UNIT 3 Sets and Functions

Students' learning outcomes

At the end of the unit, the students will be able to:

- Recall:
- Describe mathematics as the study of patterns, structure and relationships.
- Identify sets and apply operations on three sets (Subsets overlapping sets and disjoint sets), using Venn diagrams.
- Solve problems on classification and cataloguing by using Venn diagrams for scenarios involving two sets and three sets. Further application of sets.
- Verify and apply properties/laws of union and intersection of three sets through analytical and Venn diagram methods.
- Apply concepts from set theory to real-world problems (such as in demographic classification, categorizing products in shopping malls)
- Explain product, binary relations and its domain and range.
- Recognize that a relation can be represented by a table, ordered pair and graphs.
- Recognize notation and determine the value of a function and its domain and range.
- Identify types of functions (into, onto, one-to-one, injective, surjective and bijective) by using Venn diagrams.

N = The set of natural numbers $= \{1,2,3,\ldots\}$

W = The set of whole numbers $= \{0,1,2,\ldots\}$

Z = The set of integers $= \{0, \pm1, \pm2,\ldots\}$

O = The set of odd integers $= \{\pm1,\pm3,\pm5,\ldots\}$

E = The set of even integers $= \{0, \pm2, \pm4,\ldots\}$

P = The set of prime numbers $= \{2,3,5,7,11,13,17,\ldots\}$

Q = The set of all rational numbers $= \left\{x \mid x = \dfrac{p}{q} \text{ where } p,q \in Z \text{ and } q \neq 0\right\}$

Q' = The set of all irrational numbers $= \left\{x \mid x \neq \dfrac{p}{q}, \text{where } p,q \in Z \text{ and } q \neq 0\right\}$

R = The set of all real numbers $= Q \cup Q'$

A set with only one element is called a **singleton set.** For example, {3}, {a}, and {Saturday} are singleton sets. The set with no elements (zero number of elements) is called an **empty set, null set,** or **Void set.** The empty set is denoted by the symbol $\varnothing$ or { }.

Remember!
The set {0} is a singleton set having zero as its only element, and not the empty set.

Equal sets: Two sets A and B are equal if they have exactly the same elements or if every element of set A is an element of set B. If two sets A and B are equal, we write $A=B$. Thus, the sets {1,2,3} and {2, 1, 3} are equal.

Equivalent sets: Two sets A and B are equivalent if they have the same number of elements. For example, if $A = \{a, b, c, d, e\}$ and $B = \{1, 2, 3, 4, 5\}$, then A and B are equivalent sets. The symbol ~ is used to represent equivalent sets. Thus, we can write $A \sim B$.

Subset: If every element of a set A is an element of set B Symbolically this is written as $A \subseteq B$ (A is a subset of B). In such a case, we say B is a superset of A. Symbolically this is written as:

$B \supseteq A$ (B is a superset of A).

Remember!
The subset of a set can also be stated as follows: $A \subseteq B$ iff $\forall\ x \in A \Rightarrow x \in B$

Proper subset: If A is a subset of B and B contains at least one element that is not an element of A, then A is said to be a proper subset of B. In such a case, we write:

$A \subset B$ (A is a proper subset of B).

Improper subset: If A is a subset of B and $A = B$, then we say that A is an improper subset of B. From this definition, it also follows that every set A is a subset of itself and is called an improper subset. For example, let $A = \{a, b, c\}$, $B = \{c, a, b\}$ and $C = \{a, b, c, d\}$, then clearly

$$A \subset C,\ B \subset C \text{ but } A = B.$$

Remember!

When we do not want to distinguish between proper and improper subsets, we may use the symbol $\subseteq$ for the relationship. It is easy to see that: $N \subset W \subset Z \subset O \subset R$

Notice that each of sets A and B is an improper subset of the other because $A = B$.

Universal set: The set that contains all objects or elements under consideration is called the universal set or the universe of discourse. It is denoted by U.

Power set: The power set of a set S denoted by $P(S)$ is the set containing all the possible subsets of S. For Example:

(i) If $C = \{a, b, c, d\}$, then

$P(C) = \{ \varnothing, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, \{a, b, c, d\}\}$.

(ii) If $D = \{a\}$, then $P(D) = \{ \varnothing, \{a\}\}$

If S is a finite set with $n(S) = m$ representing the number of elements of the set 5, then $n\{P(S)\} = 2^m$ is the number of the elements of the power set.

EXERCISE 3.1

1. Write the following sets in set builder notation:

(i) $\{1, 4, 9, 16, 25, 36, \ldots, 484\}$

(ii) $\{2, 4, 8, 16, 32, 64, \ldots, 150\}$

(iii) $\{0, \pm 1, \pm 2, \ldots, \pm 1000\}$

(iv) $\{6, 12, 18, \ldots, 120\}$

(v) $\{100, 102, 104, \ldots, 400\}$

(vi) $\{1, 3, 9, 27, 81, \ldots\}$

(vii) $\{1, 2, 4, 5, 10, 20, 25, 50, 100\}$

(viii) $\{5, 10, 15, \ldots, 100\}$

(ix) The set of all integers between -100 and 1000.

Solution: Here are the sets written in **set-builder notation:**

(i) $\{1, 4, 9, 16, 25, 36, \ldots, 484\}$

This is the set of all perfect squares up to 484.

$$A = \{x \mid x = n^2, n \in \mathbb{N} \wedge 1 \leq n \leq 22\}$$

(ii) $\{2, 4, 8, 16, 32, 64, \ldots, 150\}$

This is the set of all powers of 2 less than or equal to 150.

$$B = \{x \mid x = 2^n, n \in \mathbb{N} \wedge 1 \leq 2^n \leq 150\}$$

(iii) $\{0, +1, +2, \ldots, +1000\}$

This is the set of all integers from 0 to 1000.

$$C = \{x \mid x \in \mathbb{Z} \wedge 0 \leq x \leq 1000\}$$

(iv) $\{6, 12, 18, \ldots, 120\}$

This is the set of all multiples of 6 up to 120.

$$D = \{x \mid x = 6n, n \in \mathbb{N} \wedge 1 \leq n \leq 20\}$$

(v) $\{100, 102, 104, \ldots, 400\}$

This is the set of all even numbers between 100 and 400, inclusive.

$$E = \{x \mid x = 100 + 2, n \in \mathbb{N} \wedge 0 \leq n \leq 150\}$$

(vi) $\{1, 3, 9, 27, 81, \ldots\}$

This is the set of all powers of 3.

$$F = \{x \mid x = 3^n, n \in W\}$$

(vii) $\{1, 2, 4, 5, 10, 20, 25, 50, 100\}$

This is the set of all divisors of 100.

$$G = \{x \mid x \text{ is a divisor of } 100\}$$

(viii) $\{5, 10, 15, \ldots, 100\}$

This is the set of all multiples of 5 up to 100.

$$H = \{x \mid x = 5n, n \in \mathbb{N} \wedge 1 \leq n \leq 20\}$$

(ix) The set of all integers between –100 and 1000

$$I = \{x \mid x \in \mathbb{Z} \wedge -100 < x < 1000\}$$

2. Write each of the following sets in tabular forms:

(i) $\{x \mid x \text{ is a multiple of } 3 \wedge x \leq 35\}$

(ii) $\{x \mid x \in R \wedge 2x + 1 = 0\}$

(iii) $\{x \mid x \in P \wedge x < 12\}$

(iv) $\{x \mid x \text{ is a divisor of } 128\}$

(v) $\{x \mid x = 2^n, n \in N \wedge n < 8\}$

(vi) $\{x \mid x \in N \wedge x + 4 = 0\}$

(vii) $\{x \mid xN \wedge x = x\}$

(viii) $\{x \mid x \in Z \wedge 3x + 1 = 0\}$

(i) $\{x \mid x \text{ is a multiple of } 3 \wedge x < 35\}$

Sol. This is the set of multiples of 3 less than 35:

$$\{3,6,9,12,15,18,21,24,27,30,33\}$$

(ii) $\{x \mid x \in \mathbb{R} \wedge 2x + 1 = 0\}$

Sol. Solve for x from the equation:

$$2x + 1 = 0 \quad \Rightarrow \quad 2x = -1 \quad \Rightarrow \quad x = -\frac{1}{2}$$

Thus, the set is:

$$\left\{-\frac{1}{2}\right\}$$

(iii) $\{x \mid x \in \mathbb{P} \wedge x < 12\}$

This is the set of prime numbers less than 12:

$$\{2,3,5,7,11\}$$

(iv) $\{x \mid x \text{ is a divisor of } 128\}$

The divisors of 128 are:

$$\{1,2,4,8,16,32,64,128\}$$

(v) $\{x \mid x = 2^n, n \in \mathbb{N} \wedge n < 8\}$

This is the set of powers of 2 where n is a natural number less than 8:

$$\{2,4,8,16,32,64,128\}$$

(vi) $\{x \mid x \in \mathbb{N} \wedge x + 4 = 0\}$

This is an impossible equation for natural numbers, as no natural number x satisfies $x + 4 = 0$. $x = -4$, $-4 \notin N$. Hence, the set is $\{\ \}$.

(vii) $\{x \mid x \in \mathbb{N} \wedge x = x\}$

This set is simply the set of all natural numbers:

$$\mathbb{N} = \{1,2,3,4,5,\dots\}$$

(viii) $\{x \mid x \in \mathbb{Z} \wedge 3x + 1 = 0\}$

Solve for x from the equation:

$$3x + 1 = 0 \quad \Rightarrow \quad 3x = -1 \quad \Rightarrow \quad x = -\frac{1}{3}$$

Since $x = -\frac{1}{3}$ is not an integer $x \notin \mathbb{Z}$, the set is: $\{\ \}$.

3. **Write two proper subsets of each of the following sets:**

(i) $\{a, b, c\}$ (ii) $\{0, 1\}$ (iii) N

(iv) Z (v) Q (vi) R

(vii) $\{x \mid x \in Q \wedge 0 < x \le 2\}$

Solution: Here are two proper subsets for each of the given sets:

(i) $\{a, b, c\}$

Proper subsets are subsets of the given set that are not equal to the original set. Two proper subsets:

$$\{a,b\}, \{b,c\}$$

(ii) $\{0, 1\}$

Two proper subsets:

$$\{0\}, \{1\}$$

(iii) $\mathbb{N}$ **(Set of natural numbers)**

Two proper subsets:

$$\{1,2,3\}, \{5,10,15\}$$

(iv) $\mathbb{Z}$ **(Set of integers)**

Two proper subsets:

$$\{-1,0,1\}, \{-10,-5,5\}$$

(v) $\mathbb{Q}$ **(Set of rational numbers)**

Two proper subsets:

$$\left\{\frac{1}{2}, \frac{2}{3}, 1\right\}, \left\{-1, 0, \frac{3}{4}\right\}$$

(vi) $\mathbb{R}$ **(Set of real numbers)**

Two proper subsets:

$$\{x \in \mathbb{R} \mid x > 0\}, \{x \in \mathbb{R} \mid x < 1\}$$

(vii) $\{x \mid x \in \mathbb{Q}, 0 < x \le 2\}$ **(Set of rational numbers between 0 and 2, inclusive)**

Two proper subsets:

$$\left\{\frac{1}{2}, 1\right\}, \left\{\frac{3}{4}, 2\right\}$$

4. **Is there any set which has no proper subset? If so, name that set.**

Solution: Yes, there is a set that has no proper subset: the empty set, denoted by $\varnothing$ or {}.

Explanation:

- A **proper subset** of a set A is any subset of A that is not equal to A itself.
- The empty set $\varnothing$ is a subset of every set, including itself.
- However, $\varnothing$ does not contain any elements, so it cannot have a subset that is different from itself.
- Thus, the empty set has no proper subsets.

5. **What is the difference between $\{a, b\}$ and $\{\{a, b\}\}$?**

Solution: The sets $\{a, b\}$ and $\{\{a, b\}\}$ are fundamentally different because they contain different elements.

Explanation:

- Set $\{a, b\}$:
  - This set contains two elements: a and b.
  - Example: If $a = 1$ and $b = 2$, then $\{a, b\} = \{1,2\}$.
- Set $\{\{a, b\}\}$:
  - This set contains only one element, which is itself a set: $\{a, b\}$.
  - Example: If $a = 1$ and $b = 2$, then $\{\{a, b\}\} = \{\{1,2\}\}$.

