UNIT 5

Linear Equations and Inequalities

Students' learning outcomes

At the end of the unit, the students will be able to:

- Solve linear equations and inequalities with rational coefficients and represent the solution set on a real line.
- Solve two linear inequalities with two unknowns simultaneously.
- Interpret and identify regions in plane bounded by two linear inequalities in two unknowns.
- Find maximum and minimum values of a function using points in the feasible solution.

Example 1: Solve the following equations and represent their solutions on real line.

(i) $3x-5=7$ (ii) $\frac{x-2}{5}-\frac{x-4}{2}=2$

Solution

(i)

$$3x-5=7$$
$$3x-5+5=7+5$$
$$3x=12$$
$$x=\frac{12}{3}=4$$

Check: Substitute $x = 4$ back into the original equation

$$3(4)-5=7$$
$$12-5=7$$
$$7=7$$

So, $x = 4$ is a solution because it makes the original equation true.

Representation of the solution on a number line:

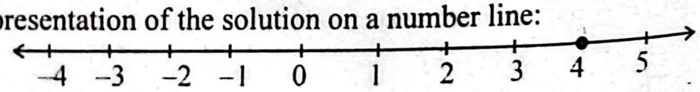


Fig. 5.1

(ii) $\frac{x-2}{5}-\frac{x-4}{2}=2$

$$\frac{2(x-2)-5(x-4)}{10}=2$$
$$\frac{2x-4-5x+20}{10}=2$$
$$\frac{-3x+16}{10}=2$$
$$-3x+16=2\times10$$
$$-3x+16=20$$
$$-3x+16-16=20-16$$
$$-3x=4$$
$$x=\frac{-4}{3}$$

Check: Substitute $x=-\frac{4}{3}$ back into the original equation

$$\frac{-\frac{4}{3}-2}{5}-\frac{-\frac{4}{3}-4}{2}=2$$
$$\Rightarrow \frac{\frac{-4-6}{3}}{5}-\frac{\frac{-4-12}{3}}{2}=2$$
$$\Rightarrow \frac{-10}{15}-\frac{-16}{6}=2$$
$$\Rightarrow -\frac{2}{3}+\frac{8}{3}=2$$
$$\Rightarrow \frac{-2+8}{3}=2$$
$$\Rightarrow \frac{6}{3}=2$$
$$\Rightarrow 2=2$$

So, $x = -\frac{4}{3}$ is the solution of given equation.

Representation of the solution on a number line:

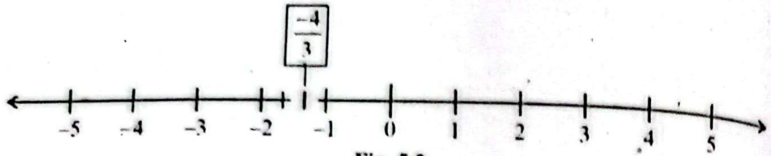


Fig. 5.2

Example 2: Find solution of $\frac{2}{3}x - 1 < 0$ and also represent it on a real line.

Solution: $\frac{2}{3}x - 1 < 0$ ...(i)

$\Rightarrow \quad \frac{2}{3}x < 1$

$\Rightarrow \quad 2x < 3$

$\Rightarrow \quad x < \frac{3}{2}$

It means that all real numbers less than $\frac{3}{2}$ are in the solution of (i)

Thus the interval $(-\infty, \frac{3}{2})$ or $-\infty < x < \frac{3}{2}$ is the solution of the given inequality which is shown in figure 5.3

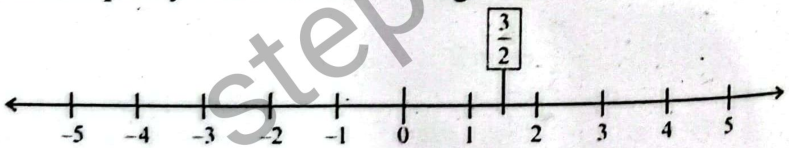


Fig. 5.3

We conclude that the solution set of an inequality consists of all solutions of the inequality.

Example 3: Graph the inequality $x + 2y < 6$.

Solution: The associated equation of the inequality

$x + 2y < 6$ (i)

is $x + 2y = 6$ (ii)

The line (ii) intersects the X-axis and Y-axis at (6,0) and (0,3) respectively. As no point of the line (ii) is a solution x of the inequality (i), so the graph of the line (ii) is shown by using dashes. We take $O(0, 0)$ as a test point because it is not on the line (ii).

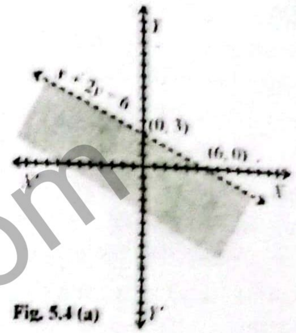


Fig. 5.4 (a)

Substituting $x = 0, y = 0$ in the expression $x + 2y$ gives $0 - 2(0) = 0 < 6$. So, the point (0, 0) satisfies the inequality (i). Any other point below the line (ii) satisfies the inequality (i), that is all points in the half plane containing the point (0,0) satisfy the inequality (i).

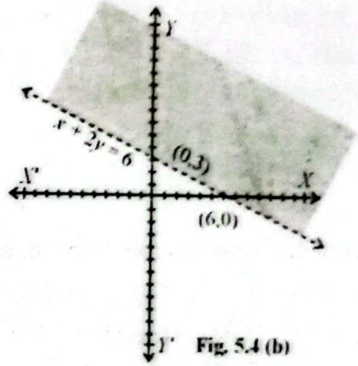


Fig. 5.4 (b)

Thus, the graph of the solution set of inequality (i) is a region on the origin-side of the line (ii), that is, the region below the line (ii). A portion of the open half plane below the line (ii) is shown as shaded region in figure 5.4(a)

All points above the dashed line satisfy the inequality $x + 2y > 6$ (iii)

A portion of the open half plane above the line (ii) is shown by shading in figure 5.4(b)

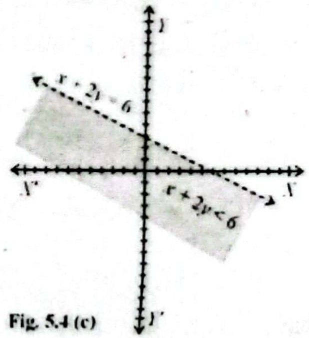


Fig. 5.4 (c)

Note: 1. The graph of the inequality $x + 2y \le 6$ ...(iv) includes the graph of the line (ii), so the open half-plane below the line (ii) including the graph of the line (ii) is the graph of the inequality (iv). A portion of the graph of the inequality (iv) is shown by shading in fig. 5.4(c).

Note: 2 All points on the line (ii) and above the line (ii) satisfy

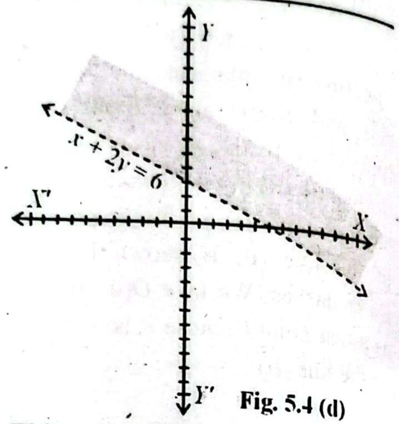


Fig. 5.4 (d)

the inequality $x + 2y \ge 6$ .... (v). This means that the solution set of the inequality (v) consists of all points above the line (ii) and all points on the lines (ii). The graph of the inequality (v) is partially shown as shaded region in fig. 5.4(d).

Note: 3 The graphs of $x + 2y \le 6$ and $x + 2y \ge 6$ are closed half planes.

Example 4. Graph the following linear inequalities in *xy*-plane:

(i) $2x \ge -3$ (ii) $y \le 2$

Solution:(i) The inequality (i) in *xy*-plane is considered as $2x + 0y \ge -3$ and its solution set consists of all point (x, y) such that $x, y \in R$ and $x \ge -\dfrac{3}{2}$

The corresponding equation of the inequality (i) is

$2x = -3$ (1)

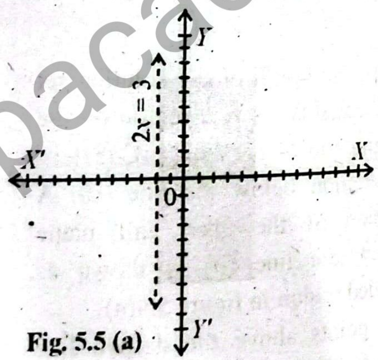


Fig. 5.5 (a)

which a vertical line (parallel to the y-axis) and its graph is drawn in figure 5.5(a).

The graph of the inequality $2x \ge -3$ is the open half plane to the right of the line (1).

Thus, the graph of $2x \ge -3$ is the closed half-plane to the right of the line (1).

(ii) The associated equation of the inequality (ii) is

$y = 2$ (2)

which is a horizontal line (parallel to the *x*-axis) and its graph is shown in figure 5.5(b). Here the solution set of the inequality $y < 2$ is the open half plane below the boundary line $y = 2$. Thus, the graph of $y \le 2$ consists of the boundary line and the open half plane below it.

