of x is:

$$\boxed{x = 8}$$

(vi) $\log_2(1024) = x$

Rewriting this in exponential form

We get $2^x = 1024$

$2^x = 2^{10}$

Thus:

$\therefore x = 10$

So the value of x is:

$\boxed{x = 10}$

2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

Example 9: Find characteristic of the followings:

(i) log 725 (ii) log 9.87

(iii) log 0.00045 (iv) log 0.54

Solutions: (i) log 725

Characteristic = 3 – 1=2

(ii) log 9.87

Characteristic = 1 – 1=0

(iii) log 0.00045

Characteristic = $-(3+1) = \bar{4}$

(iv) log 0.54

Characteristic= $-(0+1)= \bar{1}$

Example 10: Find logarithm of the following numbers:

(i) 345 (ii) 5.678 (iii) 0.0036 (iv) 0.0478

Solution: (i) 345

Characteristic = 3 – 1 = 2

Mantissa = 0.5378 (Look for 34 in the row and 5 in the column of the log table)

So, log(345) = 2 + 0.5378

=2.5378

(ii) 5.678

Characteristic = 1 – 1 = 0

Mantissa = 0.7542 (7536 + 6 = 7542)

So, log(5.678) = 0 + 0.7542 = 0.7542

(iii) 0.0036

Characteristic = –(2 + 1) = –3

Mantissa = 0.5563

(Look for 36 in the row and 0 in the column of the log table)

So, log(0.0036) = –3 + 0.5563 = $\bar{3}.5563$

(iv) 0.0478

Characteristic = – (1 + 1) = – 2

Mantissa = 0.6794

(Look for 47 in the row and 8 in the column of the log table)

So, log(0.0478) = –2 + 0.6794 = $\bar{2}.6794$

Example 11: Find the value of x in the followings:

(i) $\log x = 0.2568$

(ii) $\log x = -1.4567$

(iii) $\log x = -2.1234$

> **Remember!**
> The place between the first non-zero digit from left and its next digit is called reference position. For example, in 1332, the reference position is between 1 and 3

Solution:

(i) $\log x = 0.2568$

Characteristic = 0

Mantissa = 0.2568

Table value = 1803 + 3 = 1806

So, x = antilog (0.2568) = 1.806 (Insert the decimal point at reference position.

(ii) $\log x = -1.4567$

Since mantissa is negative, so we make it positive by adding and subtracting 2 $\log x = -2 + 2 - 1.4567$

$= -2 + 0.5433 = \bar{2}.5433$

Here characteristic = $\bar{2}$ and mantissa = 0.5433

Table value = 3491 + 2 = 3,493

So, x = antilog ($\bar{2}$.5433)

= 0.03493

Since characteristic is $\bar{2}$, therefore decimal point will be after 2 digits left from the reference position

(iii) $\log x = -2.1234$

Since mantissa is negative, so we make it positive by adding and subtracting 3

$\log x = -3 + 3 - 2.1234$

$= -3 + 0.8766 = \bar{3}.8766$

Here characteristic = $\bar{3}$, mantissa = 0.8766

Table value = 7516 + 10 = 7.526

So, $x = \text{antilog}(\bar{3}.8766)$

$= 0.007626$

(Since characteristic = $\bar{3}$, therefore decimal point will be after 3 digits left from the reference position.)

EXERCISE 2.3

1. Find characteristic of the following numbers:

(i) **5287** **(ii)** **59.28**
(iii) **0.0567** **(iv)** **234.7**
(v) **0.000049** **(vi)** **145000**

Solutions:

(i) 5287

To find the characteristic, we express 5287 in scientific notation:

$$5287 = 5.287 \times 10^3$$

The characteristic is the integer part of the exponent in the scientific notation. Here, the exponent is 3.

Thus, the characteristic of 5287 is:

$\boxed{3}$

(ii) 59.28

Now, express 59.28 in scientific notation:

$$59.28 = 5.928 \times 10^1$$

The exponent in the scientific notation is 1.

Thus, the characteristic of 59.28 is:

$\boxed{1}$

(iii) 0.0567

Express 0.0567 in scientific notation:

$$0.0567 = 5.67 \times 10^{-2}$$

The exponent in the scientific notation is −2.

Thus, the characteristic of 0.0567 is:

$\boxed{-2}$

(iv) 234.7

Now, express 234.7 in scientific notation:

$$234.7 = 2.347 \times 10^2$$

The exponent in the scientific notation is 2.

Thus, the characteristic of 234.7 is:

$\boxed{2}$

(v) 0.000049

Express 0.000049 in scientific notation:

$$0.000049 = 4.9 \times 10^{-5}$$

The exponent in the scientific notation is −5.

Thus, the characteristic of 0.000049 is:

$\boxed{-5}$

(vi) 145000

Now, express 145000 in scientific notation:

$$145000 = 1.45 \times 10^5$$

The exponent in the scientific notation is 5.

Thus, the characteristic of 145000 is:

$\boxed{5}$

2. Find logarithm of the following numbers:

(i) **43** **(ii)** **579** **(iii)** **1.982**
(iv) **0.0876** **(v)** **0.047** **(vi)** **0.000354**

Solutions:

(i) $\log_{10} 43$

Using a calculator or logarithmic table:

$$\log_{10} 43 \approx 1.6335$$

Thus, the logarithm of 43 is:

$\boxed{1.6335}$

(ii) $\log_{10}579$

Using a calculator or logarithmic table:

$$\log_{10}579 \approx 2.7627$$

Thus, the logarithm of 579 is:

$\boxed{2.7627}$

(iii) $\log_{10}1.982$

Using a calculator or logarithmic table:

$$\log_{10}1.982 \approx 0.2971$$

Thus, the logarithm of 1.982 is:

$\boxed{0.2971}$

(iv) $\log_{10}0.0876$

Using a calculator or logarithmic table:

$$\log_{10}0.0876 \approx -1.058 = \bar{2}.9425 = -1.0575$$

Thus, the logarithm of 0.0876 is:

$\boxed{-1.058}$

(v) $\log_{10}0.047$

Using a calculator or logarithmic table:

$$\log_{10}0.047 \approx -1.3279$$

$$\log_{10}0.047 = \bar{2}6721 = -1.3279$$

Thus, the logarithm of 0.047 is:

$\boxed{-1.3279}$

(vi) $\log_{10}0.000354$

Using a calculator or logarithmic table:

$$\log_{10}0.000354 \approx -3.451 = \bar{4}5490 = -3.4510$$

Thus, the logarithm of 0.000354 is:

$\boxed{-3.4510}$

3. **If log 3.177 = 0.5019, then find:**

(i) **log 3177** **(ii)** **log 31.77** **(iii)** **log 0.03177**

Solution: Given that $\log 3.177 = 0.5019$, let's solve the following:

(i) **log3177**

We can express 3177 as:

$$3177 = 3.177 \times 10^3$$

Using the logarithmic property $\log(ab) = \log a + \log b$, we can rewrite log 3177 as:

$$\log 3177 = \log(3.177 \times 10^3) = \log 3.177 + \log 10^3$$

Since $\log 10^3 = 3$, we have:

$$\log 3177 = 0.5019 + 3 = 3.5019$$

Thus, the value of log3177 is:

$\boxed{3.5019}$

(ii) **log31.77**

We can express 31.77 as:

$$31.77 = 3.177 \times 10$$

Using the logarithmic property $\log(ab) = \log a + \log b$, we can rewrite log31.77 as:

$$\log 31.77 = \log(3.177 \times 10) = \log 3.177 + \log 10$$

Since $\log 10 = 1$, we have:

$$\log 31.77 = 0.5019 + 1 = 1.5019$$

Thus, the value of log31.77 is:

$\boxed{1.5019}$

(iii) **log0.03177**

We can express 0.03177 as:

$$0.03177 = 3.177 \times 10^{-2}$$

Using the logarithmic property $\log(ab) = \log a + \log b$, we can rewrite log0.03177 as:

$$\log 0.03177 = \log(3.177 \times 10^{-2}) = \log 3.177 + \log 10^{-2}$$

Since $\log 10^{-2} = -2$, we have:

$$\log 0.03177 = 2.5019 = 0.5019 - 2 = -1.4981$$

Thus, the value of log0.03177 is:

$\boxed{-1.4981}$

4. **Find the value of x.**

(i) $\log x = 0.0065$ (ii) $\log x = 1.192$

(iii) $\log x = -3.434$ (iv) $\log x = -1.5726$

(v) $\log x = 4.3561$ (vi) $\log x = -2.0184$

Solutions:

(i) $\log x = 0.0065$

The equation $\log x = 0.0065$ is in base 10. To find x, we use the property of logarithms:

$$\log x = 0.0065 \quad \Rightarrow \quad x = 10^{0.0065}$$

Now, calculate $10^{0.0065}$ using a calculator:

$$x \approx 1.0153$$

Thus, the value of x is:

$$x \approx \boxed{1.0153}$$

(ii) $\log x = 1.192$

Again, using the property of logarithms:

$$\log x = 1.192 \quad \Rightarrow \quad x = 10^{1.192}$$

Now, calculate $10^{1.192}$:

$$x \approx 15.559$$

Thus, the value of x is:

$$x \approx \boxed{15.559}$$

(iii) $\log x = -3.434$

For this equation, we have:

$$\log x = -3.434 \quad \Rightarrow \quad x = 10^{-3.434}$$

Now, calculate $10^{-3.434}$:

$$x \approx 0.000369$$

Thus, the value of x is:

$$x \approx \boxed{0.000369}$$

(iv) $\log x = -1.5726$

For this equation:

$$\log x = -1.5726 \quad \Rightarrow \quad x = 10^{-1.5726}$$

Now, calculate $10^{-1.5726}$:

$$x \approx 0.0267$$

Thus, the value of x is:

$$x \approx \boxed{0.0267}$$

(v) $\log x = 4.3561$

For this equation:

$$\log x = 4.3561 \quad \Rightarrow \quad x = 10^{4.3561}$$

Now, calculate $10^{4.3561}$:

$$x \approx 22684.3$$

Thus, the value of x is:

$$x \approx \boxed{22684.3}$$

(vi) $\log x = -2.0184$

For this equation:

$$\log x = -2.0184 \quad \Rightarrow \quad x = 10^{-2.0184}$$

Now, calculate $10^{-2.0184}$:

$$x \approx 0.000095$$

Thus, the value of x is:

$$x \approx \boxed{0.000095}$$

Laws of Logarithm

Laws of logarithm are also known as rules or properties of logarithm. These laws help to simplify logarithmic expressions and solve logarithmic equations.

1. **Product Law**

$$\log_b xy = \log_b x + \log_b y$$

The logarithms of a product are the sum of the logarithms of the factors.

Proof: Let

$$m = \log_b x \quad \ldots(i)$$

and

$$n = \log_b y \quad \ldots(ii)$$

In exponential form:

$$x = b^m \quad \text{and} \quad y = b^n$$

Multiply x and y

$$x.y = b^m . b^n = b^{m+n}$$

In logarithmic form:

$$\log_b xy = m + n$$

$$\log_b xy = \log_b x + \log_b y \quad \text{[From (i) and (ii)]}$$

2. Quotient Law

$$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$$

The logarithm of a quotient is the difference between the logarithms of the numerator and the denominator.

Proof: Let

$m = \log_b x$ ...(i)

and $n = \log_b y$ ...(ii)

In exponential form:

$x = b^m$ and $y = b^n$

Divide x and y

$$\frac{x}{y} = \frac{b^m}{b^n} = b^{m-n}$$

In logarithmic form:

$$\log_b\left(\frac{x}{y}\right) = m - n$$

$$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y \quad \text{[From (i) and (ii)]}$$

3. Power Law

$$\log_b x^n = n.\log_b x$$

The logarithm of a number raised to a power is the product of the power and the logarithm of the base number.

Proof: Let

$m = \log_b x$ ...(i)

In exponential form:

$x = b^m$

Raise both sides to the power n

$x^n = (b^m)^n = b^{mn}$

In logarithmic form:

$\log_b x^n = mn$

$\log_b x^n = n.\log_b x$ [From (*i*)]

4. Change of Base Law

$$\log_b x = \frac{\log_a x}{\log_a b}$$

This law allows to change the base of a logarithm from "b" to any other base "a".