6. **What is the number of elements of the power set of each of the following sets?**

(i)	{ }	(ii)	{0, 1}
(iii)	{1, 2, 3, 4, 5, 6, 7}	(iv)	{0, 1, 2, 3, 4, 5, 6, 7}
(v)	{a, {b, c}}	(vi)	$\{\{a, b\}, \{b, c\}, \{d, e\}\}$

Solution:

(i) {}:

- The empty set has $n = 0$ elements.
- The power set has $2^0 = 1$ element.
- Power set: P (1).

Answer: 1 element.

(ii) $\{0, 1\}$:

- The set has $n = 2$ elements.
- The power set has $2^2 = 4$ elements.
- Power set: {{}, {0}, {1}, {0,1}}.

Answer: 4 elements.

(iii) $\{1, 2, 3, 4, 5, 6, 7\}$:

- The set has $n = 7$ elements.
- The power set has $2^7 = 128$ elements.

Answer: 128 elements.

(iv) $\{0, 1, 2, 3, 4, 5, 6, 7\}$:

- The set has $n = 8$ elements.
- The power set has $2^8 = 256$ elements.

Answer: 256 elements.

(v) $\{a, \{b, c\}\}$:

- The set has $n = 2$ elements (a and $\{b, c\}$ are distinct elements).
- The power set has $2^2 = 4$ elements.
- Power set: $\{\{\}, \{a\}, \{\{b, c\}\}, \{a, \{b, c\}\}\}$.

Answer: 4 elements.

(vi) $\{\{a, b\}, \{b, c\}, \{d, e\}\}$:

- The set has $n = 3$ elements ($\{a, b\}$, $\{b, c\}$, and $\{d, e\}$ are distinct elements).
- The power set has $2^3 = 8$ elements.
- Power set:

$\{\{\}, \{\{a, b\}\}, \{\{b, c\}\}, \{\{d, e\}\}, \{\{a, b\}, \{b, c\}\}, \{\{a, b\}, \{d, e\}\},$
- $\{\{b, c\}, \{d, e\}\}, \{\{a, b\}, \{b, c\}, \{d, e\}\}\}$.

Answer: 8 elements.

7. **Write down the power set of each of the following sets:**

(i)	{9, 11}	(ii)	{+, – ×, ÷}
(iii)	$\{\phi\}$	(iv)	{a, {b, c}}

Solution:

(i) $\{9, 11\}$:

The set has 2 elements: 9 and 11. The power set contains $2^2 = 4$ subsets.

Power set:

{{}, {9}, {11}, {9,11}}

(ii) $\{+,-,\times,\div\}$:

The set has 4 elements: $+,-,\times,\div$. The power set contains $2^4 = 16$ subsets.

Power set:

$\{\{\},\{+\},\{-\},\{\times\},\{\div\},\{+,-\},\{+,\times\},\{+,\div\},\{-,\times\},\{-,\div\},\{\times\div\},$
$\{+,-,\times\},\{+,-,\div\},\{+,\times,\div\},\{-,\times,\div\},\{+,-,\times,\div\}\}$

(iii) $\{\}$ (empty set):

The set has no elements. The power set contains $2^0 = 1$ subset.

Power set:

$\{\{\}\}$

(iv) $\{a,\{b,c\}\}$:

The set has 2 elements: a and $\{b,c\}$. The power set contains $2^2 = 4$ subsets.

Power set: $\{\{\},\{a\},\{\{b,c\}\},\{a,\{b,c\}\}\}$

Properties of union and intersection

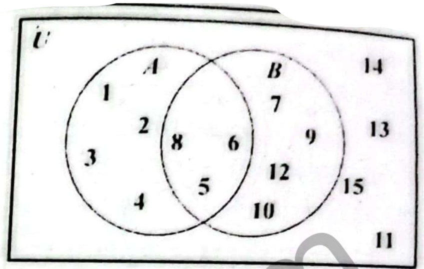
We now state the fundamental properties of union and intersection of two or three sets.

Properties

(i) $A \cup B = B \cup A$ (Commutative property of Union)
(ii) $A \cap B = B \cap A$ (Commutative property of Intersection)
(iii) $A \cup (B \cup C) = (A \cup B) \cup C$ (Associative property of Union)
(iv) $A \cap (B \cap C) = (A \cap B) \cap C$ (Associative property of Intersection)
(v) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ (Distributivity of Union over intersection)
(vi) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ (Distributivity of intersection over Union)
(vii) $(A \cup B)' = A' \cap B'$
(viii) $(A \cap B)' = A' \cup B'$ (De Morgan's Laws)

Example 1: Consider the adjacent Venn diagram illustrating two non-empty sets, A and B.

(a) Determine the number of elements common to sets A and B.
(b) Identify all the elements exclusively in set 5 and not in set A.
(c) Calculate the union of sets A and B.



Solution: From the information provided in the Venn diagram, we have: Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$
$A = \{1,2, 3, 4, 5, 6,8\}$
$B = \{5, 6, 7, 8, 9, 10, 12\}$

(a) The elements in both sets A and B are the intersection of the sets: $A \cap B = \{5, 6, 8\}$

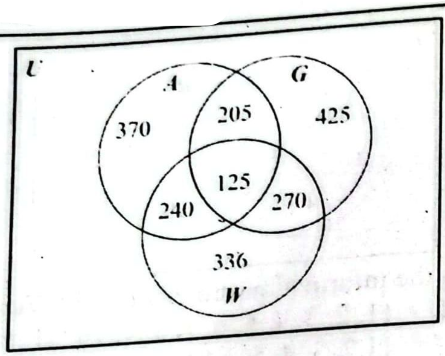
(b) The elements that are only in set B, not in set A, is the sets' differences. $B - A = \{7, 9, 10, 12\}$

(c) $A \cup B = \{1, 2, 3, 4, 5, 6, 8\} \cup \{5, 6, 7, 8, 9, 10, 12\}$
$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12\}$

Example 2: Consider the adjacent Venn diagram representing the students enrolled in different courses in an IT institution.

U = {Students enrolled in IT institutions}
A = {Students enrolled in an Applied Robotics}
G = {Students enrolled in a Game Development}
W = {Students enrolled in a Web Designing}

(a) How many students enrolled in the applied Robotics course?
(b) Determine the total number of Students enrolled in a Game Development.
(c) How many students are enrolled in the Game development and Web designing course?
(d) Identify the students enrolled in Web development but not Applied Robotics,
(e) How many students are enrolled in IT institutions?
(f) How many students enrolled in all three courses?



Solution:

(a) Set A represents the total number of students enrolled in the Applied Robotics program.

$$\text{Total} = 370 + 205 + 125 + 240 = 940$$

So, the total number of students in the Applied Robotics course is 940.

(b) The total number of students enrolled in a Game Development is represented by the set G.

$$\text{Total} = 205 + 125 + 270 + 425 = 1025$$

Thus, the Students enrolled in a Game Development is 1025

(c) Total students are enrolled in both the Game development and Web designing. The course is the intersection of G and W.

$$G \cap W = 125 + 270 = 395$$

Therefore, 395 students are enrolled in both the Game development and Web designing Course.

(d) The students who are enrolled in Web development but not in Applied Robotics is the sum of values 336 and 270 in set W.

$\text{Total} = 336 + 270 = 606$

So, there are 606 students who enrolled in Web development courses but not in Applied Robotics.

(e) The total number of students enrolled in all three courses is represented by all the values inside the circles.

$$\text{Total} = 370 + 205 + 125 + 240 + 425 + 270 + 336 = 1971$$

There are a total of 1971 students enrolled in IT Institutions.

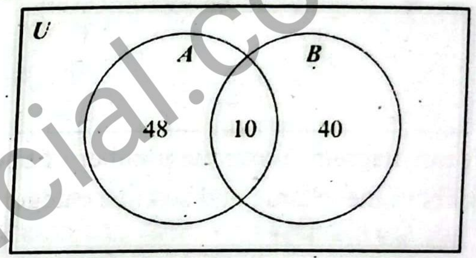
(f) The students who enrolled in all three courses are the intersection of all the circles are represented by the value 125.

Example 3: There are 98 secondary school students in a sports club. 58 students join the swimming club, and 50 join the tug-of-war club. How many students participated in both games?

Solution: Let U = {total student in a sports club of school}

A = {students who participated in swimming club}

B = {students who participated in tug-of-war club} From the statement of problems, we have $n(U) = n(A \cup B) = 98$, $n(A) = 58$, $n(B) = 50$.



We want to find the total number of students who participated in both clubs.

$n(A \cap B) = ?$

Using the principles of inclusion and exclusion for two sets:

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$\Rightarrow n(A \cap B) = n(A) + n(B) - n(A \cup B)$

$= 58 + 50 - 98$

$= 10$

Thus, 10 students participated in both clubs.

The adjacent Venn diagram shows the number of students in each sports club.

Example 4: Mr. Saleem, a school teacher, has a small library in his house containing 150 books. He has two main categories for these books: islamic and science. He categorized 70 books as islamic books and 90 books as science books. There are 15 books that neither belong to the islamic nor science books category. How many books are classified under both the islamic and science categories?

From the statement of problems, we have

$n(U) = 130$, $n(G) = 57$, $n(C) = 50$, $n(E) = 46$, $n(G \cap C) = 31$, $n(G \cap E) = 25$, $n(C \cap E) = 21$ and $n(G \cap C \cap E) = 12$.

(a) We want to find the total number of customers who have bought at least one of the products: garments, cosmetics, or electronics.

We are to find $n(G \cup C \cup E)$.

Using the principle of inclusion and exclusion for three sets:

$n(G \cup C \cup E) = n(G) + n(C) + n(E) - n(G \cap C) - n(G \cap E) - n(C \cap E) + n(G \cap C \cap E)$

$= 57 + 50 + 46 - 31 - 25 - 21 + 12 = 88$

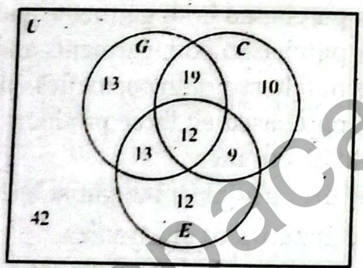
Thus, 88 customers bought at least one of the products: garments, cosmetics, or electronics.

(b) Customers who bought only garments.

$= n(G) - n(G \cap C) - n(G \cap E) + n(G \cap C \cap E)$

$= 57 - 31 - 25 + 12$

$= 13$



Customers who bought only cosmetics

$= n(C) - n(G \cap C) - n(C \cap E) + n(G \cap C \cap E)$

$= 50 - 31 - 21 + 12 = 10$

Customers who bought only electronics

$= n(E) - n(G \cap E) - n(C \cap E) + n(G \cap C \cap E)$

$= 46 - 25 - 21 + 12 = 12$

Therefore, the customers bought only one of the products: garments, cosmetics, or electronics $= 13 + 10 + 12 = 35$

(c) Since the total number of Customers surveyed was 130, and 88 customers bought at least one of the products: garments, cosmetics, or electronics. The customers who did not buy any of the three products can be calculated as:

$n(G \cup C \cup E)' = n(U) - n(G \cup C \cup E)$

$= 130 - 88 = 42$

So, 42 customers did not buy any of the three products.