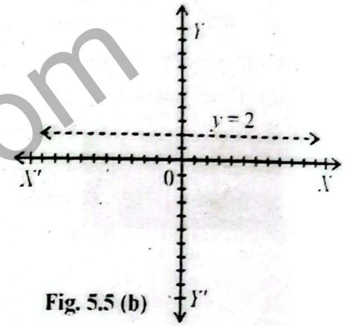


Fig. 5.5 (b)

Example 5: Find the solution region by drawing the graph the system of inequalities

$$x - 2y \le 6$$
$$2x + y \ge 2$$

Solution: $x - 2y \le 6$ ...(i)

$2x + y \ge 2$ ...(ii)

The associated equation of (i) is

$x - 2y = 6$ ...(iii)

For *x*-intercept, put $y = 0$ in (iii), we get

$x - 2(0) = 6$

$x - 0 = 6$

$\Rightarrow$ $x = 6$, so the point is (6, 0)

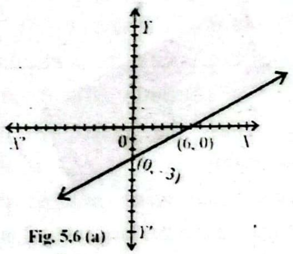


Fig. 5.6 (a)

For *y*-intercept, put $x = 0$ in (iii), we get

$0 - 2y = 6$

$\Rightarrow$ $-2y = 6$

$\Rightarrow$ $y = \dfrac{6}{-2} = -3$, so the point is (0, –3)

The graph of the line $x-2y=6$ is drawn by joining the point (6, 0) and (0, -3). The point (0,0) satisfies the inequality $x-2y<6$ because $0-2(0)=0<6$. Thus, the graph of $x-2y\leq 6$ is the upper half-plane including the graph of the line $x-2y=6$. The closed half-plane is partially shown by shading in figure 5.6(a). The associated equation of (ii) is

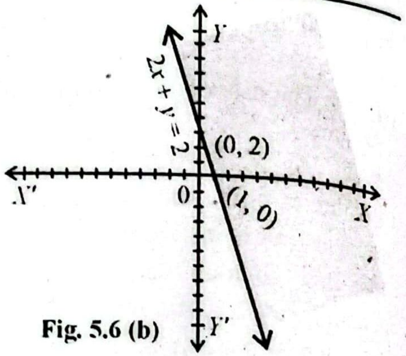


Fig. 5.6 (b)

$$2x+y=2 \quad \ldots(iv)$$

For x-intercept, put $y=0$ in (iv), we get $2x+0=2$

$\Rightarrow \quad 2x=2$

$\Rightarrow \quad x=1$, so the point is (1, 0)

For y-intercept, put $x=0$ in (iv), we get

$$2(0)+y=2$$

$\Rightarrow \quad y=2$, so the point is (0, 2)

We draw the graph of the line $2x+y=2$ joining the points (1, 0) and (0, 2). The point (0, 0) does not satisfy the inequality $2x+y>2$ because $2(0)+0=0 \not> 2$. Thus, the graph of the inequality $2x+y\geq 2$ is the closed half-plane not on the origin-side of the line $2x+y=2$ and partially shown by shading in figure 5.6(b).

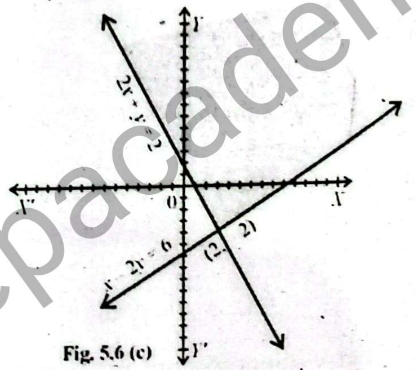


Fig. 5.6 (c)

The solution region of the given system of inequalities is the intersection of the graphs indicated in figures 5.6(a) and 5.6(b) is shown as shaded region in figure 5.6(c).

EXERCISE 5.1

1. Solve and represent the solution set on a real line.

(i) $12x+30=-6$ (ii) $\frac{x}{3}+6=-12$

(iii) $\frac{x}{2}-\frac{3x}{4}=\frac{1}{12}$ (iv) $2=7(2x+4)+12x$

(v) $\frac{2x-1}{3}-\frac{3x}{4}=\frac{5}{6}$ (vi) $\frac{-5x}{10}=9-\frac{10}{5}x$

Solution:

(i) $\mathbf{12x+30=-6}$

$$12x+30-30=-6-30$$
$$12x=-36$$
$$x=\frac{-36}{12}=-3$$

CHECK THE SOLUTION.

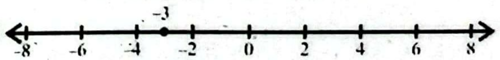
Substitute $x=-3$ into the original equation:

$$12(-3)+30=-6$$
$$-36+30=-6$$
$$-6=-6 \quad \text{(True)}$$

SOLUTION: $x=-3$

Represent the solution on the real line:

Real line: • −3

$x=-3$ 

(ii) $\frac{x}{3}+6=-12$

$$\frac{x}{3}+6-6=-12-6$$
$$\frac{x}{3}=-18$$
$$x=-18\times 3=-54$$

Check the solution:

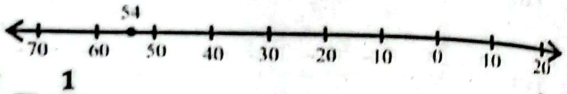
Substitute $x=-54$ into the original equation:

$$\frac{-54}{3} + 6 = -12$$
$$-18 + 6 = -12$$
$$-12 = -12 \quad \text{(True)}$$

SOLUTION: $x = -54$

Represent the solution on the real line:

Real line: • -54

$x = -54$ 

(iii) $\frac{x}{2} - \frac{3x}{4} = \frac{1}{12}$

$$\frac{2x}{4} - \frac{3x}{4} = \frac{1}{12}$$
$$\frac{2x - 3x}{4} = \frac{1}{12}$$
$$\frac{-x}{4} = \frac{1}{12}$$
$$12 \times \frac{-x}{4} = 12 \times \frac{1}{12}$$
$$-3x = 1$$
$$x = \frac{1}{-3} = -\frac{1}{3}$$

CHECK THE SOLUTION.

Substitute $x = -\frac{1}{3}$ into the original equation:

$$\frac{-\frac{1}{3}}{2} - \frac{3 \times -\frac{1}{3}}{4} = \frac{1}{12}$$
$$-\frac{1}{6} + \frac{1}{4} = \frac{1}{12}$$

Convert fractions to have a common denominator of 12:

$$-\frac{2}{12} + \frac{3}{12} = \frac{1}{12}$$
$$\frac{1}{12} = \frac{1}{12} \quad \text{(True)}$$

SOLUTION: $x = -\frac{1}{3}$

Represent the solution on the real line:

Real line: • $-\frac{1}{3}$

$x = -\frac{1}{3}$ 

(iv) $2 = 7(2x + 4) + 12x$

$$2 = 7(2x + 4) + 12x$$
$$2 = 7 \times 2x + 7 \times 4 + 12x$$
$$2 = 14x + 28 + 12x$$
$$2 = 14x + 12x + 28$$
$$2 = 26x + 28$$

Subtract 28 from both sides: $2 - 28 = 26x$

$$-26 = 26x$$

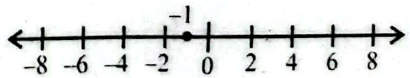
Divide both sides by 26: $x = \frac{-26}{26} = -1$

Solution:

The solution to the equation is: $x = -1$

Representing on a Real Line:

To represent the solution on a real line, plot a single point at $x = -1$.

$x = -1$ 

(v) $\frac{2x-1}{3} - \frac{3x}{4} = \frac{5}{6}$

$$\frac{4(2x - 1)}{12} - \frac{9x}{12} = \frac{10}{12}$$

$$\frac{8x - 4 - 9x}{12} = \frac{10}{12}$$

$$\frac{-x - 4}{12} = \frac{10}{12}$$

$$-x - 4 = 10$$

$$-x = 14$$

$$x = -14$$

CHECK THE SOLUTION.

Substitute $x = -14$ into the original equation:

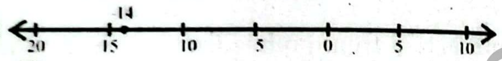
$$\frac{2(-14) - 1}{3} - \frac{3(-14)}{4} = \frac{5}{6}$$

Simplifying this shows that the equation holds true.

SOLUTION: $x = -14$

Represent the solution on the real line:

Real line: • -14

$x = -14$ 

(vi) $\frac{-5x}{10} = 9 - \frac{10}{5}x$

$$\frac{-5x}{10} = 9 - \frac{10}{5}x$$

$$-\frac{x}{2} = 9 - 2x$$

$$2 \times \left(-\frac{x}{2}\right) = 2 \times (9 - 2x)$$

$$-x = 18 - 4x$$

$$-x + 4x = 18 - 4x + 4x$$

$$3x = 18$$

Now, divide both sides by 3:

$$x = \frac{18}{3} = 6$$

Solution:

The solution to the equation is:

$$x = 6$$

Representing on a Real Line:

To represent the solution on a real line, plot a single point at $x = 6$.