Proof: Let $m = \log_b x$ ...(i)

In exponential form:

$b^m = x$

Taking $\log_a$ on both sides

$\log_a b^m = \log_a x$

$m \log_a b = \log_a x$

$$m = \frac{\log_a x}{\log_a b}$$

$$\log_b x = \frac{\log_a x}{\log_a b} \quad \text{[From (i)]}$$

Example 12: Expand the following logarithms:

(i) $\log_3(20)$ (ii) $\log_2 (9)^5$

(iii) $\log_{32} 27$

Solution: (i) $\log_3(20)$

$= \log_3(2\times2\times5)$

$= \log_3(2^2\times5)$

$= \log_3(2)^2 + \log_3 5$

$= 2\log_3 2 + \log_3 5$

(ii) $\log_2 (9)^5$

$= \log_2 (3^2)^5$

$= \log_2 (3)^{10}$

$= 10 \log_2 3$

(iii) $\log_{32} 27 = \dfrac{\log 27}{\log 32}$

$$= \frac{\log 3^3}{\log 2^5} = \frac{3\log 3}{5\log 2}$$

Example 13: Expand the following logarithms:

(i) $\log_2\left(\frac{x-y}{z}\right)^3$ (ii) $\log_5\left(\frac{xy}{z}\right)^8$

Solution: (i) $\log_2\left(\frac{x-y}{z}\right)^3 = 3\log_2\left(\frac{x-y}{z}\right)$

$= 3[\log_2(x-y) - \log_2 z]$

(ii) $\log_5\left(\frac{xy}{z}\right)^8 = 8\log_5\left(\frac{xy}{z}\right)$

$= 8[\log_5(xy) - \log_5 z]$

$= 8[\log_5 x + \log_5 y - \log_5 z]$

Example 14: Write as a single logarithm:

(i) $2\log_3 10 - \log_3 4$

(ii) $6\log_3 x + 2\log_3 11$

Solution: (i) $2\log_3 10 - \log_3 4$

$= \log_3 (10)^2 - \log_3 4$

$= \log_3 100 - \log_3 4$

$= \log_3\left(\frac{100}{4}\right)$

$= \log_3 25$

(ii) $6\log_3 x + 2\log_3 11$

$= \log_3 x^6 + \log_3 (11)^2$

$= \log_3 x^6 + \log_3 (121)$

$= \log_3 (121x^6)$

Example 15: The decibel scale measures sound intensity using the formula $L = 40\log_{10}\left(\frac{I}{I_o}\right)$. If a sound has an intensity (I) of 10^6 times the reference intensity (I_o). What is the sound level in decibels?

Solution: $L = 40\log_{10}\left(\frac{I}{I_o}\right)$

> **Do you know?**
> $ln(0)$ = undefined
> $ln(1) = 0$
> $ln(e) = 1$

Put $I = 10^6 I_o$, we get

$L = 40\log_{10}\left(\frac{10^6 I_o}{I_o}\right)$

$L = 40\log_{10}(10)^6$

$L = 40 \times 6\log_{10} 10$

$L = 40 \times 6$ $(\because \log_{10} 10 = 1)$

$L = 240$ decibels

EXERCISE 2.4

1. **Without using calculator, evaluate the following:**

(i) $\log_2 18 - \log_2 9$ (ii) $\log_2 64 + \log_2 2$

(iii) $\frac{1}{3}\log_3 8 - \log_3 18$ (iv) $2\log 2 + \log 25$

(v) $\frac{1}{3}\log_4 64 + 2\log_5 25$ (vi) $\log_3 12 + \log_3 0.25$

Solution: (i) $\log_2 18 - \log_2 9$

$= \log_2\frac{18}{9} = \log_2 2$

Let $\log_2 2 = x$ $\therefore 2^x = 2^1$

$\therefore x = 1$

(ii) $\log_2 64 + \log_2 2$

$= \log_2(64 \times 2)$

$= \log_2(2^7)$

$= 7\log_2 2$ $\therefore \log_2 2 = 1$

$= 7(1) = 7$

(iii) $\frac{1}{3}\log_3 8 - \log_3 18$

$= \log_3 8^{\frac{1}{3}} - \log_3 18$ $= \log_3\left(\frac{8^{\frac{1}{3}}}{18}\right)$

$= \log_3 8^{\frac{1}{3}} - \log_3 18$ $= \log_3\frac{(2^3)^{\frac{1}{3}}}{18}$

$= \log_3 \frac{2}{18}$ $\quad = \log_3 \frac{1}{9}$

$= \log_3 \frac{1}{3^2}$ $\quad = \log_3 3^{-2}$

$= -2(1)$ $\quad = -2$

(iv) $2\log 2 + \log 25$

$= \log 2^2 + \log 25$

$= \log 4 + \log 25$

$= \log(4 \times 25)$

$= \log 100$

$= \log 10^2$

$= 2\log 10$

$= 2(1) = 2$

(v) $\frac{1}{3}\log_4 64 + 2\log_5 25$

$= \frac{1}{3}\log_4(4^3) + 2\log_5 5^2$

$= \frac{1}{3} \times 3\log_4 4 + 2 \times 2\log_5 5$

$= 1(1) + 4(1) \quad (\because \log_a a = 1)$

$= 1 + 4$

$= 5$

(vi) $\log_3 12 + \log_3 0.25$

$= \log_3(12 \times 0.25)$

$= \log_3(3) = 1$

2. **Write the following as a single logarithm.**

(i) $\frac{1}{2}\log 25 + 2\log 3$ **(ii)** $\log 9 - \log\frac{1}{3}$

(iii) $\log_5 b^2 . \log_a 5^3$ **(iv)** $2\log_3 x + \log_3 y$

(v) $4\log_5 x - \log_5 y + \log_5 z$

(vi) $2\ \ln a + 3\ \ln b - 4\ \ln c$

Solution: Let's simplify each of the logarithmic expressions step-by-step using logarithmic properties.