EXERCISE 3.2

1. Consider the universal set $U = \{x : x$ is multiple of 2 and $0 < x \leq 30\}$, $A = \{x : x$ is a multiple of 6$\}$ and $B = \{x : x$ is a multiple of 8$\}$

(i) List all elements of sets A and B in tabular form

(ii) Find $A \cap B$ (iii) Draw a Venn diagram

Solution:

Given:

- Universal set $U = \{2,4,6,8,10,12,14,16,18,20,22,24,26,28\}$
- $A = \{x: x$ is a multiple of 6$\}$
- $B = \{x: x$ is a multiple of 8$\}$

(i) List all elements of A and B:

ELEMENTS OF A:

- Multiples of 6 in U: 6,12,18,24.
- $n(A) = \{6,12,18,24\}$

ELEMENTS OF B:

- Multiples of 8 in U: 8,16,24.
- $B = \{8,16,24\}$

TABULAR REPRESENTATION:

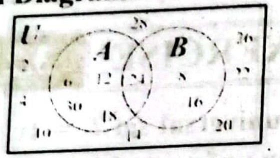
Set	Elements
A	$\{6,12,18,24\}$
B	$\{8,16,24\}$

(ii) Find $A \cap B$:

The intersection of A and B is the set of elements common to both A and B.

$$A \cap B = ?$$
$$A \cap B = \{6, 12, 18, 24, 30\} \cap \{8, 16, 24\}$$
$$= \{24\}$$

(iii) Draw a Venn Diagram:



2. Let, $U = \{x : x \text{ is an integer and } 0 < x \leq 150\}$, $G = \{x : x = 2^m \text{ for integer } m \text{ and } 0 \leq m \leq 12\}$ and $H = \{x : x \text{ is a square}\}$

(i) List all elements of sets G and H in tabular form

(ii) Find $G \cup H$ (iii) Find $G \cap H$

Sol. (i) Tabular Form

$$U = \{1, 2, 3, 4, \ldots, 150\}$$
$$G = \{1, 2, 4, 8, 16, 32, 64, 128\}$$
$$H = \{1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144\}$$

(ii) Find $G \cup H$:

$G \cup H = \{1, 2, 4, 8, 16, 32, 64, 128\} \cup \{1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144\}$

$G \cup H = \{1, 2, 4, 8, 9, 16, 25, 32, 36, 49, 64, 81, 100, 121, 128, 144\}$

(iii) Find $G \cap H$:

Sol. $G \cap H = \{1, 2, 4, 8, 16, 32, 64, 128\} \cap \{1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144\}$

$G \cap H = \{1, 4, 16, 64\}$

3. Consider the sets $P = \{x : x \text{ is a prime number and } 0 < x \leq 20\}$ and $Q = \{x : x \text{ is a divisor of 210 and } 0 < x < 20\}$

(i) Find $P \cap Q$ (ii) Find $P \cup Q$

Solution: We are working with the sets:

- $P = \{x : x \text{ is a prime number and } 0 < x \leq 20\}$
- $Q = \{x : x \text{ is a divisor of 210 and } 0 < x < 20\}$

$$P = \{2,3,5,7,11,13,17,19\}$$
$$Q = \{1,2,3,5,6,7,10,14,15\}$$

(i) Find $P \cap Q$:

The intersection $P \cap Q$ is the set of elements common to both P and Q.

$$P \cap Q = \{2,3,5,7,11,13,17,19\} \cap \{1,2,3,5,6,7,10,14,15\}$$
$$P \cap Q = \{2,3,5,7\}$$

(ii) Find $P \cup Q$:

The union $P \cup Q$ is the set of elements that belong to either P, Q, or both.

$$P \cup Q = \{2,3,5,7,11,13,17,19\} \cup \{1,2,3,5,6,7,10,14,15\}$$
$$P \cup Q = \{1,2,3,5,6,7,10,11,13,14,15,17,19\}$$

4. Verify the commutative properties of union and intersection for the following pairs of sets:

(i) $A = \{1, 2, 3, 4, 5\}$, $B = \{4, 6, 8, 10\}$

(ii) N, Z (iii) $A = \{x \mid x \in R \wedge x\}$, $B = R$

Solution: **The commutative properties of union and intersection state that:**

- **Union:** $A \cup B = B \cup A$
- **Intersection:** $A \cap B = B \cap A$

We will verify these properties for the given pairs of sets.

(i) $A = \{1, 2, 3, 4, 5\}$, $B = \{4, 6, 8, 10\}$

VERIFY UNION:

$$A \cup B = \{1,2,3,4,5\} \cup \{4,6,8,10\} = \{1,2,3,4,5,6,8,10\}$$
$$B \cup A = \{4,6,8,10\} \cup \{1,2,3,4,5\} = \{1,2,3,4,5,6,8,10\}$$

$\therefore A \cup B = B \cup A.$

VERIFY INTERSECTION:

$$A \cap B = \{1,2,3,4,5\} \cap \{4,6,8,10\} = \{4\}$$
$$B \cap A = \{4,6,8,10\} \cap \{1,2,3,4,5\} = \{4\}$$

$\therefore A \cap B = B \cap A.$

(ii) $A = \mathbb{N}$ **(natural numbers),** $B = \mathbb{Z}$ **(integers)**

VERIFY UNION:

$$A \cup B = \mathbb{N} \cup \mathbb{Z} = \mathbb{Z}$$
$$B \cup A = \mathbb{Z} \cup \mathbb{N} = \mathbb{Z}$$

$\therefore A \cup B = B \cup A.$

VERIFY INTERSECTION:

$$A \cap B = \mathbb{N} \cap \mathbb{Z} = \mathbb{N}$$
$$B \cap A = \mathbb{Z} \cap \mathbb{N} = \mathbb{N}$$

$\therefore A \cap B = B \cap A.$

(iii) $A = \{x \mid x \in \mathbb{R}$ **and** $x \geq 0\}, B = \mathbb{R}$

NOTE ON A**:**

The set $A = \{x \mid x \in \mathbb{R}$ and $x \geq 0\}$

then, if $A = \mathbb{R}$:

- Union: $A \cup B = \mathbb{R} \cup \mathbb{R} = \mathbb{R}$
- Intersection: $A \cap B = \mathbb{R} \cap \mathbb{R} = \mathbb{R}$.

$\therefore\ A \cup B = B \cup A$ and $A \cap B = B \cap A$

Q.5. **Let** $U = \{a, b, c, d, e, f, g, h, i, j\}$
$A = \{a, b, c, d, g, h\}, B = \{c, d, e, f, j\}$
Verify De Morgan's Laws for these sets. Draw Venn diagram.

Sol. $U = \{a, b, c, d, e, f, g, h, i, j\}$
$A = \{a, b, c, d, g, h\}$
$B = \{c, d, e, f, j\}$

De Morgan's Laws

(i) $(A \cup B)' = A' \cap B'$ (ii) $(A \cap B)' = A' \cup B'$

$A' = U - A$
$= \{a, b, c, d, e, f, g, h, i, j\} - \{a, b, c, d, g, h\}$
$= \{e, f, i, j\}$

$B' = U - B$
$= \{a, b, c, d, e, f, g, h, i, j\} - \{c, d, e, f, j\}$
$= \{a, b, c, d, e, f, g, h, j\}$

$A \cup B = \{a, b, c, d, g, h\} \cup \{c, d, e, f, j\}$
$= \{a, b, c, d, e, f, g, h, j\}$

$A \cap B = \{a, b, c, d, g, h\} \cap \{c, d, e, f, j\}$
$= \{c, d\}$

(i) $(A \cup B)' = A' \cap B'$

$(A \cup B)' = U - A \cup B$
$= \{a, b, c, d, e, f, g, h, i, j\} - \{a, b, c, d, e, f, g, h, j\}$
$= \{i\}$

$A' \cap B' = \{e, f, i, j\} \cap \{a, b, g, h, i\}$
$= \{i\}$

$\therefore\ (A \cup B)' = A' \cap B'$

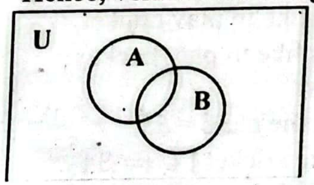
(ii) $(A \cap B)' = A' \cup B'$

$(A \cap B)' = U - A \cap B$
$= \{a, b, c, d, e, f, g, h, i, j\} - \{c, d\}$
$= \{a, b, c, d, e, f, g, h, i, j\}$

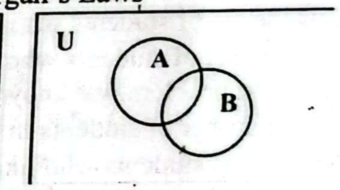
$A' \cup B' = \{e, f, i, j\} \cup \{a, b, c, d, e, f, g, h, j\}$
$= \{a, b, c, d, e, f, g, h, i, j\}$

$\therefore\ (A \cap B)' = A' \cup B'$

Hence, verified De Morgan's Laws



$(A \cup B)' = A' \cap B'$



$(A \cap B)' = A' \cup B'$

Q.6. **If** $U = \{1, 2, 3, \ldots, 20\}$ **and** $A = \{1, 3, 5, \ldots, 19\}$**, verify the following:**

(i) $A \cup A' = U$ **(ii)** $A \cap U = A$ **(iii)** $A \cap A' = \phi$

(i) $A \cup A' = U$

Sol. $A = U - A$
$= \{1, 2, 3, \ldots, 20\} - \{1, 3, 5, \ldots, 19\}$
$= \{2, 4, 6, \ldots, 20\}$

$A \cup A' = \{1, 3, 5, \ldots, 19\} \cup \{2, 4, 6, \ldots, 20\}$
$= \{1, 2, 3, \ldots, 20\}$
$= U$

Hence, proved that $A \cup A' = U$

(ii) $A \cap U = A$

Sol. $A \cap U = \{1, 3, 5, \ldots, 19\} \cap \{1, 2, 3, \ldots, 20\}$
$= \{1, 3, 5, \ldots, 19\}$
$= A$

Hence, proved that $A \cap U = A$

(iii) $A \cap A' = \phi$

Sol. $A' = U - A$
$= \{1, 2, 3, \ldots, 20\} - \{1, 3, 5, \ldots, 20\}$
$= \{2, 4, 6, \ldots, 20\}$
$A \cap A' = \{1, 3, 5, \ldots, 19\} \cap \{2, 4, 6, \ldots, 20\}$
$= \{\ \} = \phi$

Hence, proved that $A \cap A' = \phi$

7. In a class of 55 students, 34 like to play cricket and 30 like to play hockey. Also each students likes to play at least one of the two games. How many students like to play both games?

Solution: We can solve this problem using the principle of inclusion-exclusion. Let's define:

- C as the set of students who like to play cricket,
- H as the set of students who like to play hockey.

From the problem, we know:

- Total number of students in the class = 55
- Number of students who like cricket $|C| = 34$
- Number of students who like hockey $|H| = 30$
- Every student likes at least one of the two games, so $|C \cup H| = 55$.

The formula for the union of two sets is:

$$|C \cup H| = |C| + |H| - |C \cap H|$$

Where $|C \cap H|$ represents the number of students who like both cricket and hockey.

Substitute the known values into the formula:

$$55 = 34 + 30 - |C \cap H|$$

Simplifying:

$$55 = 64 - |C \cap H|$$

Solving for $|C \cap H|$:

$$|C \cap H| = 64 - 55 = 9$$

So, 9 students like to play both cricket and hockey.

8. In a group of 500 employees, 250 can speak Urdu, 150 can speak English, 50 can speak Punjabi, 40 can speak Urdu and English, 30 can speak both English and Punjabi and 10 can speak both Urdu and Punjabi. How many can speak all three languages?

Solution: To solve this problem, we can use the principle of inclusion-exclusion. Let's define the following:

- U as the set of employees who can speak Urdu,
- E as the set of employees who can speak English,
- P as the set of employees who can speak Punjabi.

We are given the following information:

- $|U| = 250$ (employees who can speak Urdu),
- $|E| = 150$ (employees who can speak English),
- $|P| = 50$ (employees who can speak Punjabi),
- $|U \cap E| = 40$ (employees who can speak both Urdu and English),
- $|E \cap P| = 30$ (employees who can speak both English and Punjabi),
- $|U \cap P| = 10$ (employees who can speak both Urdu and Punjabi).

We need to find $|U \cap E \cap P|$, the number of employees who can speak all three languages. We can use the inclusion-exclusion formula for three sets:

$$|U \cup E \cup P| = |U| + |E| + |P| - |U \cap E| - |E \cap P| - |U \cap P| + |U \cap E \cap P|$$

Since there are 500 employees in total, we know that:

$$|U \cup E \cup P| = 500$$

Now substitute the known values into the formula:

$$500 = 250 + 150 + 50 - 40 - 30 - 10 + |U \cap E \cap P|$$

Simplifying:

$$500 = 450 - 80 + |U \cap E \cap P|$$
$$500 = 370 + |U \cap E \cap P|$$

Solving for $|U \cap E \cap P|$:

$$|U \cap E \cap P| = 500 - 370 = 130$$

Thus, **130 employees** can speak all three languages.

9. In sports events, 19 people wear blue shirts, 15 wear green shirts, 3 wear blue and green shirts, 4 wear a cap and blue shirts, and 2 wear a cap and green shirts. The total number of people with either a blue or green shirt or cap is 27. How many people are wearing caps?

Solution:

Let A, B and C be three showing people who wear green, blue shirts and cap respectively.

15 wear green shirts $= n(A) = 15$

19 wear blue shirts $= n(B) = 19$

Let x wear caps $= n(c) = x$

3 wear blue and green shirts $= n(A \cap B) = 3$

4 wear cap and blue shirts $= n(B \cap C) = 4$

2 wear cap and green shirts $= n(A \cap C) = 2$

34 wear either blue, green or cap

$$n(A \cup B \cup C) = 34$$

According to information no one wear all the three items,

So, $n(A \cap B \cap C) = 0$

Using principle of inclusion and exclusion:

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cup B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

$34 = 15 + 19 + x - 3 - 4 - 2 + 0$

$34 = 34 + x - 9$

$\cancel{34} - \cancel{34} + 9 = x$

$9 = x$

$\Rightarrow \boxed{x = 9}$

Thus 9 people are wearing cap.