$x = 6$ 

2. Solve each inequality and represent the solution on a real line.

(i) $x - 6 \leq -2$ (ii) $-9 > -16 + x$

(iii) $3 + 2x \geq 3$ (iv) $6(x + 10) \leq 0$

(v) $\frac{5}{3}x - \frac{3}{4} < \frac{-1}{12}$ (vi) $\frac{1}{4}x - \frac{1}{2} \leq -1 + \frac{1}{2}x$

Solution:

(i) $x - 6 \leq -2$

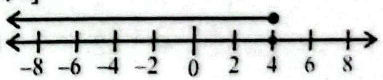
Solution:

$$x - 6 \leq -2$$

$$x \leq -2 + 6$$

$$x \leq 4$$

Solution set: $(-\infty, 4]$

$x \leq 4$ 

(ii) $-9 > -16 + x$

Solution:

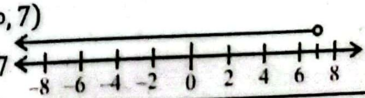
$$-9 > -16 + x$$

$$-9 + 16 > x$$

$$7 > x$$

$$x < 7$$

Solution set: $(-\infty, 7)$

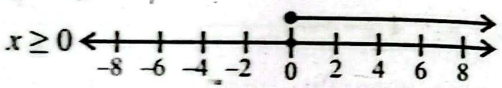
$x < 7$ 

(iii) $3 + 2x \geq 3$

Solution:

$$3 + 2x \geq 3$$
$$2x \geq 3 - 3$$
$$2x \geq 0$$
$$x \geq 0$$

Solution set: $[0, \infty)$

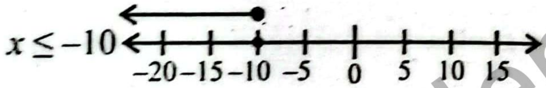


(iv) $6(x + 10) \leq 0$

Solution:

$$6(x + 10) \leq 0$$
$$x + 10 \leq 0$$
$$x \leq -10$$

Solution set: $(-\infty, -10]$



(v) $\frac{5}{3}x - \frac{3}{4} < \frac{-1}{12}$

Solution:

$$\frac{5}{3}x - \frac{3}{4} < \frac{-1}{12}$$

Add $\frac{3}{4}$ to both sides:

$$\frac{5}{3}x < \frac{-1}{12} + \frac{3}{4}$$

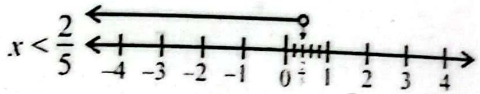
Convert $\frac{3}{4}$ to $\frac{9}{12}$:

$$\frac{5}{3}x < \frac{-1}{12} + \frac{9}{12} = \frac{8}{12} = \frac{2}{3}$$

Multiply both sides by $\frac{3}{5}$ to solve for x:

$$x < \frac{2}{3} \times \frac{3}{5} = \frac{2}{5}$$

Solution set: $\left(-\infty, \frac{2}{5}\right)$



(vi) $\frac{1}{4}x - \frac{1}{2} \leq -1 + \frac{1}{2}x$

Solution:

$$\frac{1}{4}x - \frac{1}{2} \leq -1 + \frac{1}{2}x$$

Add $\frac{1}{2}$ to both sides:

$$\frac{1}{4}x \leq -\frac{1}{2} + \frac{1}{2}x$$

Subtract $\frac{1}{4}x$ from both sides: $+\frac{1}{2} \leq -\frac{1}{4}x$

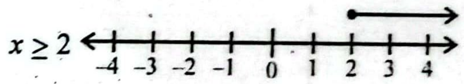
Multiply both sides by -4 (which reverses the inequality):

$$2 \geq -x$$

or

$$x \geq 2$$

Solution set: $x\ (2, \infty)$



3. Shade the solution region for the following linear inequalities in *xy*-plane:

(i)	$2x + y < 6$	**(ii)**	$3x + 7y \geq 21$
(iii)	$3x - 2y \geq 6$	**(iv)**	$5x - 4y \leq 20$
(v)	$2x + 1 \geq 0$	**(vi)**	$3y - 4 \leq 0$

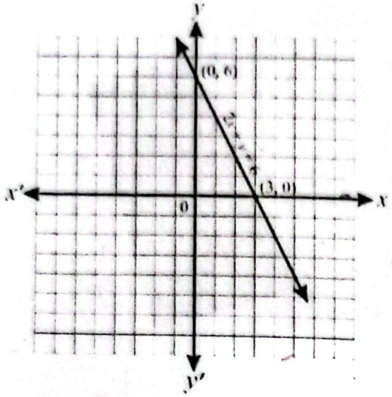
Solution:

(i) $2x + y < 6$

- **Boundary Line:** $2x + y = 6$.
  - Points: When $x = 0, y = 6$; when $y = 0, x = 3$.
  - Line: Solid line connecting (0,6) and (3,0).
- **Test Point:** Substitute (0,0):

  $2(0) + 0 \leq 6 \Rightarrow 0 \leq 6$ (true).

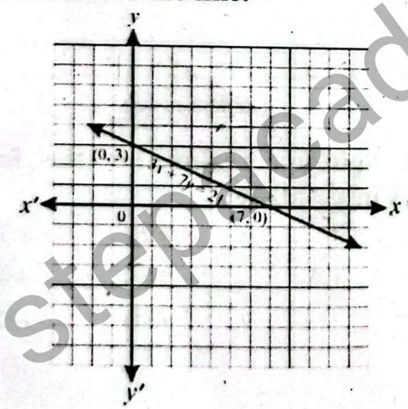
- Shade below the line.



(ii) $3x + 7y \geq 21$

- **Boundary Line**: $3x + 7y = 21$.
  - Points: When $x = 0, y = 3$; when $y = 0, x = 7$.
  - Line: Solid line connecting $(0,3)$ and $(7,0)$.
- **Test Point**: Substitute $(0,0)$:

  $3(0) + 7(0) \geq 21 \quad \Rightarrow \quad 0 \geq 21 \quad$ (false).
  - Shade above the line.

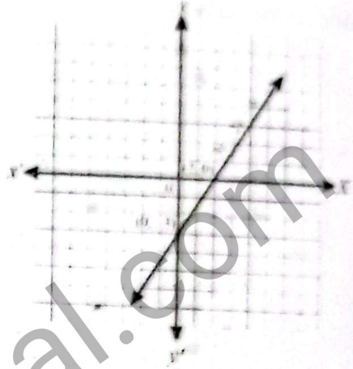


(iii) $3x - 2y \geq 6$

- **Boundary Line**: $3x - 2y = 6$.
  - Points: When $x = 0, y = -3$; when $y = 0, x = 2$.
  - Line: Solid line connecting $(0, -3)$ and $(2,0)$.
- **Test Point**: Substitute $(0,0)$:

  $3(0) - 2(0) \geq 6 \quad \Rightarrow \quad 0 \geq 6 \quad$ (false).

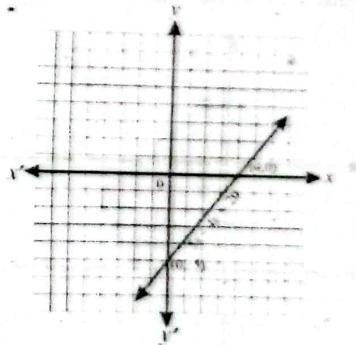
- Shade above the line.



(iv) $5x - 4y \leq 20$

- **Boundary Line**: $5x - 4y = 20$.
  - Points: When $x = 0, y = -5$; when $y = 0, x = 4$.
  - Line: Solid line connecting $(0, -5)$ and $(4,0)$.
- **Test Point**: Substitute $(0,0)$:

  $5(0) - 4(0) \leq 20 \quad \Rightarrow \quad 0 \leq 20 \quad$ (true).
  - Shade below the line.

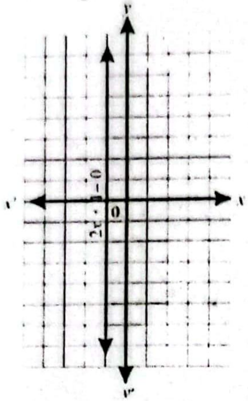


(v) $2x + 1 > 0$

- **Boundary Line**: $2x + 1 = 0$.
  - Point: $x = -\frac{1}{2}$; vertical line passing through $x = -\frac{1}{2}$.
- **Test Point**: Substitute $(0,0)$:

  $2(0) + 1 > 0 \quad \Rightarrow \quad 1 > 0 \quad$ (true).