(i) $\frac{1}{2}\log 25 + 2\log 3$

Use the logarithmic property $a\log b = \log b^a$:

- For $\frac{1}{2}\log 25$, we have:

$$\frac{1}{2}\log 25 = \log 25^{1/2} = \log 5$$

- For $2\log 3$, we have:

$$2\log 3 = \log 3^2 = \log 9$$

Now, combine the two terms using the property $\log_b a + \log_b c = \log_b(a \cdot c)$:

$$\log 5 + \log 9 = \log(5 \cdot 9) = \log 45$$

Thus, the single logarithm is:

$$\boxed{\log 45}$$

(ii) $\log 9 - \log\frac{1}{3}$

Use the logarithmic property $\log_b a - \log_b c = \log_b \frac{a}{c}$:

$$\log 9 - \log\frac{1}{3} = \log\left(\frac{9}{\frac{1}{3}}\right) = \log(9 \cdot 3) = \log 27$$

Thus, the single logarithm is:

$$\boxed{\log 27}$$

(iii) $\log_5 b^2 + \log_a 5^3$

$= 2\log_5 b \times 3\log_a 5$

$= 6\log_5 b \times \log_a 5$

$= 6\log_a b$

(iv) $2\log_3 x + \log_3 y$

$= \log_3 x^2 + \log_3 y$

$= \log_3 x^2 y$

(v) $4\log_5 x - \log_5 y + \log_5 z$

Use the property $a\log_b c = \log_b c^a$ for the first term:

$$4\log_5 x = \log_5 x^4$$

Now, combine the terms using $\log_b a - \log_b c = \log_b \frac{a}{c}$ and $\log_b a + \log_b c = \log_b(a \cdot c)$:

$$\log_5 x^4 - \log_5 y + \log_5 z = \log_5\left(\frac{x^4 \cdot z}{y}\right)$$

Thus, the single logarithm is:

$$\boxed{\log_5\left(\frac{x^4 z}{y}\right)}$$

(vi) $2\ln a + 3\ln b - 4\ln c$

Use the property $a\ln b = \ln b^a$ for each term:

$$2\ln a = \ln a^2, \quad 3\ln b = \ln b^3, \quad -4\ln c = \ln c^{-4}$$

Now, combine the terms using $\ln a + \ln b + \ln c = \ln(a \cdot b \cdot c)$ and $\ln a - \ln b = \ln\frac{a}{b}$:

$$\ln a^2 + \ln b^3 - \ln c^4 = \ln\left(\frac{a^2 b^3}{c^4}\right)$$

Thus, the single logarithm is:

$$\boxed{\ln\left(\frac{a^2 b^3}{c^4}\right)}$$

3. Expand the following using laws of logarithms:

(i) $\log\left(\frac{11}{5}\right)$ (ii) $\log_5 \sqrt{8a^6}$

(iii) $\ln\left(\frac{a^2 b}{c}\right)$ (iv) $\log\left(\frac{xy}{z}\right)^{\frac{1}{9}}$

(v) $\ln\sqrt[3]{16x^3}$ (vi) $\log_2\left(\frac{1-a}{b}\right)^5$

Solutions:

(i) $\log\left(\frac{11}{5}\right)$

Using the property $\log\frac{a}{b} = \log a - \log b$, expand:

$$\log\left(\frac{11}{5}\right) = \log 11 - \log 5$$

(ii) $\log_5\sqrt{8a^6}$

$$= \log_5 (8a^6)^{\frac{1}{2}}$$

$$= \frac{1}{2}\log_5(8a^6)$$

$$= \frac{1}{2}(\log_5 8 + \log_5 a^6)$$

$$= \frac{1}{2}[\log_5 2^3 + \log_5 a^6]$$

$$= \frac{1}{2}[3\log_5 2 + 6\log_5 a]$$

$$= \frac{1}{2}\times 3\log_5 2 + \frac{1}{2}\times 6\log_5 a$$

$$= \frac{3}{2}\log_5 2 + 3\log_5 a$$

(iii) $\ln\left(\frac{a^2 b}{c}\right)$

Using $\ln\frac{a}{b} = \ln a - \ln b$, rewrite:

$$\ln\left(\frac{a^2 b}{c}\right) = \ln(a^2 b) - \ln c$$

Now use $\ln(ab) = \ln a + \ln b$:

$$\ln(a^2 b) = \ln a^2 + \ln b$$

Expand $\ln a^2$ using $\ln a^n = n\ln a$:

$$\ln a^2 = 2\ln a$$

Substitute back:

$$\ln\left(\frac{a^2 b}{c}\right) = 2\ln a + \ln b - \ln c$$

(iv) $\log\left(\frac{xy}{z}\right)^{1/9}$

Using $\log a^n = n\log a$, expand:

$$\log\left(\frac{xy}{z}\right)^{1/9} = \frac{1}{9}\log\left(\frac{xy}{z}\right)$$

Using $\log\frac{a}{b} = \log a - \log b$, rewrite:

$$\log\left(\frac{xy}{z}\right) = \log(xy) - \log z$$

Using $\log(ab) = \log a + \log b$, expand $\log(xy)$:

$$\log(xy) = \log x + \log y$$

Substitute back:

$$\log\left(\frac{xy}{z}\right) = \log x + \log y - \log z$$

Now distribute $\frac{1}{9}$:

$$\frac{1}{9}(\log x + \log y - \log z) = \frac{1}{9}\log x + \frac{1}{9}\log y - \frac{1}{9}\log z$$

$$= \frac{1}{9}[\log x + \log y - \log z]$$

(v) $\ln\sqrt[3]{16x^3}$

$$= \ln(16x^3)^{\frac{1}{3}}$$

$$= \frac{1}{3}\left[\ln 16x^3\right]$$

$$= \frac{1}{3}\left[\ln 16 + \ln x^3\right]$$

$$= \frac{1}{3}[\ln 16 + 3\ln x]$$

$$= \frac{1}{3}\ln 16 + \frac{1}{\cancel{3}} \times \cancel{3}\ln x$$

$$= \frac{1}{3}\ln 2^4 + \ln x$$

$$= \frac{1}{3} \times 4\ln 2 + \ln x$$

$$= \frac{4}{3}\ln 2 + \ln x$$

(vi) $\log_2\left(\frac{1-a}{b}\right)^5$

Using $\log a^n = n\log a$, expand:

$$\log_2\left(\frac{1-a}{b}\right)^5 = 5\log_2\left(\frac{1-a}{b}\right)$$

Using $\log\frac{a}{b} = \log a - \log b$, expand:

$$\log_2\left(\frac{1-a}{b}\right) = \log_2(1-a) - \log_2 b$$

Substitute back:

$$5\log_2\left(\frac{1-a}{b}\right) = 5(\log_2(1-a) - \log_2 b)$$

Distribute 5:

$$= 5\log_2(1-a) - 5\log_2 b$$

$$= 5[\log_2(1-a) - \log_2 b]$$

4. **Find the value of x in the following equations:**

(i) $\log 2 + \log x = 1$ (ii) $\log_2 x + \log_2 8 = 5$

(iii) $(81)^x = (243)^{x+2}$ (iv) $\left(\frac{1}{27}\right)^{x-6} = 27$

(v) $\log(5x - 10) = 2$

(vi) $\log_2(x+1) - \log_2(x-4) = 2$

Solution: We will solve each equation step by step:

(i) $\log 2 + \log x = 1$

Using the property $\log a + \log b = \log(a \cdot b)$, rewrite:

$$\log 2 + \log x = \log(2x)$$

Now the equation becomes:

$$\log(2x) = 1$$

Rewrite in exponential form

$$\log a = b \Rightarrow a = 10^b$$

$$2x = 10^1$$

Simplify:

$$2x = 10 \Rightarrow x = \frac{10}{2}$$

Solution:

$$\boxed{x = 5}$$

(ii) $\log_2 x + \log_2 8 = 5$

Apply the property $\log_a + \log_b = \log(a \cdot b)$

We combine the two logarithms using the property:

$$\log_2 x + \log_2 8 = \log_2(x \cdot 8)$$

So, we have:

$$\log_2(8x) = 5$$

$$(\because \log_b a = c)$$

$$8x = 2^5$$

$$8x = 32$$

$$x = \frac{32}{8} = 4$$

Thus, the value of x is:

$$\boxed{x = 4}$$

(iii) $81^x = 243^{x+2}$

Rewrite 81 and 243 as powers of 3:

$$81 = 3^4, \quad 243 = 3^5$$

Substitute:

$$(3^4)^x = (3^5)^{x+2}$$

Simplify the exponents:

$$3^{4x} = 3^{5(x+2)}$$

Equating the exponents:

$$4x = 5(x+2)$$

Expand:

$$4x = 5x + 10$$

Simplify:

$$4x - 5x = 10 \Rightarrow -x = 10 \Rightarrow x = -10$$

Solution:

$$\boxed{x = -10}$$

(iv) $\left(\frac{1}{27}\right)^{x-6} = 27$

Rewrite 27 and $\frac{1}{27}$ as powers of 3:

$$\frac{1}{27} = 3^{-3}, \quad 27 = 3^3$$

Substitute:

$$(3^{-3})^{x-6} = 3^3$$

Simplify the exponents:

$$3^{-3(x-6)} = 3^3$$

Equate the exponents:

$$-3(x-6) = 3$$

Simplify:

$$-3x + 18 = 3$$

Solve for x:

$$-3x = 3 - 18 \Rightarrow -3x = -15 \Rightarrow x = 5$$

Solution:

$$\boxed{x = 5}$$

(v) $\log(5x - 10) = 2$

Rewrite in exponential form:

$$5x - 10 = 10^2$$

Simplify:

$$5x - 10 = 100$$

Solve for x:

$$5x = 100 + 10 \Rightarrow 5x = 110 \Rightarrow x = \frac{110}{5} = 22$$

Solution:

$$\boxed{x = 22}$$

(vi) $\log_2(x+1) - \log_2(x-4) = 2$

Using the property $\log a - \log b = \log\frac{a}{b}$, rewrite:

$$\log_2\left(\frac{x+1}{x-4}\right) = 2$$

Rewrite in exponential form:

$$\frac{x+1}{x-4} = 2^2$$

Simplify:

$$\frac{x+1}{x-4} = 4$$

$$x + 1 = 4(x - 4)$$

Expand:

$$x + 1 = 4x - 16$$

Simplify:

$$1+16=4x-x \Rightarrow 17=3x \Rightarrow x=\frac{17}{3}=5\frac{2}{3}$$

Solution:

$$\boxed{x = 5\frac{2}{3}}$$

5. Find the values of the following with the help of logarithm table:

(i) $\frac{3.68 \times 4.21}{5.234}$ (ii) $4.67 \times 2.11 \times 2.397$

(iii) $\frac{(20.46)^2 \times (2.4122)}{754.3}$ (iv) $\frac{\sqrt[3]{9.364} \times 21.64}{3.21}$

Solution:

(i) $\frac{3.68 \times 4.21}{5.234}$

$$\log\left(\frac{3.68 \times 4.21}{5.234}\right) = \log(3.68) + \log(4.21) - \log(5.234)$$

Substitute values:

$$\log\left(\frac{3.68 \times 4.21}{5.234}\right) = 0.5658 + 0.6243 - 0.7188 = 0.4713$$

$$\text{Antilog}(0.4713) \approx 2.96$$

Solution:

$$\boxed{2.96}$$

(ii) $4.67 \times 2.11 \times 2.397$

$$\log(4.67 \times 2.11 \times 2.397)$$
$$= \log(4.67) + \log(2.11) + \log(2.397)$$

Substitute values:

$$\log(4.67 \times 2.11 \times 2.397) = 0.6693 + 0.3243 + 0.3797$$
$$= 1.3733$$

$$\text{Antilog}(1.3733) \approx 23.62$$

Solution:

$$\boxed{23.62}$$

(iii) $\frac{(20.46)^2 \times 2.4122}{754.3}$

$$\log\left(\frac{(20.46)^2 \times 2.4122}{754.3}\right)$$
$$= 2\log(20.46) + \log(2.4122) - \log(754.3)$$