10. In a training session, 17 participants have laptops, 11 have tablets, 9 have laptops and tablets, 6 have laptops and books, and 4 have both tablets and books. Eight participants have all three items. The total number of participants with laptops, tablets, or books is 35. How many participants have books?

Sol: Let's define the sets:

- L represents the set of participants with laptops.
- T represents the set of participants with tablets.
- B represents the set of participants with books.

Given information:

- $|L| = 17$ (participants with laptops)
- $|T| = 11$ (participants with tablets)
- $|L \cap T| = 9$ (participants with both laptops and tablets)
- $|L \cap B| = 6$ (participants with both laptops and books)
- $|T \cap B| = 4$ (participants with both tablets and books)
- $|L \cap T \cap B| = 8$ (participants with laptops, tablets, and books)
- $|L \cup T \cup B| = 35$ (total number of participants with laptops, tablets, or books)

We need to find $|B|$, the number of participants with books.

Step 1: Use the inclusion-exclusion principle.

The inclusion-exclusion formula for three sets is:

$$|L \cup T \cup B| = |L| + |T| + |B| - |L \cap T| - |L \cap B| - |T \cap B| + |L \cap T \cap B|$$

Substitute the known values:

$$35 = 17 + 11 + |B| - 9 - 6 - 4 + 8$$

Simplifying: $35 = 36 - 19 + |B|$

$$35 = 17 + |B|$$
$$|B| = 35 - 17$$
$$|B| = 18$$

Conclusion:

The number of participants who have books is $\boxed{18}$.

11. A shopping mall has 150 employees labelled 1 to 150, representing the Universal set U. The employees fall into the following categories:

- **Set A: 40 employees with a salary range of 30k-45k, labelled from 50 to 89.**
- **Set B: 50 employees with a salary range of 50k-80k, labelled from 101 to 150.**
- **Set C: 60 employees with a salary range of 100k-150k, labelled from 1 to 49 and 90 to 100.**

(a) Find $(A' \cup B') \cap C$ (b) Find $n\{A \cap (B' \cap C')\}$

Solution:

$U = \{1, 2, 3, 4, \ldots, 150\}, n(U) = 150$

$A = \{50, 51, 52, \ldots, 89\}, n(A) = 40$

$B = \{101, 102, 103, \ldots, 150\}, n(B) = 50$

$C = \{1,2,3,4, \ldots, 49, 90, 91, 92, \ldots, 100\}, n(C) = 60$

(a) Find $(A' \cup B') \cap C$

$A' = U - A$

$= \{1,2,3,4,5,\ldots,150\} - \{50,51,52,\ldots,89\}$

$= \{1,2,3,4,\ldots, 49, 90, 91, 92, 93, \ldots, 150\}$

Now, $B' = U - B$

$= \{1, 2, 3, 4, \ldots, 150\} - \{101, 102, 103, \ldots, 150\}$

$B' = \{1, 2, 3, 4, \ldots, 100\}$

Now, $A' \cup B' = \{1, 2, 3, 4, \ldots, 49, 90, 91, 92, 93, \ldots, 150\} \cup \{1, 2, 3, 4, \ldots, 100\}$

$A' \cup B' = \{1, 2, 3, 4, \ldots, 150\}$

Now,

$(A' \cup B') \cap C = \{1, 2, 3, 4, \ldots, 150\} \cap \{1, 2, 3, 4, \ldots, 49, 90, 91, 92, \ldots, 100\}$

$(A' \cup B') \cap C = \{1, 2, 3, 4, \ldots, 49, 90, 91, 93, \ldots, 100\}$

(b) Find $n\{A \cap (B' \cap C')\}$

$B' = U - B$

$B' = \{1,2,3,4,5,\ldots,150\} - \{101,102,103,\ldots,150\}$

$= \{1,2,3,4,\ldots,100\}$

Now, $C' = U - C$

$C' = \{1,2,3,4,\ldots,150\} - \{1,2,3,4,\ldots,49,90,91,92,\ldots,150\}$

Now,

$B' \cap C' = \{50, 51, 52, \ldots, 89\} = A$

Now,

$A \cap (B' \cap C') = A \cap A$

$A \cap (B' \cap C') = A$

$n\{(A \cap (B' \cap C')\} = n(A) = 40$

12. In a secondary school with 125 students participate in at least one of the following sports: Cricket, football or hockey.

- **60 students play cricket.**
- **70 students play football.**
- **40 students play hockey.**
- **25 students play both cricket and football.**
- **15 students play both football and hockey.**
- **10 students play both cricket and hockey.**

(a) How many students play all three sports?

(b) Draw a Venn diagram showing the distribution of sports participation in all the games.

Solution: Let's solve the problem step-by-step.

Given Data:

- Total students: 125
- Students who play cricket (C): 60
- Students who play football (F): 70
- Students who play hockey (H): 40
- Students who play both cricket and football ($C \cap F$): 25
- Students who play both football and hockey ($F \cap H$): 15
- Students who play both cricket and hockey ($C \cap H$): 10

Part (a): How many students play all three sports?

We need to find the number of students who play all three sports, which is the intersection of all three sets: $C \cap F \cap H$.

We will use the principle of inclusion and exclusion to calculate the total number of students who play at least one sport.

FORMULA FOR INCLUSION-EXCLUSION:

$$n(C \cup F \cup H) = n(C) + n(F) + n(H) - n(C \cap F) - n(F \cap H) - n(C \cap H) + n(C \cap F \cap H)$$

We are given the following values:

- $n(C \cup F \cup H) = 125$ (total number of students)
- $n(C) = 60$
- $n(F) = 70$
- $n(H) = 40$
- $n(C \cap F) = 25$
- $n(F \cap H) = 15$
- $n(C \cap H) = 10$

We need to find $n(C \cap F \cap H)$, the number of students who play all three sports.

SUBSTITUTING INTO THE FORMULA:

$$125 = 60 + 70 + 40 - 25 - 15 - 10 + n(C \cap F \cap H)$$

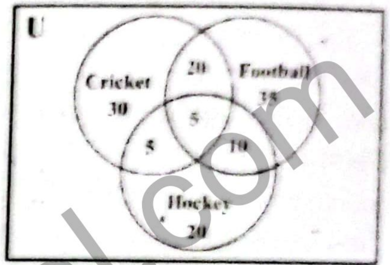
Simplifying:

$$125 = 170 - 50 + n(C \cap F \cap H)$$
$$125 = 120 + n(C \cap F \cap H)$$
$$n(C \cap F \cap H) = 125 - 120$$
$$n(C \cap F \cap H) = 5$$

So, **5 students** play all three sports.

Part (b): Draw a Venn diagram

- 30 students play only cricket.
- 35 students play only football.
- 20 students play only hockey.
- 20 students play both cricket and football, but not hockey
- 10 students play both football and hockey, but not cricket.
- 5 students play all three sports.



13. A survey was conducted in which 130 people were asked about their favourite foods. The survey results showed the following information:

- **40 people said they liked nihari.**
- **65 people said they liked biryani.**
- **50 people said they liked korma.**
- **20 people said they liked nihari and biryani.**
- **35 people said they liked biryani and korma.**
- **27 people said they liked nihari and korma.**
- **12 people said they liked all three foods nihari, biryani and korma.**

(a) At least how many people like nihari, biryani or korma?

(b) How many people did not like nihari or korma?

(c) How many people like only one the following foods: nihari, biryani or korma?

(d) Draw a Venn diagram.

Sol. Let's solve the questions step by step using the principle of inclusion-exclusion.

Given:

- Total number of people: 130
- People who like:
  - Nihari (N): 40
  - Biryani (B): 65

- Korma (K): 50
- Nihari and Biryani ($N \cap B$): 20
- Biryani and Korma ($B \cap K$): 35
- Nihari and Korma ($N \cap K$): 27
- All three foods ($N \cap B \cap K$): 12

(a) AT LEAST HOW MANY PEOPLE LIKE NIHARI, BIRYANI, OR KORMA?

To find the number of people who like at least one of the three foods, we use the inclusion-exclusion principle:

$$|N \cup B \cup K| = |N| + |B| + |K| - |N \cap B| - |B \cap K| - |N \cap K| + |N \cap B \cap K|$$

Substitute the given values:

$$|N \cup B \cup K| = 40 + 65 + 50 - 20 - 35 - 27 + 12$$

Now, calculate:

$$|N \cup B \cup K| = 40 + 65 + 50 - 20 - 35 - 27 + 12 = 85$$

So, **at least 85 people** like Nihari, Biryani, or Korma.

(b) HOW MANY PEOPLE DID NOT LIKE NIHARI OR KORMA?

People who did not like Nihari or Korma = 130 – 85
= 45

So, 45 people did not like Nihari or Korma.

(c) HOW MANY PEOPLE LIKE ONLY ONE OF THE FOLLOWING FOODS: NIHARI, BIRYANI, OR KORMA?

To find the number of people who like only one of the foods, we need to subtract the people who like two or more foods from the total number of people who like each food.

- **People who like only Nihari:**

$$|N| - |N \cap B| - |N \cap K| + |N \cap B \cap K|$$
$$40 - 20 - 27 + 12 = 5$$

- **People who like only Biryani:**

$$|B| - |N \cap B| - |B \cap K| + |N \cap B \cap K|$$
$$65 - 20 - 35 + 12 = 22$$

- **People who like only Korma:**

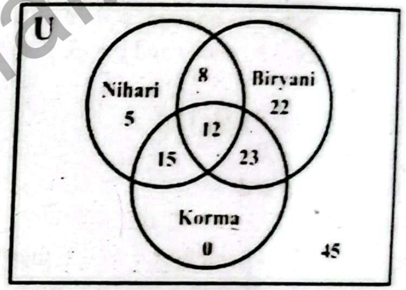
$$|K| - |N \cap K| - |B \cap K| + |N \cap B \cap K|$$
$$50 - 27 - 35 + 12 = 0$$

Now, add the people who like only one of the foods:

5 (only Nihari) + 22 (only Biryani) + 0 (only Korma) = 27

So, **27 people like only one of the foods: Nihari, Biryani, or Korma.**

Draw a Venn diagram



Example 7: Let c_1, c_2, c_3 be three children and m_1, m_2 be two men such that the father of both $c_1\, c_2$ is m_1 and father of c_3 is m_2. Find the relation {(child, father)}

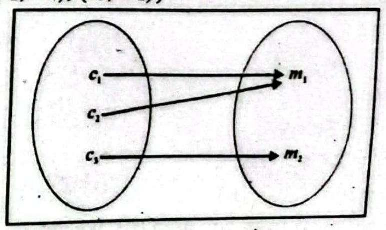
Solution: C = Set of children = $\{c_1, c_2, c_3\}$ and F = set of fathers = $\{m_1, m_2\}$

The Cartesian product of C and F:

$C \times F = \{(c_1, m_1), (c_1, m_2), (c_2, m_1), (c_2, m_2), (c_3, m_1), (c_3, m_2)\}$

r = set of ordered pairs (child, father)

$= \{(c_1, m_1), (c_2, m_1), (c_3, m_2)\}$



Dom $r = \{c_1, c_2, c_3\}$, Range $r = \{m_1, m_2\}$

The relation is shown diagrammatically in adjacent figure.

Example 8: Let $A = \{1, 2, 3\}$. Determine the relation r such that $x\,r\,y$ iff $x < y$.

Solution: $A \times A = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$

Clearly, required relation is:

$r = \{(1,2), (1,3), (2, 3)\}$, Dom $r = \{1, 2\}$, Range $r = \{2,3\}$

Example 9: If $A = \{0, 1, 2, 3, 4\}$ and $B = \{3, 5, 7, 9, 11\}$, define a function $f : A \rightarrow B$, $f = \{(x, y) \mid y = 2x + 3, x \in A \text{ and } y \in B\}$, Find the value of function f, its domain, codomain and range.