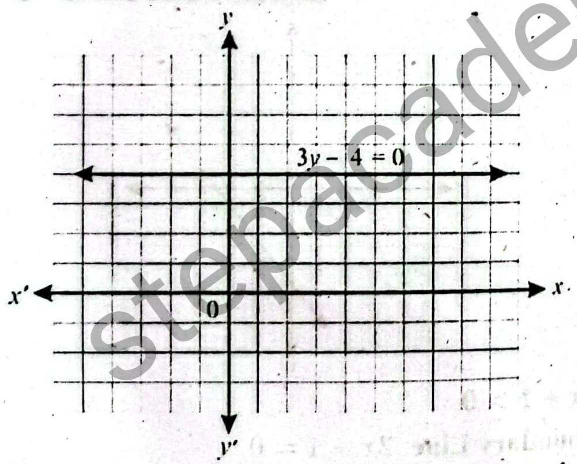
- Shade to the right of the line.



(vi) $3y - 4 \leq 0$

- **Boundary Line**: $3y - 4 = 0$.
  - Point: $y = \frac{4}{3}$; horizontal line passing through $y = \frac{4}{3}$.
- **Test Point**: Substitute (0,0):

  $3(0) - 4 \leq 0 \quad \Rightarrow \quad -4 \leq 0$ (true).
  - Shade below the line.



4. Indicate the solution region of the following linear inequalities by shading:

(i) $2x - 3y \leq 6$, $2x + 3y \leq 12$ **(ii)** $x + y \geq 5$, $-y + x \leq 1$

(iii) $3x + 7y \geq 21$, $x - y \leq 2$ **(iv)** $4x - 3y \leq 12$, $x \geq -\frac{3}{2}$

(v) $3x - 7y \geq 21$, $y \leq 4$ **(vi)** $5x + 7y \leq 35$, $x - 2y \leq 2$

Solution:

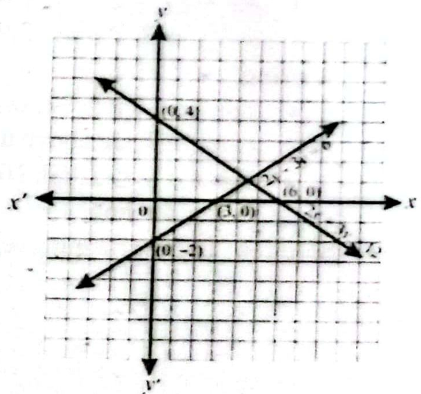
(i) $2x - 3y \leq 6$ **and** $2x + 3y \leq 12$

- **Boundary Lines**:
  - $2x - 3y = 6$: Points (3,0) and (0, −2).
  - $2x + 3y = 12$: Points (6,0) and (0,4).
- **Shading**:

a. For $2x - 3y \leq 6$, test (0,0): $2(0) - 3(0) \leq 6$ (true). Shade below the line.

b. For $2x + 3y \leq 12$, test (0,0): $2(0) + 3(0) \leq 12$ (true). Shade below the line.

Solution: The overlapping shaded region is below both lines.

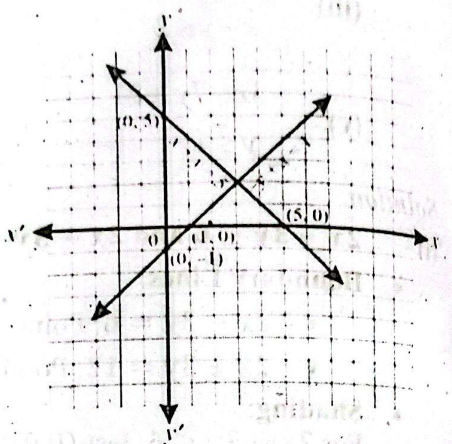


(ii) $x + y \geq 5$ **and** $-y + x \leq 1$

- **Boundary Lines**:
- $x + y = 5$: Points (5,0) and (0,5).
- $-y + x = 1$: Points (1,0) and (0, −1).
- **Shading**:
- For $x + y \geq 5$, test (0,0): $0 + 0 \geq 5$ (false). Shade above the line.

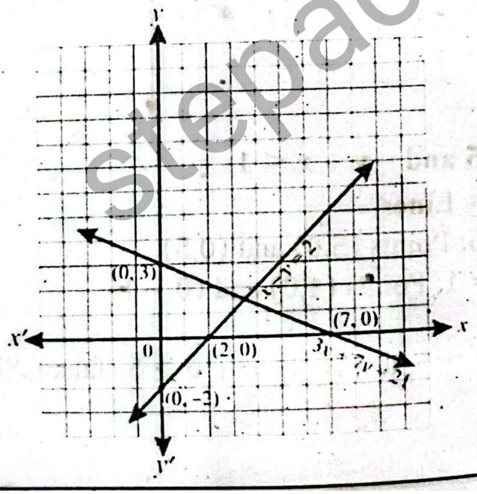
- For $-y + x \leq 1$, test $(0,0)$: $0 - 0 \leq 1$ (true). Shade below the line.

Solution: The region where $x + y \geq 5$ and $-y + x \leq 1$ overlap.



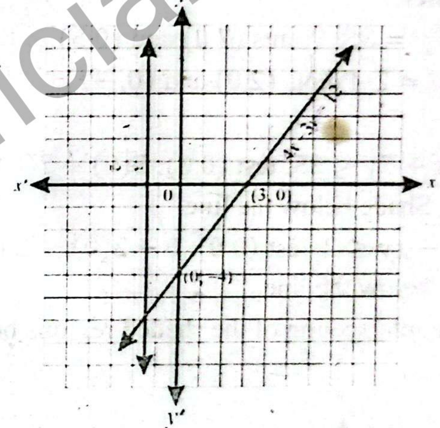
(iii) $\mathbf{3x + 7y \geq 21}$ **and** $\mathbf{x - y \leq 2}$

- **Boundary Lines:**
  - $3x + 7y = 21$: Points $(7,0)$ and $(0,3)$.
  - $x - y = 2$: Points $(2,0)$ and $(0,-2)$.
- **Shading:**
  - For $3x + 7y \geq 21$, test $(0,0)$: $3(0) + 7(0) \geq 21$ (false). Shade above the line.
  - For $x - y \leq 2$, test $(0,0)$: $0 - 0 \leq 2$ (true). Shade below the line.
- **Solution:** The overlapping region of the shaded areas.



(iv) $\mathbf{4x - 3y \leq 12}$ **and** $\mathbf{x \geq -\frac{3}{2}}$

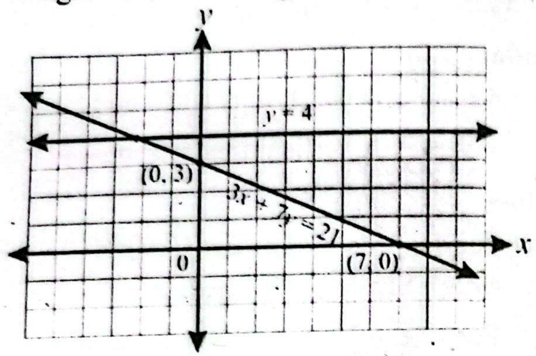
- **Boundary Lines:**
  - $4x - 3y = 12$: Points $(3,0)$ and $(0,-4)$.
  - $x = -\frac{3}{2}$: A vertical line at $x = -\frac{3}{2}$.
- **Shading:**
  - For $4x - 3y \leq 12$, test $(0,0)$: $4(0) - 3(0) \leq 12$ (true). Shade below the line.
  - For $x \geq -\frac{3}{2}$, shade to the right of the vertical line.
- **Solution:** The intersection of the two regions.



(v) $\mathbf{3x - 7y \geq 21}$ **and** $\mathbf{y \leq 4}$

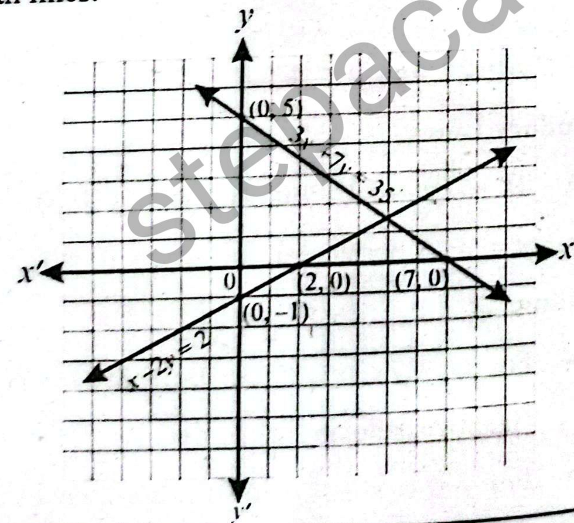
- **Boundary Lines:**
  - $3x - 7y = 21$: Points $(7,0)$ and $(0,3)$.
  - $y = 4$: A horizontal line at $y = 4$.
- **Shading:**
  - For $3x - 7y \geq 21$, test $(0,0)$: $3(0) - 7(0) \geq 21$ (false). Shade above the line.
  - For $y \leq 4$, shade below the horizontal line.

Solution: The region above $3x - 7y = 21$ and below $y = 4$.