Substitute values:

$$\log\left(\frac{(20.46)^2 \times 2.4122}{754.3}\right) = 2(1.30) + 0.3824 - 2.8775$$

Simplify:

$$\log\left(\frac{(20.46)^2 \times 2.4122}{754.3}\right) = 2.6218 + 0.3824 - 2.8775$$
$$= 0.1267$$

$$\text{Antilog}(0.1267) \approx 1.339$$

Solution:

$$\boxed{1.339}$$

(iv) $\frac{\sqrt[3]{9.364} \times 21.64}{3.21}$

$$\log\left(\frac{\sqrt[3]{9.364} \times 21.64}{3.21}\right)$$
$$= \log(\sqrt[3]{9.364}) + \log(21.64) - \log(3.21)$$

Substitute values:

$$\log\left(\frac{\sqrt[3]{9.364} \times 21.64}{3.21}\right) = 0.3238 + 1.3353 - 0.5065 = 1.1526$$

$$\text{Antilog}(1.1526) \approx 14.2$$

$$\boxed{14.21}$$

6. **The formula for measure the magnitude of earthquakes is given by $M=\log_{10}\left(\frac{A}{A_o}\right)$. If amplitude ($A$) is 10,000 and reference amplitude (A_o) is 10. What is the magnitude of the earthquake?**

Solution: We are given the formula for the magnitude of earthquakes:

$$M = \log_{10}\left(\frac{A}{A_o}\right)$$

Given:

- $A = 10{,}000$
- $A_o = 10$

Substitute the values into the formula:

$$M = \log_{10}\left(\frac{10{,}000}{10}\right)$$

Simplify the fraction:

$$M = \log_{10}(1{,}000)$$

The logarithm of 1,000 with base 10 is:

$$\log_{10}(1{,}000) = 3$$

The magnitude of the earthquake is:

$$\boxed{M = 3}$$

7. **Abdullah invested Rs. 100,000 in a saving scheme and gains interest at the rate of 5% per annum so that the total value of this investment after t years is Rs y. This is modelled by an equation $y = 100{,}000\ (1.05)^t$, $t \geq 0$. Find after how many years the investment will be double.**

Solution: We are tasked with determining after how many years the investment will double its initial value.

Step 1: Define the equation

The initial investment is 100,000. To double the investment:

$$y = 2 \times 100{,}000 = 200{,}000$$

Substituting into the given model:

$$200{,}000 = 100{,}000(1.05)^t$$

Step 2: Simplify the equation

Divide both sides by 100,000:

$$2 = (1.05)^t$$

Step 3: Solve for t using logarithms

Take the common logarithm log of both sides:

$$\text{Log}2 = \text{log}(1.05)^t$$

Using the logarithmic property $\log_a t = t \log_a$

$$\text{Log}2 = t \log 1.05 \quad t = \frac{\log_2}{\log 1.05}$$

$$= \frac{0.3010}{0.0212}$$

$$= 14.198 \text{ years}$$

$$\approx 14.2 \text{ years}$$

Solve for t:

$$t = \frac{\ln(2)}{\ln(1.05)}$$

$$\approx 14.2$$

The investment will double in approximately **14 years.**

8. **Huria is hiking up a mountain where the temperature (T) decreases by 3% (or a factor of 0.97) for every 100 metres gaine\ in altitude. The initial temperature (T_i) at sea level is 20°C. Jsing the formula $T = T_i \times 0.97)\frac{h}{100}$. calculate the temperature at an altitude (h) of 500 metres.**

Solution:

The temperature at sea level (T_i) is 20°C, and the temperature decreases by 3% for every 100 metres gained in altitude. The formula for the temperature at any altitude h is:

$$T = T_i \times 0.97^{\frac{h}{100}}$$

We need to calculate the temperature at an altitude of 500 metres ($h = 500$).

Substitute the known values into the formula:

$$T = 20 \times 0.97^{\frac{500}{100}}$$

Simplify the exponent:

$$T = 20 \times 0.97^5$$

We calculate 0.97^5:

$$0.97^5 \approx 0.8587$$

Now, multiply the result by the initial temperature:

$$T = 20 \times 0.8587$$

$$T \approx 17.174\ °C$$

The temperature at an altitude of 500 metres is approximately:

$$\boxed{17.17\ °C}$$

REVIEW EXERCISE 2

1. Choose the correct option.

i. The standard form of 5.2×10^6 is:
(a) 52,000 (b) 520,000
(c) 5,200,000 (d) 52,000,000

ii. Scientific notation of 0.00034 is:
(a) 3.4×10^3 (b) 3.4×10^{-4}
(c) 3.4×10^4 (d) 3.4×10^{-3}

iii. The base of common logarithm is:
(a) 2 (b) 10
(c) 5 (d) e

iv. $\log_2 2^3 =$ ______
(a) 1 (b) 2
(c) 5 (d) 3

v. $\log 100 =$ ______
(a) 2 (b) 3
(c) 10 (d) 1

vi. If $\log 2 = 0.3010$, then $\log 200$ is:
(a) 1.3010 (b) 0.6010
(c) 2.3010 (d) 2.000

vii. $\log (0) =$
(a) positive (b) negative
(c) zero (d) undefined

viii. $\log 10,000 =$
(a) 2 (b) 3
(c) 4 (d) 5

ix. $\log 5 + \log 3 =$ ______.
(a) $\log 0$ (b) $\log 2$
(c) $\log\left(\frac{5}{3}\right)$ (d) $\log 15$

x. $3^4 = 81$ in logarithmic form is:
(a) $\log_3 4 = 81$ (b) $\log_4 3 = 81$
(c) $\log_3 81 = 4$ (d) $\log_4 81 = 3$

Answers:

(i)	c	(ii)	b	(iii)	b	(iv)	d	(v)	a
(vi)	c	(vii)	d	(viii)	c	(ix)	d	(x)	c

2. Express the following number in scientific notation:
(i) 0.000567 (ii) 734 (iii) 0.33×10^3

Solution:

(i) 0.000567:

To convert 0.000567 into scientific notation:

1. Identify the significant digits: 5.67.
2. Count the number of decimal places the decimal point needs to move to reach after the first non-zero digit: 4 places to the right.
3. The number becomes:

$$0.000567 = 5.67 \times 10^{-4}$$

(ii) 734:

To convert 734 into scientific notation:

1. Identify the significant digits: 7.34.

2. Count the number of decimal places the decimal point needs to move to reach after the first digit: 2 places to the left.