Solution: Given: $y = 2x + 3$; $x \in A$ and $y \in B$, then value of function,

$f = \{(0,3), (1,5), (2, 7), (3, 9), (4, 11)\}$

Dom $f = \{0, 1, 2, 3, 4\} = A$

$\Rightarrow$ Co-domain $f = B$ and

$\Rightarrow$ Range $f = \{3, 5, 7, 9, 11\} \subseteq B$

Example 10: If $f(x) = 2x - 1$ and $g(x) = x^2 - 3$, then find:

(i) $f(1)$ (ii) $f(-3)$ (iii) $f(7)$ (iv) $g(1)$

(v) $g(-3)$ (vi) $g(4)$

Solution: (i) $f(1) = 2 \times 1 - 1 = 1$

(ii) $f(-3) = 2 \times (-3) - 1 = -7$

(iii) $f(7) = 2 \times 7 - 1 = 13$ (iv) $g(1) = (1)2 - 3 = -3$

(v) $g(-3) = (-3)^2 - 3 = 6$ (vi) $g(4) = (4)^2 - 3 = 13$

Example 11: Consider $f(x) = ax + b + 3$, where a and b are constant numbers. If $f(1) = 4$ and $f(5) = 9$, then find the value of a and b. **Solution:** Given function $f(x) = ax + b + 3$

If $f(1) = 4$

Then $a \times 1 + b + 3 = 4$

$\Rightarrow$ $a + b = 1$ ...(i)

Similarly, $f(5) = 9$

$\Rightarrow$ $1 + 6 + 3 = 4$

$\Rightarrow$ $5a + b = 6$ ... (ii)

Subtract equation (i) from equation (ii), we get.

$$(5a + b) - (a + b) = 6 - 1$$
$$5a + b - a - b = 5$$
$$4a = 5 \Rightarrow a = \frac{5}{4}$$

Substitute $a = \frac{5}{4}$ in the equation (i)

$$\frac{5}{4} + b = 1$$
$$b = -\frac{5}{4}$$
$$\Rightarrow b = -\frac{1}{4}$$

Thus, $a = \frac{5}{4}$ and $b = -\frac{1}{4}$

EXERCISE 3.3

1. For $A = \{1, 2, 3, 4\}$, find the following relations in A. State the domain and range of each relation.

(i) $\{(x, y) \mid y = x\}$ **(ii)** $\{(x, y) \mid y + x = 5\}$

(iii) $\{(x, y) \mid x + y < 5\}$ **(iv)** $\{(x, y) \mid x + y > 5\}$

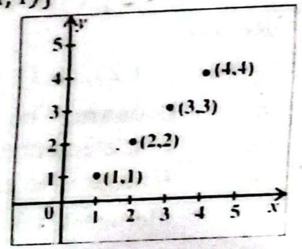
Sol: Let's go through each of the given relations one by one, finding the domain and range for each.

(i) Relation: $\{(x, y) \mid y = x\}$

This relation represents all pairs where the second component (y) is equal to the first component (x). Since $A = \{1,2,3,4\}$, the relation will consist of pairs where $x = y$.

Relation:

$$\{(1,1), (2,2), (3,3), (4,4)\}$$



- **Domain:** The set of all possible first elements, which is A, i.e., $\{1,2,3,4\}$.
- **Range:** The set of all second elements, which is also A, i.e., $\{1,2,3,4\}$.

(ii) Relation: $\{(x, y) \mid y + x = 5\}$

This relation represents all pairs where the sum of x and y is 5.
We can solve for y in terms of x:

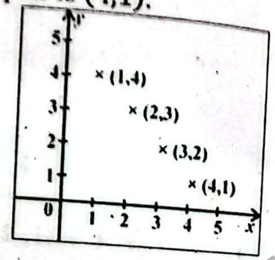
$$y = 5 - x$$

Given $A = \{1,2,3,4\}$, we will check the pairs:

- For $x = 1, y = 5 - 1 = 4$, so the pair is $(1,4)$.
- For $x = 2, y = 5 - 2 = 3$, so the pair is $(2,3)$.
- For $x = 3, y = 5 - 3 = 2$, so the pair is $(3,2)$.
- For $x = 4, y = 5 - 4 = 1$, so the pair is $(4,1)$.

Relation:

$\{(1,4), (2,3), (3,2), (4,1)\}$



- **Domain**: The set of all possible first elements, which is $\{1,2,3,4\}$.
- **Range**: The set of all second elements, which is also $\{4, 3, 2, 1\}$.

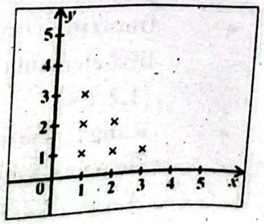
(iii) Relation: $\{(x, y) \mid x + y < 5\}$

This relation represents all pairs where the sum of x and y is less than 5.
We check each possible pair:

- For $x = 1$, y can be 1,2,3 (since $1 + 4 = 5$, which is not less than 5).
- For $x = 2$, y can be 1,2 (since $2 + 3 = 5$, which is not less than 5).
- For $x = 3$, y can only be 1 (since $3 + 2 = 5$, which is not less than 5).
- For $x = 4$, no values of y work because all pairs give sums greater than or equal to 5.

Relation:

$\{(1,1), (1,2), (1,3), (2,1), (2,2), (3,1)\}$



- **Domain**: The set of all possible first elements, which is $\{1,2,3\}$.
- **Range**: The set of all second elements, which is $\{1,2,3\}$.

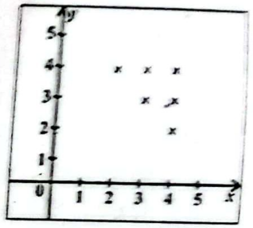
(iv) Relation: $\{(x, y) \mid x + y > 5\}$

This relation represents all pairs where the sum of x and y is greater than 5.
We check each possible pair:

- For $x = 1$, no values of y work because all sums are less than or equal to 5.
- For $x = 2$, no values of y work because all sums are less than or equal to 5.
- For $x = 3$, y can only be 2,3,4 (since $3 + 2 = 5$, $3 + 3 = 6$ and $3 + 4 = 7$).
- For $x = 4$, y can be 2,3,4 (since $4 + 2 = 6$, $4 + 3 = 7$, and $4 + 4 = 8$).

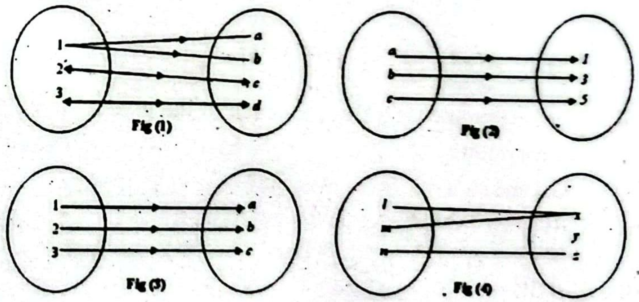
Relation:

$\{(2,4), (3,3), (3,4), (4,2), (4,3), (4,4)\}$



- **Domain**: The set of all possible first elements, which is $\{2,3,4\}$.
- **Range**: The set of all second elements, which is $\{4,3,2\}$.

2. Which of the following diagrams represents functions and of which type?



Solution: Fig (1)

$A = \{1, 2, 3\}, B = \{a, b, c, d\}$

$R = \{(1, a), (1, b), (2, c), (3, d)\}$

Since, first elements in first two ordered pairs are same i.e., 1, 1, so the relation is not a function.

Fig (2)

$C = \{a, b, c\}, D = \{1, 3, 5\}$

$R = \{(a, 1), (b, 3), (c, 5)\}$

Range $(R) = \{1, 3, 5\} = D$

Since the relation is an onto and one-one function. So, the relation is a bijective function.

Fig (3)

$E = \{1, 2, 3\}, F = \{a, b, c\}$

$R = \{(1, a), (2, b), (3, c)\}$

Range $(R) = \{a, b, c\} = F$

Since, the relation is onto function and one-one function. So, the relation is a bijective function.

Fig (4)

$G = \{l, m, n\}, H = \{x, y, z\}$

$R = \{(l, x), (m, x), (x, z)\}$

Range $(f) = \{x, z\} \neq H$

Since, range $(f) \subseteq H$, So, function into function.

3. If $g(x) = 3x + 2$ and $h(x) = x^2 + 1$, then find:

(i) $g(0)$ **(ii)** $g(-3)$

(iii) $g\left(\frac{2}{3}\right)$ **(iv)** $h(1)$

(v) $h(-4)$ **(vi)** $h\left(-\frac{1}{2}\right)$

Sol: Let's compute the values for each of the given expressions:

Given:

- $g(x) = 3x + 2$
- $h(x) = x^2 + 1$

(i) $g(0)$

Substitute $x = 0$ into $g(x)$:

$$g(0) = 3(0) + 2 = 0 + 2 = 2$$

(ii) $g(-3)$

Substitute $x = -3$ into $g(x)$:

$$g(-3) = 3(-3) + 2 = -9 + 2 = -7$$

(iii) $g\left(\frac{2}{3}\right)$

Substitute $x = \frac{2}{3}$ into $g(x)$:

$$g\left(\frac{2}{3}\right) = 3\left(\frac{2}{3}\right) + 2 = 2 + 2 = 4$$

(iv) $h(1)$

Substitute $x = 1$ into $h(x)$:

$$h(1) = 1^2 + 1 = 1 + 1 = 2$$

(v) $h(-4)$

Substitute $x = -4$ into $h(x)$:

$$h(-4) = (-4)^2 + 1 = 16 + 1 = 17$$

(vi) $h\left(-\frac{1}{2}\right)$

Substitute $x = -\frac{1}{2}$ into $h(x)$:

$$h\left(-\frac{1}{2}\right) = \left(-\frac{1}{2}\right)^2 + 1 = \frac{1}{4} + 1 = \frac{1}{4} + \frac{4}{4} = \frac{5}{4}$$

4. Given that $f(x) = ax + b + 1$, where a and b are constant numbers. If $f(3) = 8$ and $f(6) = 14$, then find the values of a and b.

Sol. We are given that $f(x) = ax + b + 1$, and we are also given the following values:

- $f(3) = 8$
- $f(6) = 14$

We can use these values to form two equations and solve for a and b.

Step 1: Use $f(3) = 8$

Substitute $x = 3$ into the function $f(x) = ax + b + 1$:

$$f(3) = a(3) + b + 1 = 8$$
$$3a + b + 1 = 8$$
$$3a + b = 7 \quad \text{(Equation 1)}$$

Step 2: Use $f(6) = 14$

Substitute $x = 6$ into the function $f(x) = ax + b + 1$:

$$f(6) = a(6) + b + 1 = 14$$
$$6a + b + 1 = 14$$
$$6a + b = 13 \quad \text{(Equation 2)}$$

Step 3: Solve the system of equations

We now have the system of equations:

- $3a + b = 7$
- $6a + b = 13$

To eliminate b, subtract Equation 1 from Equation 2:

$$(6a + b) - (3a + b) = 13 - 7$$
$$6a - 3a = 6$$
$$3a = 6$$
$$a = 2$$

Step 4: Find b

Now substitute $a = 2$ into Equation 1:

$$3(2) + b = 7$$
$$6 + b = 7$$
$$b = 1$$

5. Given that $g(x) = ax + b + 5$, where a and b are constant numbers. If $g(-1) = 0$ and $g(2) = 10$, find the values of a and b.

Sol. We are given that $g(x) = ax + b + 5$, and we are also given the following values:

- $g(-1) = 0$
- $g(2) = 10$

We can use these values to form two equations and solve for a and b.

Step 1: Use $g(-1) = 0$

Substitute $x = -1$ into the function $g(x) = ax + b + 5$:

$$g(-1) = a(-1) + b + 5 = 0$$
$$-a + b + 5 = 0$$
$$-b + a = 5 \quad \text{(Equation 1)}$$

Step 2: Use $g(2) = 10$

Substitute $x = 2$ into the function $g(x) = ax + b + 5$:

$$g(2) = a(2) + b + 5 = 10$$
$$2a + b + 5 = 10$$
$$2a + b = 5 \quad \text{(Equation 2)}$$

Step 3: Solve the system of equations

We now have the system of equations:

- $-a + b = -5$
- $\underline{2a + b = -5}$

$-3a \quad = -10$ $\qquad a = \frac{10}{3}$

To eliminate b, subtract Equation 1 from Equation 2:

$$-\frac{10}{3} + b + 5 = 0$$
$$b = -5 + \frac{10}{3}$$
$$= \frac{-5+10^3}{3} = -\frac{5}{3}$$

6. Consider the function defined by $f(x) = 5x + 1$. If $f(x) = 32$, find the x value.

Solution: We are given the function $f(x) = 5x + 1$ and the equation $f(x) = 32$. We need to find the value of x that satisfies this equation.

Step 1: Set up the equation

$$f(x) = 32$$

Substitute $f(x) = 5x + 1$ into this equation:

$$5x + 1 = 32$$

Step 2: Solve for x

Subtract 1 from both sides:

$$5x = 32 - 1$$
$$5x = 31$$

Now, divide both sides by 5:

$$x = \frac{31}{5} = 6.2$$

7. Consider the function $f(x) = cx^2 + d$, where c and d are constant numbers. If $f(1) = 6$ and $f(-2) = 10$, then find the values of c and d.