(vi) $\mathbf{5x + 7y \leq 35}$ **and** $\mathbf{x - 2y \leq 2}$

- **Boundary Lines:**
  - $5x + 7y = 35$: Points (7,0) and (0,5).
  - $x - 2y = 2$: Points (2,0) and $(0, -1)$.
- **Shading:**
  - For $5x + 7y \leq 35$, test (0,0): $5(0) + 7(0) \leq 35$ (true). Shade below the line.
  - For $x - 2y \leq 2$, test (0,0): $0 - 2(0) \leq 2$ (true). Shade below the line.
- **Solution**: The intersection of the shaded regions below both lines.



Example 6: Shade the feasible region and ınd the corner points for the following system of inequalities (or subject to the following constraints).

$$x - y \leq 3$$
$$x + 2y \leq 6, \quad x \geq 0, \quad y \geq 0$$

Solution: The associated equations for the inequalities

$x - y \leq 3$ (i) and $x + 2y \leq 6$ (ii)

are $x - y = 3$ (1) and $x + 2y = 6$ (2)

As the point (3, 0) and (0,–3) are on the line (1), so the graph of $x - y = 3$ is drawn by joining the points (3, 0) and (0,–3) by solid line.

Similarly, line (2) is graphed by joining the points (6, 0) and (0, 3) by solid line. For $x = 0$ and $y = 0$, we have;

$0 - 0 = 0 < 3$ and $0 + 2(0) = 0 < 6$

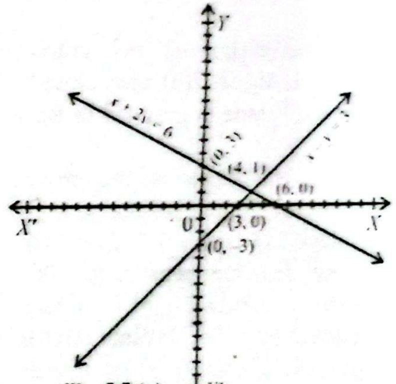


Fig. 5.7 (a)

So, both the closed half-planes are on the origin sides of the lines (1) and (2). The intersection of these closed half-planes is partially displayed as shaded region in fig. 5.7(a).

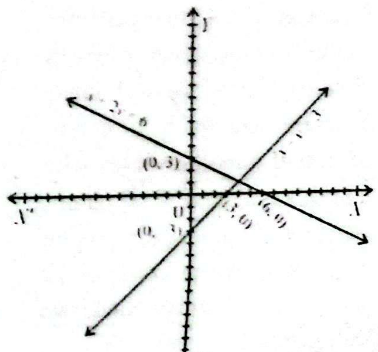


Fig. 5.7 (b)

The graph of $y \geq 0$, will be the closed upper half plane.

The intersection of graph shown in figure 5.7(a) and closed upper half plane is partially displayed as shaded region in figure 5.7(b). The graph of $x \geq 0$ will be closed right half plane.

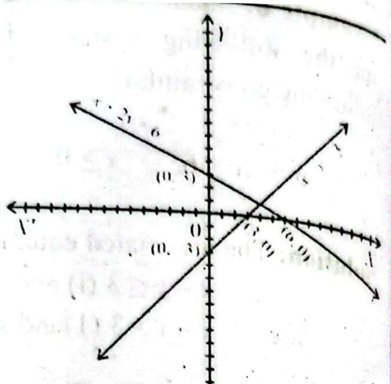


Fig. 5.7 (d)

The intersection of the graph shown in fig. 5.7(a) and closed right half plane is graphed in fig. 5.7(c).

Finally, the graph of the given system of linear inequalities is displayed in figure 5.7 (d) which is the feasible region for the given system of linear inequalities. The points (0,0), (3,0), (4, 1) and (0, 3) are corner points of the feasible region.

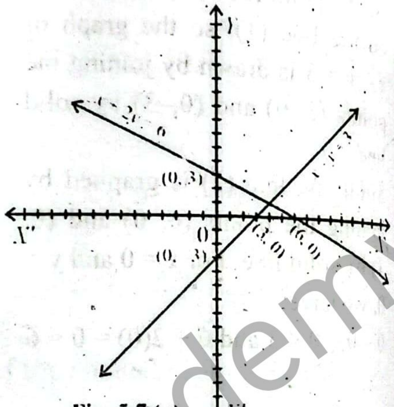


Fig. 5.7 (c)

Example 7: A manufacturer wants to make two types of concrete. Each bag of A grade concrete contains 8 kilograms of gravel (small pebbles with coarse sand) and 4 kilograms of cement while each bag of B-grade concrete contains 12 kilograms of gravel and two kilograms of cement. If there are 1920 kilograms of gravel and 480 kilograms of cement, then graph the feasible region under the given restrictions and find corner points of the feasible region.

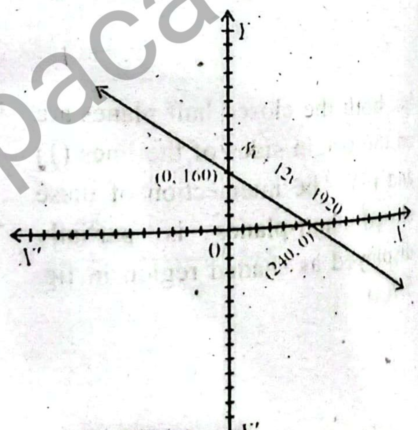


Fig. 5.8 (a)

Solution: Let x be the number of bags of A-grade concrete produced and y denote the number of bags of B-grade concrete produced, then $8x$ kilograms of gravel will be used for A-grade concrete and $12y$ kilograms of gravel will be required for B-grade concretes so $8x + 12y$ should not exceed 1920, that is,

$$8x + 12y \leq 1920$$

Similarly, the linear constraint for cement will be $4x + 2y \leq 480$

Now we have to graph the feasible region for the linear constraints

$$8x + 12y \leq 1920$$

$$4x + 2y \leq 480, \quad x \geq 0, y \geq 0$$

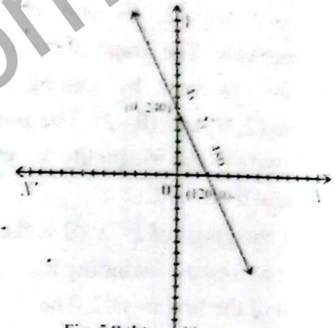


Fig. 5.8 (b)

Taking the one unit along x-axis and y-axis equal to 40 we draw the graph of the required feasible region.

The shaded region of figure 5.8(a) shows the graph of $8x + 12y \leq 1920$ including the non-negative constraints $x \geq 0$ and $y \geq 0$

In the figure 5.8(b), the graph of $4x + 2y \leq 480$ including the non-negative constraints $x \geq 0$ and $y \geq 0$ is displayed as shaded region.

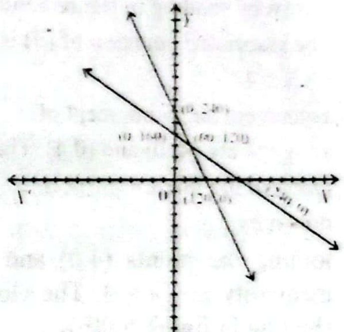


Fig. 5.8 (c)

The intersection of these two graphs is shown as shaded region in figure 5.5(c), which is the feasible region for the given linear constraints.

The point (0, 0), (120, 0), (60, 120) and (0, 160) are the corner points of the feasible region.

Example 8: Find the maximum and minimum values of the function defined as:

$f(x,y)=2x+3y$

subject to the constraints; $x-y \leq 2$

$x + y \leq 4$

$x \geq 0,\ y \geq 0$

Solution:

$x - y \leq 2$ ...(i)

$x + y \leq 4$ ...(ii)

The associated equation of (i) is

$x - y = 2$

x-intercept and y-intercept of $x - y = 2$ are (2, 0) and (0,–2) respectively. The graph of the line $x - y = 2$ is draw by joining the points (2, 0) and (0,–2). The point (0,0) satisfy the inequality $x - y \leq 2$ because $0 - 0 = 0 < 2$.

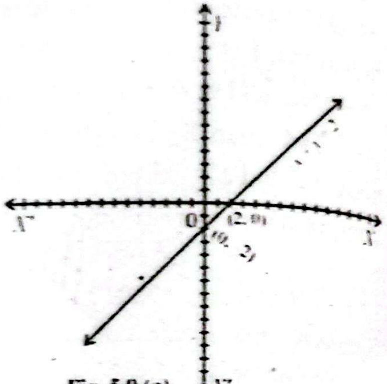


Fig. 5.9 (a)

Thus, the graph of $x - y \leq 2$ is the upper half-plane including the graph of the line $x - y = 2$. The closed half-plane is partially shown by shading in figure 5.9(a).

The associated equation of (ii) is

$x + y = 4$

x-intercept and y-intercept of $x + y = 4$ are (4, 0) and (0,4), The graph of the line $x + y = 4$ is drawn by joining the points (4,0) and (0,4). The point (0,0) satisfies the inequality $x + y \leq 4$. The closed half-plane is partially shown by shading in figure 5.9(b).