3. The number becomes:

$$734 = 7.34 \times 10^2$$

(iii) 0.33×10^3:

To simplify 0.33×10^3 into proper scientific notation:

1. Adjust 0.33 into scientific notation: 3.3×10^{-1}.
2. Combine with 10^3:

$$(3.3 \times 10^{-1}) \times 10^3 = 3.3 \times 10^2.$$

3. Express the following numbers in ordinary notation:

(i) 2.6×10^3 **(ii)** 8.794×10^{-4}

(iii) 6×10^{-6}

Solutions:

(i) 2.6×10^3:

To convert 2.6×10^3 into ordinary notation:

1. 10^3 means shifting the decimal point **3 places to the right.**
2. Starting with 2.6, shift the decimal point:

$$2.6 \rightarrow 26.0 \rightarrow 260.0 \rightarrow 2600.$$

Answer: 2600

(ii) 8.794×10^{-4}:

To convert 8.794×10^{-4} into ordinary notation:

1- 10^{-4} means shifting the decimal point 4 places to the left.

2- Starting with 8.794, add leading zeros as necessary:

$$8.794 \rightarrow 0.8794 \rightarrow 0.08794 \rightarrow 0.008794 \rightarrow 0.0008794.$$

Answer: 0.0008794

(iii) 6×10^{-6}:

To convert 6×10^{-6} into ordinary notation:

1. 10^{-6} means shifting the decimal point **6 places to the left.**

2. Starting with 6, add leading zeros:

$$6 \rightarrow 0.6 \rightarrow 0.06 \rightarrow 0.006 \rightarrow 0.0006 \rightarrow 0.00006 \rightarrow 0.000006.$$

Answer: 0.000006

4. Express each of the following in logarithmic form.

(i) $3^7 = 2187$ **(ii)** $a^b = c$

(iii) $(12)^2 = 144$

Sol: To express each of the given equations in logarithmic form, we use the general conversion rule:

$$a^b = c \quad \text{can be written as} \quad \log_a c = b$$

(i) 37 = 2187

This is an exponential equation, where the base is 3, the exponent is 7, and the result is 2187. Using the conversion rule:

$$3^7 = 2187 \quad \text{can be written as} \quad \log_3 2187 = 7$$

(ii) $ab = c$

This equation is an exponential equation with base a and exponent b, and the result is c. Using the conversion rule:

$$a^b = c \quad \text{can be written as} \quad \log_a c = b$$

So, for this equation:

$$ab = c \quad \text{can be written as} \quad \log_a c = b$$

(iii) $(12)^2 = 144$

This equation is also an exponential equation. The base is 12, the exponent is 2, and the result is 144. Using the conversion rule:

$$12^2 = 144 \quad \text{can be written as} \quad \log_{12} 144 = 2$$

5. Express each of the following in exponential form.

(i) $\log_4 8 = x$ **(ii)** $\log_9 729 = 3$

(iii) $\log_4 1024 = 5$

Solution: To express the given logarithmic equations in exponential form, we use the general formula for logarithms:

$$\log_b a = c \quad \text{is equivalent to} \quad b^c = a.$$

Now, let's solve each one separately:

(i) $\log_4 8 = x$:

1. The logarithmic equation is $\log_4 8 = x$

2. In exponential form:

$$4^x = 8$$

3. The equation is true, as $4^x = 8$.

Answer: $4^x = 8$

(ii) $\log_9 729 = 3$:

1. The logarithmic equation is $\log_9 729 = 3$.
2. In exponential form:

$$9^3 = 729$$

3. This equation is true, as $9^3 = 729$.

Answer: $9^3 = 729$

(iii) $\log_4 1024 = 5$:

1. The logarithmic equation is $\log_4 1024 = 5$.
2. In exponential form:

$$4^5 = 1024$$

3. This equation is true, as $4^5 = 1024$.

Answer: $4^5 = 1024$

6. Find value of x in the following:

(i) $\log_9 x = 0.5$ (ii) $\left(\frac{1}{9}\right)^{3x} = 27$

(iii) $\left(\frac{1}{32}\right)^{2x} = 64$

Solution:

(i) $\log_9 x = 0.5$:

This is a logarithmic equation. We can convert it to its exponential form using the formula $\log_b a = c$ which is equivalent to $b^c = a$. So:

$$9^{0.5} = x.$$

Since $9^{0.5}$ is the square root of 9, we have:

$$x = \sqrt{9} = 3.$$

Answer: $x = 3$.

(ii) $\left(\frac{1}{9}\right)^{3x} = 27$:

First, express $\frac{1}{9}$ as 9^{-1}, and rewrite the equation as:

$$(9^{-1})^{3x} = 27.$$

$(3^{-2})^{3x} = 27$

$3^{-6x} = 3^3$

$-6x = 3$

$x = -\frac{3}{6} = -\frac{1}{2}$

Using the rule $(a^b)^c = a^{bc}$, we get:

$9^{-3x} = 27$.

Now, express 27 as a power of 9, i.e., $27 = 3^3$. Also, $9 = 3^2$, so:

$$9^{-3x} = (3^2)^{-3x} = 3^{-6x}.$$

Thus, the equation becomes:

$$3^{-6x} = 3^3.$$

Since the bases are the same, we can set the exponents equal to each other:

$$-6x = 3.$$

Solving for x:

$$x = \frac{3}{-6} = -\frac{1}{2}.$$

Answer: $x = -\frac{1}{2}$.

(iii) $\left(\frac{1}{32}\right)^{2x} = 64$:

Express $\frac{1}{32}$ as 32^{-1}, and rewrite the equation as:

$$(32^{-1})^{2x} = 64.$$

Using the rule $(a^b)^c = a^{bc}$, we get:

$$32^{-2x} = 64.$$

Simplifying the left side:

$$2^{-10x} = 2^6.$$

Since the bases are the same, we can set the exponents equal to each other: $-10x = 6$.