Sol. We are given the function $f(x) = cx^2 + d$, where c and d are constant numbers. We are also given:

- $f(1) = 6$

- $f(-2) = 10$

We can use these values to form two equations and solve for c and d.

Step 1: Use $f(1) = 6$

Substitute $x = 1$ into the function $f(x) = cx^2 + d$:

$$f(1) = c(1)^2 + d = 6$$

$$c + d = 6 \quad \text{(Equation 1)}$$

Step 2: Use $f(-2) = 10$

Substitute $x = -2$ into the function $f(x) = cx^2 + d$:

$$f(-2) = c(-2)^2 + d = 10$$

$$4c + d = 10 \quad \text{(Equation 2)}$$

Step 3: Solve the system of equations

We now have the system of equations:

- $c + d = 6$
- $4c + d = 10$

To eliminate d, subtract Equation 1 from Equation 2:

$$(4c + d) - (c + d) = 10 - 6$$

$$4c - c = 4$$

$$3c = 4$$

$$c = \frac{4}{3}$$

Step 4: Find d

Now substitute $c = \frac{4}{3}$ into Equation 1:

$$\frac{4}{3} + d = 6$$

$$d = 6 - \frac{4}{3}$$

To subtract the fractions, express 6 as $\frac{18}{3}$:

$$d = \frac{18}{3} - \frac{4}{3} = \frac{14}{3}$$

REVIEW EXERCISE 3

1. Four options are given against each statement. Encircle the correct option.

i. The set builder form of the set $\left\{1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \ldots\right\}$ is:

(a) $\left\{x \mid x = \frac{1}{n}, n \in W\right\}$ (b) $\left\{x \mid x = \frac{1}{2n+1}, n \in W\right\}$

(c) $\left\{x \mid x = \frac{1}{n+1}, n \in W\right\}$ (d) $\{x \mid x = 2n+1, n \in W\}$

ii. If $A = \{\ \}$, then $P(A)$ is:

(a) $\{\ \}$ (b) $\{1\}$

(c) $\{\{\ \}\}$ (d) ϕ

iii. If $U = \{1, 2, 3, 4, 5\}$, $A = \{1, 2, 3\}$ and $B = \{3, 4, 5\}$, then $U - (A \cap B)$ is:

(a) $\{1, 2, 4, 5\}$ (b) $\{2, 3\}$

(c) $\{1, 3, 4, 5\}$ (d) $\{1, 2, 3\}$

iv. If A and B are overlapping sets, then $n(A - B)$ is equal to:

(a) $n(A)$ (b) $n(B)$

(c) $A \cap B$ (d) $n(A) - n(A \cap B)$

v. If $A \subseteq B$ and $B - A \neq \phi$, then $n(B - A)$

(a) 0 (b) $n(B)$

(c) $n(A)$ (d) $n(B) - n(A)$

vi. If $n(A \cup B) = 50$, $n(A) = 30$ and $n(B) = 35$, then $n(A \cap B) =$

(a) 23 (b) 15

(c) 9 (d) 40

vii. If $A = \{1, 2, 3, 4\}$ and $B = \{x, y, z\}$, then Cartesian product of A and B contains exactly ________ elements.

(a) 13 (b) 12

(c) 10 (d) 6

viii. If $f(x) = x^2 - 3x + 2$, then the value of $f(a + 1)$ is equal to:

(a) $a + 1$ (b) $a^2 + 1$

(c) $a^2 + 2a + 1$ (d) $a^2 - a$

ix. Given that $f(x) = 3x + 1$, if $f(x) = 28$, then the value of x is:

(a) 9 (b) 27

(c) 3 (d) 18

x. Let $A = \{1, 2, 3\}$ and $B = \{a, b\}$ two non-empty sets and $f: A \rightarrow B$ be a function defined as $f = \{(1,a), (2,b), (3,b)\}$, then which of the following statement is true?

(a) f is injective (b) f is surjective

(c) f is bijective (d) f is into only

Answers:

(i)	b	(ii)	c	(iii)	a	(iv)	d	(v)	d
(vi)	b	(vii)	b	(viii)	d	(ix)	a	(x)	b

2. Write each of the following sets in tabular forms:

(i) $\{x \mid x = 2n, n \in N\}$ (ii) $\{x \mid x = 2m+1, m \in N\}$

(iii) $\{x \mid x = 11n, n \in W \wedge n < 11\}$

(iv) $\{x \mid x \in E \wedge 4 < x < 6\}$ (v) $\{x \mid x \in O \wedge 5 \leq x < 7\}$

(vi) $\{x \mid x \in Q \wedge x^2 = 2\}$ (vii) $\{x \mid x \in Q \wedge x = -x\}$

(viii) $\{x \mid x \in R \wedge x \notin Q'\}$

Solution:

(i) $\{x \mid x = 2n, n \in \mathbb{N}\}$

This set represents all even natural numbers.

Since $\mathbb{N} = \{1,2,3, \dots\}$, the elements are:

$\{2,4,6,8,10,12, \dots\}$

(ii) $\{x \mid x = 2m + 1, m \in \mathbb{N}\}$

This set represents all odd natural numbers.

Since $\mathbb{N} = \{1,2,3, \dots\}$, the elements are:

$\{3,5,7,9,11,13, \dots\}$

(iii) $\{x \mid x = 11n, n \in W \wedge n < 11\}$

This set represents all multiples of 11, where $W = \{0,1,2,3, \dots\}$, and $n < 11$. The elements are:

$\{0,11,22,33,44,55,66,77,88,99,110\}$

(iv) $\{x \mid x \in E \wedge 4 < x < 6\}$

This set represents the elements from the set E (which is typically E or the set of natural even numbers) that are between 4 and 6. No even numbers in this range, the elements are:

$\{\phi\}$

(v) $\{x \mid x \in O \wedge 5 \leq x < 7\}$

This set represents the odd numbers between 5. The only odd number in this range is:

$\{5\}$

(vi) $\{x \mid x \in Q \wedge x^2 = 2\}$

This set represents the rational numbers whose squares are equal to 2. However, there are no rational numbers whose square is 2, because $\sqrt{2}$ is an irrational number. Therefore, the set is empty:

$\{\}$

(vii) $\{x \mid x \in Q \wedge x = -x\}$

This set represents the rational numbers that are their own negatives. The only number that satisfies this condition is 0, as $x = -x$ implies $x = 0$. Thus, the elements are:

$\{0\}$

(viii) $\{x \mid x \in \mathbb{R} \wedge x \notin Q'\}$

This set represents all real numbers that are not rational. These are the irrational numbers. The set can be represented as:

{irrational numbers} $\{Q'\}$

For example, it includes numbers like $\sqrt{2}, \pi, e, \dots$

3. Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{2, 4, 6, 8, 10\}$, $B = \{1, 2, 3, 4, 5\}$ and $C = \{1, 3, 5, 7, 9\}$

List the members of each of the following sets:

(i) A' (ii) B' (iii) $A \cup B$

(iv) $A - B$ (v) $A \cap C$ (vi) $A' \cup C'$

(vii) $A' \cup C$ (viii) U'

Sol. Let's find the members of each set:

Given:

- $U = \{1,2,3,4,5,6,7,8,9,10\}$
- $A = \{2,4,6,8,10\}$
- $B = \{1,2,3,4,5\}$
- $C = \{1,3,5,7,9\}$

(i) A'

Solution: $A' = U - A$

$= \{1,2,3,4,5,6,7,8,9,10\} - \{2,4,6,8,10\}$

$= \{1,3,5,7,9\}$

(ii) B'

Solution: $B' = U - B$

$= \{1,2,3,4,5,6,7,8,9,10\} - \{1,2,3,4,5\}$

$= \{6,7,8,9,10\}$

(iii) $A \cup B$

Solution:

$A \cup B = \{2,4,6,8,10\} \cup \{1,2,3,4,5\}$

$= \{1,2,3,4,5,6,8,10\}$

(iv) $A - B$

Solution:

$A - B = \{2,4,6,8,10\} - \{1,2,3,4,5\}$

$= \{6, 8, 10\}$

(v) $A \cap C$

Solution:

$A \cap C = \{2, 4, 6, 8, 10\} \cap \{1, 3, 5, 7, 9\}$

$= \phi$

(vi) $A' \cup C'$

Solution: $A' = U - A$

$= \{1,2,3,4,5,6,7,8,9,10\} - \{2, 4, 6, 8, 10\}$

$A' = \{1, 3, 5, 7, 9\}$

Now, $C' = U - C$

$= \{1,2,3,4,5,6,7,8,9,10\} - \{1, 3, 5, 7, 9\}$

$C' = \{2, 4, 6, 8, 10\}$

Now, $A' \cup C' = \{1, 3, 5, 7, 9\} \cup \{2, 4, 6, 8, 10\}$

$A' \cup C' = \{1, 2, 3, 4, \ldots, 10\}$

(vii) $A' \cup C$

Solution: $A' = U - A$

$= \{1,2,3,4,5,6,7,8,9,10\} - \{2, 4, 6, 8, 10\}$

$A' = \{1, 3, 5, 7, 9\}$

Now, $A' \cup C = \{1, 3, 5, 7, 9\} \cup \{1, 3, 5, 7, 9\}$

$A' \cup C = \{1, 3, 5, 7, 9\}$

(viii) U'

Solution: $U' = U - U$

$= \{1,2,3,4,5,6,7,8,9,10\} - \{1,2,3,4, \ldots,10\}$

$= \phi$

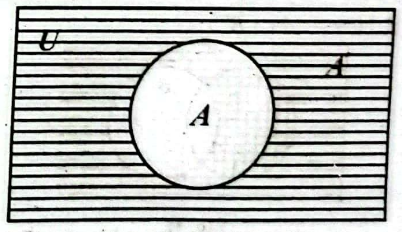
4. Using the Venn diagrams, if necessary, find the single sets equal to the following:

(i) A' (ii) $A \cap B$ (iii) $A \cup U$

(iv) $A \cup \phi$ (v) $\phi \cap \phi$

(i) A'

Solution: $A' = U - A$



Horizontally lined region shows A'.

(ii) $A \cap B$

Solution:

$A \cap B$

$A \cap B = A$ $(\because A \subset U)$

(iii) $A \cup U$

Solution:

$A \cup U = U \qquad (\because A \subset U)$

(iv) $A \cup \phi$

Solution:

$A \cup \phi = A \qquad (\because \phi \subset A)$

(v) $\phi \cap \phi$

Solution:

$\phi \cap \phi = \phi$

5. Use Venn diagrams to verify the following:

(i) $A - B = A \cup B'$ (ii) $(A - B)' \cap B = B$

Solution:

(i) $A - B = A \cup B'$

Solution: $A - B = A \cup B'$

L.H.S $= A - B$

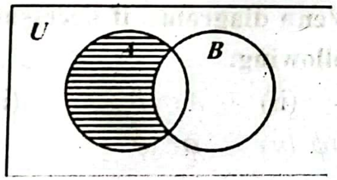


Fig. I

A – B is shown by region of horizontal line segments.

R.H.S $= A \cap B'$

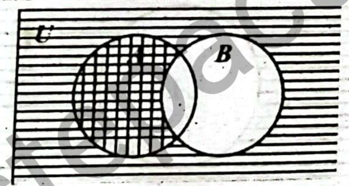


Fig. II

B' is shown by region of squares and horizontal line segments.

$A \cap B'$ is shown by region of squares. From fig. (i) and Fig. (ii) regions showing $(A - B)$ and $(A \cap B')$ are same, so

$A - B = A \cap B'$

(ii) $(A - B)' \cap B = B$

Solution:

L.H.S $= (A - B)' \cap B$

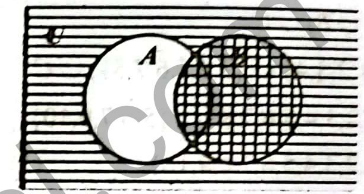


Fig. I

$(A - B)' \cap B$ **is shown by regions of squares** $(A - B)' \cap B$ **is shown by region of square.**

R.H.S $= B$

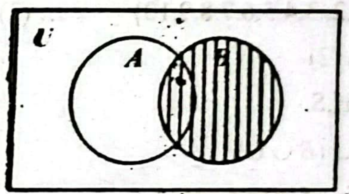


Fig. II

Regions showing $(A - B)' \cap B$ and B are same so,

$(A - B)' \cap B = B$

6. Verify the properties for the sets A, B and C given below:

(i) Associativity of Union

(ii) Associativity of intersection

(iii) Distributivity of Union over intersection

(iv) Distributivity of intersection over union.