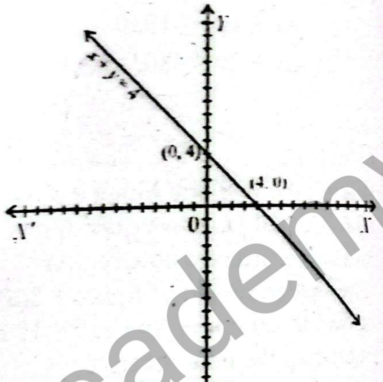


Fig. 5.9 (b)

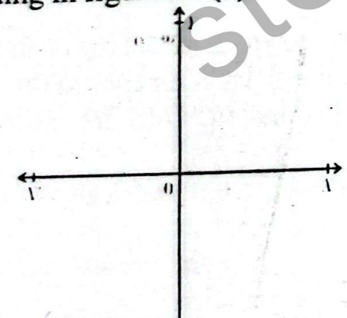


Fig. 5.9 (c)

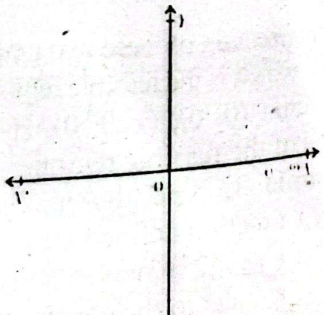


Fig. 5.9 (d)

The graph of $x \geq 0$ and $y \leq 0$ is shown by shading in figures 5.9(c) and 5.9(d) respectively.

The feasible region of the given system of inequalities is the intersection of the graphs indicated in figures 5.9(a), 5.9(b), 5.9(c) and 5.9(d) and is shown as shaded region *ABCD* in figure 5.9(e).

Corner points of the feasible region are (0,0), (2,0), (3, 1) and (0,4). Now, we find values of $f(x,y) = 2x + 3y$ at the corner points.

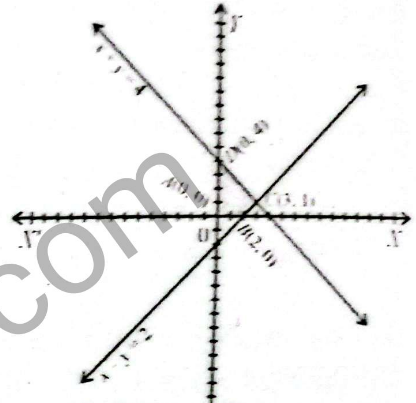


Fig. 5.9 (e)

$f(0,0) = 2(0) + 3(0) = 0$

$f(2,0) = 2(2) + 3(0) = 4$

$f(3,1) = 2(3) + 3(1) = 9$

$f(0,4) = 2(0) + 3(4) = 12$

Thus, the minimum value of f is 0 at the corner point (0,0) and maximum value of f at corner point (0,4) is 12.

EXERCISE 5.2

1. **Maximize $f(x, y) = 2x + 5y$; subject to the constraints $2y - x \leq 8$; $x - y \leq 4$; $x \geq 0$; $y \geq 0$**

Solution:

Problem:

Maximize $f(x,y) = 2x + 5y$, subject to the constraints:

$$2y - x \leq 8, \quad x - y \leq 4, \quad x \geq 0, \quad y \geq 0$$

Step 1: Associated Equations for the Inequalities

The associated equations for the inequalities are:

- $2y - x = 8$ (Equation 1)
- $x - y = 4$ (Equation 2)

Step 2: Graph the Equations

GRAPHING $2y - x = 8$ (EQUATION 1):

Rearranging, we get:

$$x = 2y - 8$$

We can find two points for plotting the line:

- When $y = 0, x = 2(0) - 8 = -8$ (but $x \geq 0$ does not hold, so the point $(-8, 0)$ is not valid.
- When $y = 4, x = 2(4) - 8 = 0$ (so the point $(0,4)$ is valid.)

Thus, the line $2y - x = 8$ passes through $(0,4)$.

GRAPHING $x - y = 4$ (EQUATION 2):

Rearranging, we get:

$$x = y + 4$$

We can find two points for plotting the line: $(4, 0)$, $(0, -4)$

- When $y = 0, x = 0 + 4 = 4$ (so the point is $(4,0)$).
- When $x = 0, y = -4$ (but $y \geq 0$, so the point $(0, -4)$ is not valid).

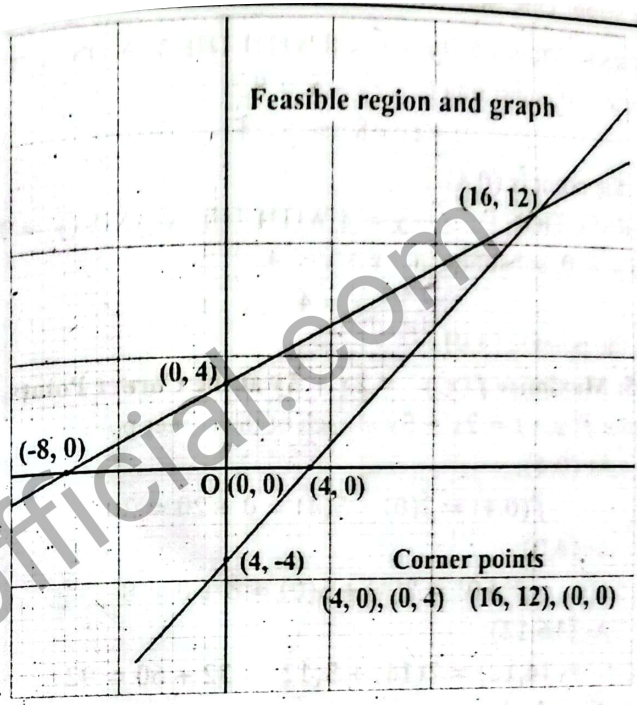
Thus, the line $x - y = 4$ passes through $(4,0)$.

Step 3: Plot the Constraints

Now, we plot the inequalities:

- For $2y - x \leq 8$, the region below the line $2y - x = 8$ (i.e., the half-plane below the line).
- For $x - y \leq 4$, the region below the line $x - y = 4$ (i.e., the half-plane below the line).
- The region for $x \geq 0$, the right half-plane.
- The region for $y \geq 0$, the upper half-plane.

These constraints, when plotted on a graph, will create a bounded region called the **feasible region**.



Step 4: Find the Corner Points of the Feasible Region

We need to determine the points where the boundaries of the feasible region intersect. These are the **corner points**.

INTERSECTION OF $2y - x = 8$ AND $x - y = 4$:

Solve the system of equations:

- $2y - x = 8$
- $x - y = 4$

From the second equation, solve for x:

$$x = y + 4$$

Substitute $x = y + 4$ into the first equation:

$$2y-(y+4)=8 \Rightarrow 2y-y-4=8 \Rightarrow y=12$$

Substitute $y = 12$ into $x = y + 4$:

$$x = 12 + 4 = 16$$

Thus, the intersection point is $(16,12)$.

INTERSECTION OF $2y - x = 8$ WITH THE Y-AXIS ($x = 0$):

When $x = 0$, substitute into $2y - x = 8$:

$$2y = 8 \Rightarrow y = 4$$

Thus, the point is (0,4).

INTERSECTION OF $x - y = 4$ WITH THE X-AXIS ($y = 0$):

When $y = 0$, substitute into $x - y = 4$:

$$x = 4$$

Thus, the point is (4,0).

Step 5: Maximize $f(x,y) = 2x + 5y$ at the Corner Points

Evaluate $f(x,y) = 2x + 5y$ at each of the corner points:

- At (0,4):
$$f(0,4) = 2(0) + 5(4) = 0 + 20 = 20$$
- At (4,0):
$$f(4,0) = 2(4) + 5(0) = 8 + 0 = 8$$
- At (16,12):
$$f(16,12) = 2(16) + 5(12) = 32 + 60 = 92$$

Step 6: Conclusion

The maximum value of $f(x,y) = 2x + 5y$ is $\boxed{92}$, and it occurs at the point (16,12).

Feasible Region and Graph:

- The feasible region is the area bounded by the lines $2y - x = 8$, $x - y = 4$, the x-axis, and the y-axis.
- The corner points of the feasible region are (0,4), (4,0), and (16,12).

2. **Maximize $f(x, y) = x + 3y$; subject to the constraints $2x + 5y \le 30$; $5x + 4y \le 20$; $x \ge 0$; $y \ge 0$**

Solution: The associated equations are

$$2x + 5y = 30 \quad \ldots \text{(i)}$$

and

$$5x + 4y = 20 \quad \ldots \text{(ii)}$$

x-intercept

Put $y = 0$ in (i)

$$2x + 5(0) = 30$$

$$2x = 30$$

$$x = 15$$

1st ordered pair = (15, 0)

y-intercept

Put $x = 0$ in (i)

$$2(0) + 5y = 30$$

$$5y = 30$$

$$y = \frac{30}{5}$$

$$y = 6$$

Second ordered pair = (0, 6)

Put $x = y = 0$ in $2x + 5y < 30$ (origin test)

$$2(0) + 5(0) < 30$$

$$0 < 30$$

Which is true, so shading lies towards origin test.