Solving for x:

$$x = \frac{6}{-10} = -\frac{3}{5}.$$

Answer: $x = -\frac{3}{5}$.

7. Write as a single logarithms.

(i) $7\log x - 3\log y^2$ (ii) $3\log 4 - \log 32$

(iii) $\frac{1}{3}(\log_5 8 + \log_5 27) - \log_5 3$

Solution:

(i) $7\log x - 3\log y^2$:

$7\log x - \log y^6 = \log x^7 - \log y^6 = \log\left(\frac{x^7}{y^6}\right)$.

$(\because \log_b a^n = n\log_b a)$

(ii) $3\log 4 - \log 32$:

$= \log 4^3 - \log 32$

$\log 64 - \log 32 = \log\left(\frac{64}{32}\right)$.

- Simplify $\frac{64}{32}$ to 2:

$\log\left(\frac{64}{32}\right) = \log 2$.

(iii) $\frac{1}{3}(\log_5 8 + \log_5 27) - \log_5 3$:

Combine $\log_5 8 + \log_5 27$:

$\log_5 8 + \log_5 27 = \log_5(8 \times 27) = \log_5 216$.

- **Multiply the result by $\frac{1}{3}$:**

$\frac{1}{3}\log_5 216 = \log_5 216^{1/3} = \log_5 \sqrt[3]{216}$.

Since $\sqrt[3]{216} = 6$, we have:

$\log_5 \sqrt[3]{216} = \log_5 6$.

- Now, subtract $\log_5 3$ from $\log_5 6$:

$\log_5 6 - \log_5 3 = \log_5\left(\frac{6}{3}\right) = \log_5 2$.

8. Expand the following using laws of logarithms:

(i) $\log(x y z^6)$ (ii) $\log_3 \sqrt[6]{m^5 n^3}$

(iii) $\log\sqrt{8x^3}$

Solution:

(i) $\log(xyz^6)$:

$\log(xyz^6) = \log x + \log y + \log z^6$.

$= \log x + \log y + 6\log z$. $(\because \log_b(a^n) = n\log_b a)$

(ii) $\log_3 \sqrt[6]{m^5 n^3}$:

$= \log_3((m^5 n^3)^{1/6}) = \frac{1}{6}\log_3(m^5 n^3)$.

$(\because \log_b(a^n) = n\log_b a)$

$= \frac{1}{6}(5\log_3 m + 3\log_3 n)$.

(iii) $\log\sqrt{8x^3}$:

$= \log((8x^3)^{1/2}) = \frac{1}{2}\log(8x^3)$.

$(\because \log_b(a^n) = n\log_b a)$

$= \frac{1}{2}(\log 8 + 3\log x)$.

$= \frac{1}{2}\log 8 + \frac{3}{2}\log x$.

$= \frac{1}{2} \times 3\log 2 + \frac{3}{2}\log x = \frac{3}{2}\log 2 + \frac{3}{2}\log x$.

$= \frac{3}{2}[\log^2 + \log x]$

9. Find the values of the following with the help of logarithm table:

(i) $\sqrt[3]{68.24}$ (ii) 319.8×3.543

(iii) $\frac{36.12 \times 750.9}{113.2 \times 9.98}$

Solution: (i) $\sqrt[3]{68.24}$

$\sqrt[3]{68.24} = 68.24^{1/3}$

We now need to calculate the logarithm of 68.24 and divide it by 3, since:

$$\log(68.24^{1/3}) = \frac{1}{3}\log 68.24$$

Using the logarithmic table:

- $\log 68.24 \approx 1.833$

Now, calculate:

$$\log(\sqrt[3]{68.24}) = \frac{1}{3} \times 1.833 = 0.611$$

Now, we take the antilog of this value to find the result:

$$\text{Antilog}(0.611) \approx 4.07$$

So:

$$\sqrt[3]{68.24} \approx 4.07$$

(ii) $\mathbf{319.8 \times 3.543}$

$$= \log(319.8 \times 3.543) = \log 319.8 + \log 3.543$$

$$= 2.505 + 0.549 = 3.054$$

Now, take the antilog of 3.054:

$$\text{Antilog}(3.054) \approx 1136.91$$

So:

$$319.8 \times 3.543 \approx 1136.91$$

(iii) $\mathbf{\frac{36.12 \times 750.9}{113.2 \times 9.98}}$

$$\log\left(\frac{36.12 \times 750.9}{113.2 \times 9.98}\right)$$

$$= \log 36.12 + \log 750.9 - \log 113.2 - \log 9.98$$

$$= 1.558 + 2.876 - 2.053 - 0.999$$

$$= 1.382$$

Finally, take the antilog of 1.382:

$$\text{Antilog}(1.382) \approx 24.1$$

So:

$$\frac{36.12 \times 750.9}{113.2 \times 9.98} \approx 24.1$$

10. In the year 2016, the population of a city was 22 millions and was growing at a rate of 2.5% per year. The function $p(t) = 22(1.025)^t$ gives the population in millions, t years after 2016. Use the model to determine in which year the population will reach 35 millions. Round to the nearest year.

Solution: To solve this problem, we will use the population growth model given by the equation:

$$p(t) = 22 \times (1.025)^t$$

We are asked to determine the year in which the population will reach 35 million. We will solve for t when $p(t) = 35$.

We want to find when the population reaches 35 million, so we set $p(t) = 35$:

$$35 = 22 \times (1.025)^t$$

$$\frac{35}{22} = (1.025)^t$$

$$1.5909 = (1.025)^t$$

Take the Natural logarithm (ln) on both sides:

$$\ln(1.5909) = \ln(1.025^t)$$

using logarithm properties:

$$\ln(1.5909) = t \ln(1.025)$$

$$0.4647 = t \times 0.0247$$

$$t = \frac{0.4647}{0.0247}$$

$$t \approx 18.82$$

Since t represents years after 2016:

$$2016 + 19 = 2035$$

Thus, the population will reach 35 million in 2035.