(a) $A = \{1, 2, 3, 4\}, B = \{3, 4, 5, 6, 7, 8\}, C = \{5, 6, 7, 9, 10\}$

(b) $A = \varnothing, B = \{0\}, C = \{0, 1, 2\}$

(c) $A = N, B = Z, C = Q$

(a) $A = \{1, 2, 3, 4\}$

$B = \{3, 4, 5, 6, 7, 8\}$

$C = \{5, 6, 7, 9, 10\}$

(i) Associativity of Union

Solution:

$(A \cup B) \cup C = A \cup (B \cup C)$

L.H.S $= (A \cup B) \cup C$

$= (\{1,2,3,4\} \cup \{3,4,5,6,7,8\}) \cup C$

$= \{1,2,3,4,5,6,7,8\} \cup \{5,6,7,9,10\}$

$= \{1,2,3,4,5,6,7,8,9,10\}$ ... (1)

Now, R.H.S $= (A \cup B) \cup C$

$= A \cup (\{3,4,5,6,7,8\} \cup \{5,6,7,9,10\})$

$= \{1,2,3,4\} \cup \{3,4,5,6,7,8,9,10\}$

$= \{1,2,3,4,5,6,7,8,9,10\}$ ... (2)

Find (1) and (2)

L.H.S. = R.H.S.

$(A \cup B) \cup C = A \cup (B \cup C)$

(ii) Associativity of intersection

Sol: $(A \cap B) \cap C = A \cap (B \cap C)$

$L.H.S = (A \cap B) \cap C$

$= (\{1,2,3,4\} \cap \{3,4,5,6,7,8\}) \cap C$

$= \{3,4\} \cap \{5,6,7,8,9,10\}$

$= \{\ \}$ ______ (i)

$R.H.S = A \cap (B \cap C)$

$= A \cap (\{3,4,5,6,7,8\} \cap \{5,6,7,9,10\})$

$= \{1,2,3,4\} \cap \{5,6,7\}$

$= \{\ \}$ ______ (ii)

From (i) and (ii) L.H.S = R.H.S

$(A \cap B) \cap C = A \cap (B \cap C)$

(iii) Distributivity of Union over intersection

Sol: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$L.H.S = A \cup (B \cap C)$

$= A \cup (\{3,4,5,6,7,8\} \cap \{5,6,7,8,9,10\})$

$= \{1,2,3,4\} \cup \{5,6,7\}$

$= \{1,2,3,4,5,6,7\}$ ______ (i)

$R.H.S = (A \cup B) \cap (A \cup C)$

$(A \cup B) = \{1,2,3,4\} \cup \{3,4,5,6,7,8\}$

$(A \cup B) = \{1,2,3,4,5,6,7,8\}$

Now, $A \cup C = \{1,2,3,4\} \cup \{5,6,7,9,10\}$

$= \{1,2,3,4,5,6,7,9,10\}$

Now, $(A \cup C) \cap (A \cup C)$

$= \{1,2,3,4,5,6,7,8\} \cap \{1,2,3,4,5,6,7,9,10\}$

$= \{1,2,3,4,5,6,7\}$ ______ (ii)

From (i) and (ii), L.H.S = R.H.S

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ Hence proved

(iv) Distributivity of intersection over union

(a) $A = \{1,2,3,4\}$, $B = \{3,4,5,6,7,8\}$,

$C = \{5,6,7,9,10\}$

Sol: $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$L.H.S = A \cap (B \cup C)$

$= A \cap (\{3,4,5,6,7,8\} \cup \{5,6,7,9,10\})$

$= \{1,2,3,4\} \cap \{3,4,5,6,7,8,9,10\}$

$= \{3,4\}$ ______ (i)

Now, $R.H.S = (A \cap B) \cup (A \cap C)$

$(A \cap B) = \{1,2,3,4\} \cap \{3,4,5,6,7,8\}$

$(A \cap B) = \{3,4\}$

Now, $(A \cap C) = \{1,2,3,4\} \cap \{5,6,7,9,10\}$

$= \{\ \}$

Now, $(A \cap B) \cup (A \cap C) = \{3,4\} \cup \{\ \}$

$= \{3,4\}$ ______ (ii)

From (i) and (ii)

$L.H.S = R.H.S$

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(b) $A = \varnothing,\ B = \{0\},\ C = \{0,1,2\}$

(i) Associativity of Union

$(A \cup B) \cup C = A \cup (B \cup C)$

L.H.S. $= (A \cup B) \cup C$

$= (\{\ \} \cup \{0\}) \cup C$

$= \{0\} \cup \{0, 1, 2\}$

$= \{0, 1, 2\}$ ...(1)

R.H.S. $= A \cup (B \cup C)$

$= A \cup (\{0\} \cup \{0, 1, 2\})$

$= \{\ \} \cup \{0, 1, 2\}$

$= \{0, 1, 2\}$ ...(2)

From (1) and (2) L.H.S = RH.S

$(A \cup B) \cup C = A \cup (B \cup C)$

(ii) Associativity of intersection

$(A \cap B) \cap C = A \cap (B \cap C)$

L.H.S. $= (A \cap B) \cap C$

$= (\{\ \} \cap \{0\}) \cap C$

$= \{\ \} \cap \{0, 1, 2\}$

$= \{\ \}$ ...(1)

R.H.S. $= A \cap (B \cap C)$

$= A \cap (\{0\} \cap \{0, 1, 2\})$

$= \{\ \} \cap \{0\}$

$= \{\ \}$ ...(2)

From (1) and (2) L.H.S = R.H.S

$(A \cap B) \cap C = A \cap (B \cap C)$

Hence proved.

(iii) Distributivity of Union over intersection

Solution:

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

L.H.S $= A \cup (B \cap C)$

$= A \cup (\{0\} \cap \{0, 1, 2\})$

$= \{\ \} \cup \{0\} = \{0\}$ ...(1)

R.H.S. $= (A \cup B) \cap (A \cup C)$

$= (\{\ \} \cup \{0\}) \cap (\{\ \} \cup \{0, 1, 2\})$

$= \{\ \} \cap \{0, 1, 2\}$

$= \{0\}$ ...(2)

From (1) and (2), L.H.S = R.H.S

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Hence proved.

(iv) Distributivity of intersection over Union

Solution:

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

L.H.S $= A \cap (B \cup C)$

$= \{\ \} \cap (\{\ \} \cup \{0, 1, 2\})$

$= \{\ \} \cap \{0, 1, 2\}$

$= \{\ \}$ ...(1)

R.H.S $= (A \cap B) \cup (A \cap C)$

$= (\{\ \} \cap \{0\}) \cup (\{\ \} \cap \{0, 1, 2\})$

$= \{\ \} \cap \{\ \}$

$= \{\ \}$ ...(2)

From (1) and (2) L.H.S = R.H.S

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(c) $A = N, B = Z, C = Q$

Solution:

$N \subset Z \subset Q$

(i) Associativity of union

Solution:

$(A \cup B) \cup C = A \cup (B \cup C)$

L.H.S $= (A \cup B) \cup C$

$= (N \cup Z) \cup Q$

$= Z \cup Q$

$= Q$ ...(1)

R.H.S $= A \cup (B \cup C)$

$= N \cup (Z \cup Q)$

$= N \cup Q$

$= Q$ ...(2)

From (1) and (2) L.H.S = R.H.S

$(A \cup B) \cup C = A \cup (B \cup C)$

Hence proved.

(ii) Associativity of intersection

Solution:

$(A \cap B) \cap C = A \cap (B \cap C)$

L.H.S $= (A \cap B) \cap C$

$= (N \cap Z) \cap Q$

$= N \cap Z$

$= N$ ...(1)

R.H.S $= A \cap (B \cap C)$

$= N \cap (Z \cap Q)$

$= N \cap Z$

$= N$ ...(2)

From (1) and (2) L.H.S. = R.H.S

$(A \cap B) \cap C = A \cap (B \cap C)$

Hence proved.

(iii) Distributivity of union over intersection

Solution:

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

L.H.S $= A \cup (B \cap C)$

$= N \cup (Z \cap Q)$

$= N \cup Z$

$= Z$ ...(i)

R.H.S $= (A \cup B) \cap (A \cup C)$

$= (N \cup Z) \cap (N \cup Q)$

$= N \cup Q$

$= Z$ ...(ii)

From (i) and (ii), L.H.S = R.H.S

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Hence proved.

(iv) Distributivity of intersection over union

Solution:

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

L.H.S $= A \cap (B \cup C)$

$= N \cap (Z \cup Q)$

$= N \cap Q$

$= N$ ...(i)

R.H.S $= (A \cap B) \cup (A \cap C)$

$= (N \cap Z) \cup (N \cap Q)$

$= N \cup N$

$= N$ ...(ii)

From (i) and (ii), L.H.S = R.H.S

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Hence proved.

7. **Verify De Morgan's Laws for the following sets:**
$U = \{1,2,3, \dots, 20\}$, $A = \{2, 4, 6, \dots, 20\}$ and $B = \{1, 3, 5, \dots, 19\}$

Solution:

$U = \{1, 2, 3, \dots, 20\}$
$A = \{2, 4, 6, \dots, 20\}$
$B = \{1, 3, 5, \dots, 19\}$

(i) $(A \cup B)' = A' \cap B'$

Solution:

$(A \cup B)' = A' \cap B'$
L.H.S $= (A \cup B)'$
$(A \cup B) = \{2, 4, 6, \dots, 20\} \cup \{1, 3, 5, \dots, 19\}$
$= \{1, 2, 3, 4, 5, 6, \dots, 19, 20\}$

Now, $(A \cup B)' = U - (A \cup B)$
$= \{1, 2, 3, 4, \dots, 20\} - \{1, 2, 3, 4, \dots, 20\}$
$= \{\ \}$ ...(1)

Now, R.H.S $= A' \cap B'$
$A' = U - A$
$A' = \{1, 2, 3, 4, 5, \dots, 20\} - \{2, 4, 6, \dots, 20\}$
$= \{1, 3, 5, 7, \dots, 19\}$

Now, $B' = U - B$
$B' = \{1, 2, 3, 4, \dots, 20\} - \{1, 3, 5, \dots, 19\}$
$= \{2, 4, 6, 8, \dots, 20\}$

R.H.S $= A' \cap B'$
$= \{1, 3, 5, \dots, 19\} \cap \{2, 4, 6, \dots, 20\}$
$= \{\ \}$ ...(2)

From (1) and (2), L.H.S = R.H.S
$(A \cup B)' = A' \cap B'$
Hence proved.

(ii) $(A \cap B)' = A' \cup B'$

L.H.S $= (A \cap B)'$
$(A \cap B) = \{2, 4, 6, \dots, 20\} \cap \{1, 3, 5, \dots, 19\}$
$= \{\ \}$

Now, $(A \cap B)' = U - (A \cap B)$
$= \{1, 2, 3, 4, \dots, 20\} - \{\ \}$
$= \{1, 2, 3, 4, \dots, 20\}$ ...(1)

R.H.S $= A' \cup B'$
$A' = U - A$
$A' = \{1, 2, 3, 4, \dots, 20\} - \{2, 4, 6, \dots, 20\}$
$= \{1, 3, 5, \dots, 19\}$

Now, $B' = U - B$
$B' = \{1, 2, 3, 4, \dots, 20\} - \{1, 3, 5, \dots, 19\}$
$= \{2, 4, 6, \dots, 20\}$

Now, $A' \cup B' = \{1, 3, 5, \dots, 19\} \cup \{2, 4, 6, \dots, 20\}$
$= \{1, 2, 3, 4, 5, \dots, 20\}$ ...(2)

From (1) and (2), L.H.S = R.H.S
$(A \cap B)' = A' \cup B'$

8. **Consider the set $P = \{x \mid x = 5m, m \in N\}$ and $Q = \{x \mid x = 2m, m \in N\}$. Find $P \cap Q$.**

Sol.
$P = \{x \mid x = 5m, m \in N\}$
$P = \{5, 10, 15, 20, 25, \dots\}$
$Q = \{x \mid x = 2m, m \in N\}$
$Q = \{2, 4, 6, 8, 10, \dots\}$

Finding $P \cap Q$
$P \cap Q = \{5, 10, 15, 20, 25, \dots\} \cap \{2, 4, 6, 8, 10, \dots\}$
$= \{10, 20, 30, 40, 50, 60, \dots\}$

9. **From suitable properties of union and intersection, deduce the following results:**

(i) $A \cap (A \cup B) = A \cup (A \cap B)$

(ii) $A \cup (A \cap B) = A \cap (A \cup B)$

Ans. To prove the given set identities, we will use the properties of union and intersection.