Now consider,

$$5x + 4y = 20$$

x-intercept

Put $y = 0$

$$5x + 4(0) = 20$$

$$5x = 20$$

$$x = \frac{20}{5} = 4$$

1st ordered pair is (4, 0)

y-intercept

Put $x = 0$

$$5(0) + 4y = 20$$

$$4y = 20$$

$$y = 5$$

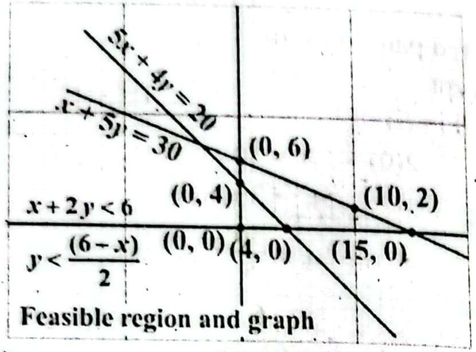
2nd ordered pair is (0, 5)

Put $x = y = 0$ in $5x + 4y < 20$

$$5(0) + 4(0) < 20$$

$$0 < 20$$

Which is true, so shading lies towards origin side.
Now we draw the lines.



Feasible region and graph

O (0, 0) A (4, 0)
B (0, 5)
OAB is the feasible region.
Corner points are $O(0, 0)$, $A(4, 0)$, $B(0, 5)$
Now, $f(x, y) = x + 3y$
$O(0, 0)$

$$f(0, 0) = 0 + 3(0) = 0$$

$A(4, 0)$

$$f(4, 0) = 4 + 3(0) = 4$$

$B(0, 5)$

$$f(0, 5) = 0 + 3(5)$$
$$= 15$$

Clearly maximum value of $f(x,y)$ is 15 at corner point (0,5).

3. **Maximize z = 2x + 3y; subject to the constraints:**

$$2x + y \le 4; \quad 4x - y \le 4; \quad x \ge 0: \; y \ge 0$$

Solution: The associated equations are

$$2x + y = 4 \quad \ldots (i)$$

and $$5x - y = 4 \quad \ldots (ii)$$

x-intercept
Put $y = 0$ in (i)

$$2x = 4$$
$$x = 2$$

1st ordered pair = (2, 0)
y-intercept
Put $x = 0$

$$2(0) + y = 4$$
$$y = 4$$

Second ordered pair = (0, 4)
Put $x = y = 0$ in $2x + y < 4$ (origin test)

$$2(0) + 0 < 4$$
$$0 < 4$$

Which is true, so shading lies towards origin test.
Now consider, $4x - y = 20$
x-intercept
Put $y = 0$ in (ii)

$$4x - 0 = 20$$
$$4x = 4$$
$$x = 1$$

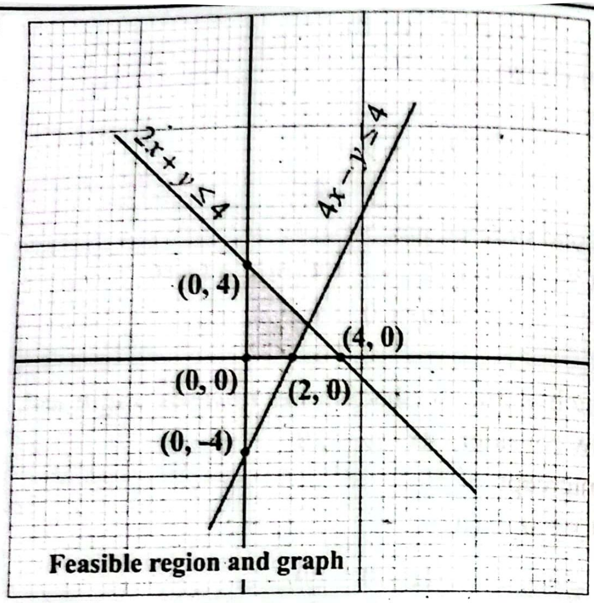
1st ordered pair is (1, 0)
y-intercept
Put $x = 0$ in (ii)

$$4(0) - y = 4$$
$$-y = 4$$
$$y = -3$$

2nd ordered pair is (0, −3)
Put $x = y = 0$ in $4x - y < 4$

$$4(0) - 0 < 4$$
$$0 < 4$$

Which is true, so shading lies towards origin side.
Now we draw the lines.



Feasible region and graph

O (0, 0), A(1, 0), B(0, 4), C(x, y)

OABC is the feasible region.

Corner points are $O(0, 0)$, $A(1, 0)$, $B(0, 4)$, $C(x, y) = ?$

Solving both equations to find corner point.

Adding (i) and (ii)

$$2x + y = 4$$
$$4x - y = 4$$
$$6x = 8$$
$$x = \frac{8}{6}$$
$$x = \frac{4}{3}$$

Put $x = \frac{4}{3}$ in 1st equation.

$$2\left(\frac{4}{3}\right) + y = 4$$
$$\frac{8}{3} + y = 4$$
$$y = 4 - \frac{8}{3}$$
$$y = \frac{12 - 8}{3} = \frac{4}{3}$$

Corner point $C\left(\frac{4}{3}, \frac{4}{3}\right)$

Putting all corner points in $z = 2x + 3y$

$f(0, 0) = 2(0) + 3(0) = 0$

$f(1, 0) = 2(1) + 3(0) = 2$

$f(0, 4) = 2(0) + 3(4) = 12$

$$f\left(\frac{4}{3}, \frac{4}{3}\right) = 2\left(\frac{4}{3}\right) + 3\left(\frac{4}{3}\right)$$
$$= \frac{8}{3} + 4$$
$$= \frac{8 + 12}{3} = \frac{20}{3} = 6.666$$

So maximum value of function is $\frac{12}{3}$ at corner point (0, 4).

4. Minimize $z = 2x + y$; subject to the constraints:

$$x + y \geq 3; \quad 7x + 5y \leq 35; \quad x \geq 0; \quad y \geq 0$$

Solution: Let's solve the given linear programming problem step by step.

Problem:

Minimize $z = 2x + y$, subject to the constraints:

$$x + y \geq 3, \quad 7x + 5y \leq 35, \quad x \geq 0, \quad y \geq 0$$

Step 1: Associated Equations for the Inequalities

The associated equations for the inequalities are:

- $x + y = 3$ (Equation 1)
- $7x + 5y = 35$ (Equation 2)

Step 2: Graph the Equations

GRAPHING $x + y = 3$ (EQUATION 1):

Rearranging, we get:

$$y = 3 - x$$

We can find two points for plotting the line:

- When $x = 0$, $y = 3 - 0 = 3$ (so the point is (0,3)).
- When $y = 0$, $x = 3 - 0 = 3$ (so the point is (3,0)).

Thus, the line $x + y = 3$ passes through the points (0,3) and (3,0).

GRAPHING $7x + 5y = 35$ (EQUATION 2):

Rearranging, we get:

$$y = \frac{35 - 7x}{5}$$

We can find two points for plotting the line:

- When $x = 0$, $y = \frac{35-7(0)}{5} = \frac{35}{5} = 7$ (so the point is (0,7)).
- When $y = 0$, $7x = 35 \Rightarrow x = 5$ (so the point is (5,0)).

Thus, the line $7x + 5y = 35$ passes through the points (0,7) and (5,0).

Step 3: Plot the Constraints

Now, we plot the inequalities on the graph:

- For $x + y \geq 3$, the region above the line $x + y = 3$.
- For $7x + 5y \leq 35$, the region below the line
- $7x + 5y = 35$.
- The region for $x \geq 0$, the right half-plane.
- The region for $y \geq 0$, the upper half-plane.

Step 4: Find the Corner Points of the Feasible Region

We need to determine the points where the boundaries of the feasible region intersect. These are the **corner points.**

INTERSECTION OF $x + y = 3$ AND $7x + 5y = 35$:

Solve the system of equations:

- $x + y = 3$
- $7x + 5y = 35$

From the first equation, solve for y:

$$y = 3 - x$$

Substitute $y = 3 - x$ into the second equation:

$$7x + 5(3 - x) = 35$$
$$7x + 15 - 5x = 35$$
$$2x = 20 \Rightarrow x = 10$$

Substitute $x = 10$ into the first equation $x + y = 3$:

$$10 + y = 3 \Rightarrow y = -7$$

Since $y = -7$ is not a valid point in the feasible region (as $y \geq 0$), this intersection point is not valid.

INTERSECTION OF $x + y = 3$ AND THE X-AXIS (I.E., $y = 0$):

Substitute $y = 0$ into the equation $x + y = 3$:

$$x + 0 = 3 \Rightarrow x = 3$$

So, the intersection point is (3,0).

INTERSECTION OF $7x + 5y = 35$ AND THE X-AXIS (I.E., $y = 0$):

Substitute $y = 0$ into the equation $7x + 5y = 35$:

$$7x + 0 = 35 \Rightarrow x = 5$$

So, the intersection point is (5,0).