(i) $A \cap (A \cup B) = A \cup (A \cap B)$

PROOF:

- **Left-hand side: $A \cap (A \cup B)$**
  This represents the set of elements that are both in A and in $A \cup B$. Since $A \cup B$ includes all elements from A as well as B,

the intersection $A \cap (A \cup B)$ will simply be all elements of A, because every element in A is also in $A \cup B$.

Hence,

$$A \cap (A \cup B) = A$$

- **Right-hand side: $A \cup (A \cap B)$**

This represents the set of elements that are either in A or in both A and B. Since the intersection $A \cap B$ contains elements that are in both A and B, the union of A and $A \cap B$ is just A, because the elements of $A \cap B$ are already included in A.

Hence,

$$A \cup (A \cap B) = A$$

Since both sides are equal, we have proved:

$$A \cap (A \cup B) = A \cup (A \cap B)$$

(ii) $A \cup (A \cap B) = A \cap (A \cup B)$

PROOF:

- **Left-hand side: $A \cup (A \cap B)$**

This represents the set of elements that are either in A or in both A and B. Since the intersection $A \cap B$ contains elements that are in both A and B, the union of A and $A \cap B$ will be exactly A, because the elements of $A \cap B$ are already included in A.

Hence,

$$A \cup (A \cap B) = A$$

- **Right-hand side: $A \cap (A \cup B)$**

This represents the set of elements that are in both A and B $A \cup B$. Since $A \cup B$ includes all elements of A, the intersection $A \cap (A \cup B)$ will simply be A, because every element in A is also in $A \cup B$.

Hence,

$$A \cap (A \cup B) = A$$

Since both sides are equal, we have proved:

$$A \cup (A \cap B) = A \cap (A \cup B)$$

10. If $g(x) = 7x - 2$ and $s(x) = 8x^2 - 3$ find:

(i) $g(0)$ (ii) $g(-1)$ (iii) $g\left(-\frac{5}{3}\right)$

(iv) $s(1)$ (v) $s(-9)$ (vi) $s\left(\frac{7}{2}\right)$

Sol. Let's calculate the values for each of the given expressions step by step.

Given Functions:

- $g(x) = 7x - 2$
- $s(x) = 8x^2 - 3$

(i) $g(0)$

To find $g(0)$, substitute $x = 0$ into $g(x)$:

$$g(0) = 7(0) - 2 = 0 - 2 = -2$$

(ii) $g(-1)$

To find $g(-1)$, substitute $x = -1$ into $g(x)$:

$$g(-1) = 7(-1) - 2 = -7 - 2 = -9$$

(iii) $g\left(-\frac{5}{3}\right)$

To find $g\left(-\frac{5}{3}\right)$, substitute $x = -\frac{5}{3}$ into $g(x)$:

$$g\left(-\frac{5}{3}\right) = 7\left(-\frac{5}{3}\right) - 2 = -\frac{35}{3} - 2 = -\frac{35}{3} - \frac{6}{3} = -\frac{41}{3}$$

Thus, $g\left(-\frac{5}{3}\right) = -\frac{41}{3}$.

(iv) $s(1)$

To find $s(1)$, substitute $x = 1$ into $s(x)$:

$$s(1) = 8(1)^2 - 3 = 8(1) - 3 = 8 - 3 = 5$$

(v) $s(-9)$

To find $s(-9)$, substitute $x = -9$ into $s(x)$:

$$s(-9) = 8(-9)^2 - 3 = 8(81) - 3 = 648 - 3 = 645$$

(vi) $s\left(\frac{7}{2}\right)$

To find $s\left(\frac{7}{2}\right)$, substitute $x = \frac{7}{2}$ into $s(x)$:

$$s\left(\frac{7}{2}\right) = 8\left(\frac{7}{2}\right)^2 - 3 = 8\left(\frac{49}{4}\right) - 3 = \frac{392}{4} - 3 = 98 - 3 = 95$$

11. Given that $f(x) = ax + b$, where a and b are constant numbers. If $f(-2) = 3$ and $f(4) = 10$, then find the values of a and b.

Sol. We are given the function $f(x) = ax + b$, where a and b are constants. The conditions provided are:

- $f(-2) = 3$
- $f(4) = 10$

Step 1: Use the condition $f(-2) = 3$

Substitute $x = -2$ into the equation $f(x) = ax + b$:

$$f(-2) = a(-2) + b = -2a + b = 3$$

Thus, we get the equation:

$$-2a + b = 3 \qquad (1)$$

Step 2: Use the condition $f(4) = 10$

Substitute $x = 4$ into the equation $f(x) = ax + b$:

$$f(4) = a(4) + b = 4a + b = 10$$

Thus, we get the equation:

$$4a + b = 10 \qquad (2)$$

Step 3: Solve the system of equations

We have the system of two equations:

- $-2a + b = 3$
- $4a + b = 10$

To eliminate b, subtract equation (1) from equation (2):

$$(4a + b) - (-2a + b) = 10 - 3$$

Simplifying:

$$4a + b + 2a - b = 7$$

$$6a = 7$$

Solving for a:

$$a = \frac{7}{6}$$

Step 4: Substitute $a = \frac{7}{6}$ into one of the original equations

Substitute $a = \frac{7}{6}$ into equation (1):

$$-2\left(\frac{7}{6}\right) + b = 3$$

Simplifying:

$$-\frac{14}{6} + b = 3$$

$$-\frac{7}{3} + b = 3$$

Now, add $\frac{7}{3}$ to both sides:

$$b = 3 + \frac{7}{3} = \frac{9}{3} + \frac{7}{3} = \frac{16}{3}$$

12. Consider the function defined by $k(x)=7x-5$. If $k(x)= 100$, find the value of x.

Sol. We are given the function $k(x) = 7x - 5$ and the equation $k(x) = 100$. We need to find the value of x.

Step 1: Set the equation equal to 100

$$7x - 5 = 100$$

Step 2: Solve for x

First, add 5 to both sides:

$$7x = 100 + 5$$

$$7x = 105$$

Now, divide both sides by 7:

$$x = \frac{105}{7}$$

$$x = 15$$

13. Consider the function $g(x) = mx^2 + n$, where m and n are constant numbers. If $g(4) = 20$ and $g(0) = 5$, find the values of m and n.

Sol. We are given the function $g(x) = mx^2 + n$, where m and n are constant numbers. We also know the values $g(4) = 20$ and $g(0) = 5$. We need to find the values of m and n.

Step 1: Use $g(0) = 5$

Substitute $x = 0$ into the function:

$$g(0) = m(0)^2 + n = 5$$

$$0 + n = 5$$

$$n = 5$$

Step 2: Use $g(4) = 20$

Substitute $x = 4$ and $n = 5$ into the function:

$$g(4) = m(4)^2 + 5 = 20$$

$$16m + 5 = 20$$

Subtract 5 from both sides:

$$16m = 15$$

Now, divide by 16:

$$m = \frac{15}{16}$$

14. A shopping mall has 100 products from various categories labeled 1 to 100, representing the universal set U. The products are categorized as follows:

- **Set A: Electronics, consisting of 30 products labeled from 1 to 30.**
- **Set B: Clothing comprises 25 products labeled from 31 to 55.**
- **Set C: Beauty Products, comprising 25 products labeled from 76 to 100.**

Write each set in tabular form, and find the union of all three sets.

Sol. Tabular Representation of the Sets

- **Set A: Electronics** (Products labeled from 1 to 30):

$$A = \{1,2,3, \ldots,30\}$$

- **Set B: Clothing** (Products labeled from 31 to 55):

$$B = \{31,32,33, \ldots,55\}$$

- **Set C: Beauty Products** (Products labeled from 76 to 100):

$$C = \{76,77,78, \ldots,100\}$$

Finding the Union of All Three Sets

$$A \cup B \cup C = \{1,2,3, \ldots,30\} \cup \{31,32,33, \ldots,55\} \cup \{76,77,78, \ldots,100\}$$

$$A \cup B \cup C = \{1,2,3, \ldots,30,31,32,33, \ldots,55,76,77,78, \ldots,100\}$$

15. Out of the 180 students who appeared in the annual examination, 120 passed the math test, 90 passed the science test and 60 passed both the math and science tests.

(a) How many passed either the math or science test?

(b) How many did not pass either of the two tests?

(c) How many passed the science test but not the math test?

(d) How many failed the science test?

We are given the following information:

- Total number of students: 180
- Number of students who passed the Math test: 120
- Number of students who passed the Science test: 90
- Number of students who passed both Math and Science: 60

Let's define the sets:

- Let M represent the set of students who passed the Math test.
- Let S represent the set of students who passed the Science test.

Sol. Using the principle of inclusion and exclusion:

The formula for the number of students who passed either the Math or Science test (i.e., the union of sets M and S) is:

$$|M \cup S| = |M| + |S| - |M \cap S|$$

Where:

- $|M|$ is the number of students who passed the Math test.
- $|S|$ is the number of students who passed the Science test.
- $|M \cap S|$ is the number of students who passed both tests.

Now, let's solve each part of the problem:

(a) How many passed either the Math or Science test?

Using the inclusion-exclusion principle:

$$|M \cup S| = 120 + 90 - 60 = 150$$

Thus, **150 students** passed either the Math or the Science test.

(b) How many did not pass either of the two tests?

The total number of students is 180, and the number of students who passed either the Math or the Science test is 150. Therefore, the number of students who did not pass either test is:

$$180 - 150 = 30$$

Thus, **30 students** did not pass either of the two tests.

(c) How many passed the Science test but not the Math test?

The number of students who passed the Science test but not the Math test is given by the difference between the total number of students who passed the Science test and the number of students who passed both tests:

$$|S \setminus M| = |S| - |M \cap S| = 90 - 60 = 30$$

Thus, **30 students** passed the Science test but not the Math test.

(d) How many failed the Science test?

The number of students who failed the Science test is the complement of the students who passed the Science test. Therefore, the number of students who failed the Science test is:

$$180 - |S| = 180 - 90 = 90$$

Thus, **90 students** failed the Science test.

16. In a software house of a city with 300 software developers, a survey was conducted to determine which programming languages are liked more. The survey revealed the following statistics:

- **150 developers like Python.**
- **130 developers like Java.**
- **120 developers like PHP.**
- **70 developers like both Python and Java.**
- **60 developers like both Python and PHP.**
- **50 developers like both Java and PHP.**
- **40 developers like all three languages: Python, Java and PHP.**

(a) How many developers use at least one of these languages?

(b) How many developers use only one of these languages?

(c) How many developers do not use any of these languages?

(d) How many developers use only PHP?

Sol.

(a) How many developers use at least one of these languages?

The number of developers who use at least one of these languages is given by the union of the sets P, J, and H.

Using the principle of inclusion and exclusion:

$$|P \cup J \cup H| = |P| + |J| + |H| - |P \cap J| - |P \cap H| - |J \cap H| + |P \cap J \cap H|$$

Substituting the given values:

$$|P \cup J \cup H| = 150 + 130 + 120 - 70 - 60 - 50 + 40$$

$$|P \cup J \cup H| = 400 - 180 + 40 = 260$$

Thus, **260 developers** use at least one of these languages.

(b) How many developers use only one of these languages?

The number of developers who use only one language can be calculated as follows:

- Developers who use only Python: $|P \setminus (J \cup H)| = |P| - |P \cap J| - |P \cap H| + |P \cap J \cap H| = 150 - 70 - 60 + 40 = 60$
- Developers who use only Java: $|J \setminus (P \cup H)| = |J| - |P \cap J| - |J \cap H| + |P \cap J \cap H| = 130 - 70 - 50 + 40 = 50$
- Developers who use only PHP: $|H \setminus (P \cup J)| = |H| - |P \cap H| - |J \cap H| + |P \cap J \cap H| = 120 - 60 - 50 + 40 = 50$

Now, summing up the developers who use only one language:

$$\text{Total} = 60 + 50 + 50 = 160$$

Thus, **160 developers** use only one of these languages.

(c) How many developers do not use any of these languages?

The number of developers who do not use any of these languages is the complement of the developers who use at least one language:

$$\text{Developers who do not use any language} = 300 - 260 = 40$$

Thus, **130 developers** do not use any of these languages.

(d) How many developers use only PHP?

We already calculated this in part (b). The number of developers who use only PHP is:

$$\text{Only PHP} = 50$$

Thus, **50 developers** use only PHP.