INTERSECTION OF $x + y = 3$ AND THE Y-AXIS (I.E., $x = 0$):

Substitute $x = 0$ into the equation $x + y = 3$:

$$0 + y = 3 \Rightarrow y = 3$$

So, the intersection point is (0,3).

INTERSECTION OF $7x + 5y = 35$ AND THE Y-AXIS (I.E., $x = 0$):

Substitute $x = 0$ into the equation $7x + 5y = 35$:

$$0 + 5y = 35 \Rightarrow y = 7$$

So, the intersection point is (0,7).

Step 5: Evaluate the Objective Function at the Corner Points

The feasible region's corner points are:

- (3,0) • (5,0)
- (0,3) • (0,7)

Evaluate the objective function $z = 2x + y$ at these points:

- At (3,0): $z = 2(3) + 0 = 6$

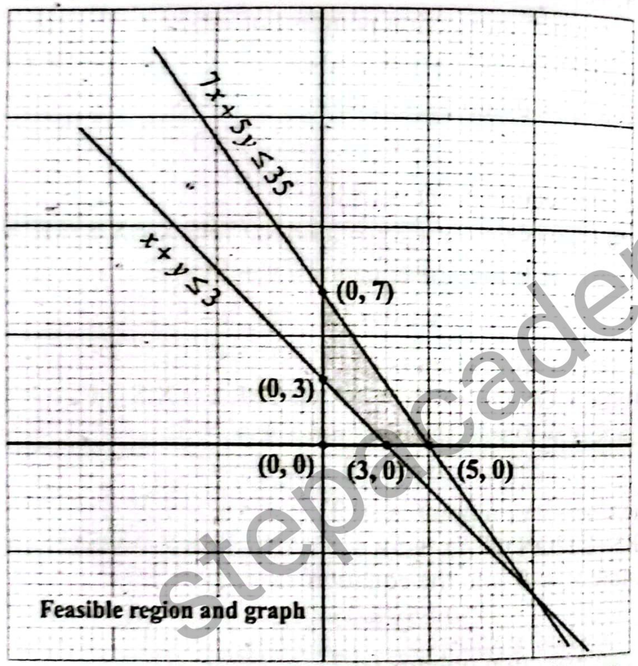
- At (5,0): $z = 2(5) + 0 = 10$
- At (0,3): $z = 2(0) + 3 = 3$
- At (0,7): $z = 2(0) + 7 = 7$

Step 6: Conclusion

The **minimum value** of $z = 2x + y$ is $\boxed{3}$, and it occurs at the point (0,3).

Feasible Region and Graph:

- The feasible region is the area bounded by the lines $x + y = 3$, $7x + 5y = 35$, the x-axis, and the y-axis.
- The corner points of the feasible region are (3,0), (5,0), (0,3), and (0,7).



Feasible region and graph

5. **Maximize the function defined as; $f(x, y) = 2x + 3y$ subject to the constraints:**

$$2x + y \le 8; \quad x + 2y \le 14; \quad x \ge 0; \quad y \ge 0$$

Solution: The associated equations are

$$2x + y = 8 \quad \dots (i)$$

$$x + 2y = 14 \quad \dots (ii)$$

x-intercept

Put $y = 0$

$$2x + 0 = 8$$
$$2x = 8$$
$$x = \frac{8}{2} = 4$$

1st ordered pair is (4, 0)

y-intercept

Put $x = 0$

$$2(0) + y = 8$$
$$y = 8$$

Second ordered pair is (0, 8)

Put $x = y = 0$

$$2x + y < 8$$
$$2(0) + 0 < 8$$
$$0 < 8$$

Which is true, so shading lies away from origin side.

x-intercept

Put $y = 0$

$$x + 2(0) = 14$$
$$x = 14$$

1st ordered pair is (14, 0)

y-intercept

Put $x = 0$

$$0 + 2y = 14$$
$$2y = 14$$
$$y = 7$$

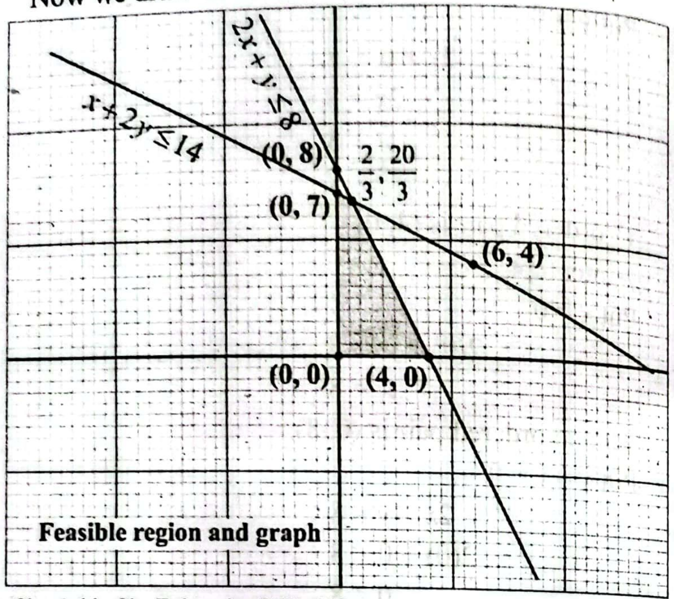
2nd ordered pair is (0, 7)

Put $x = y = 0$ in

$$x + 2y < 14$$
$$0 + 2(0) < 14$$
$$0 < 14$$

Which is true, so shading lies towards origin side.

Now we draw the lines.



Feasible region and graph

{O(0, 0), A(4, 0), B(x, y), C(0, 7)}

From diagram, common shaded region is *OABC*

Corner points are $O(0, 0)$, $A(4, 0)$, $C(0, 7)$, $B(x, y)$

We find corner point $B(x, y)$ by solving (i) and (ii) as

Multiply (i) by (ii) and subtract from (ii)

$$x + 2y = 14$$
$$\underline{4x + 2y = 16}$$
$$3x = -2$$
$$x = \frac{2}{3}$$

Put $x = \frac{2}{3}$ in 1st for y.

$$2\left(\frac{2}{3}\right) + y = 8$$
$$\frac{4}{3} + y = 8$$
$$y = 8 - \frac{4}{3}$$

$$y = \frac{24 - 4}{3} = \frac{20}{3}$$

So corner point B has coordinates as $\left(\frac{2}{3}, \frac{20}{3}\right)$

Now, $f(x, y) = 2x + 3y$

Corner point $O(0, 0)$

$$f(0, 0) = 2(0) + 3(0) = 0$$

Corner point $A(4, 0)$

$$f(4, 0) = 2(4) + 3(0) = 8$$

Corner point $B\left(\frac{2}{3}, \frac{20}{3}\right)$

$$f\left(\frac{2}{3}, \frac{20}{3}\right) = 2\left(\frac{2}{3}\right) + 3\left(\frac{20}{3}\right)$$
$$= \frac{4}{3} + 20$$
$$= \frac{4 + 60}{3} = \frac{64}{3} = 21.33$$

Corner point $C(0, 7)$

$$f(0, 7) = 2(0) + 3(7) = 21$$

we observe that $f(x, y)$ has maximum value 21.3 at corner point $\left(\frac{2}{3}, \frac{20}{3}\right)$.

6. Find minimum and maximum values of $z = 3x + y$; subject to the constraints:

$$3x + 5y \geq 15; \quad x + 6y \geq 9; \quad x \geq 0; \quad y \geq 0$$

Solution: Let's solve the linear programming problem step by step to find the minimum and maximum values of $z = 3x + y$, subject to the constraints:

Problem:

Maximize and minimize $z = 3x + y$, subject to the constraints:

$$3x + 5y \geq 15$$
$$x + 6y \geq 9$$
$$x \geq 0, \quad y \geq 0$$

Step 1: Convert Inequalities to Equalities

The associated equations for the inequalities are:

- $3x + 5y = 15$ (Equation 1)
- $x + 6y = 9$ (Equation 2)

Step 2: Graph the Equations

GRAPHING $3x + 5y = 15$ (EQUATION 1):

Rearranging the equation to solve for y:

$$y = \frac{15 - 3x}{5}$$

We can find two points for plotting the line:

- When $x = 0$, $y = \frac{15-3(0)}{5} = 3$ (so the point is $(0,3)$).
- When $y = 0$, $3x = 15 \Rightarrow x = 5$ (so the point is $(5,0)$).

Thus, the line $3x + 5y = 15$ passes through the points $(0,3)$ and $(5,0)$.

GRAPHING $x + 6y = 9$ (EQUATION 2):

Rearranging the equation to solve for y:

$$y = \frac{9 - x}{6}$$

We can find two points for plotting the line:

- When $x = 0$, $y = \frac{9-0}{6} = 1.5$ (so the point is $(0,1.5)$).
- When $y = 0$, $x = 9$ (so the point is $(9,0)$).

Thus, the line $x + 6y = 9$ passes through the points $(0,1.5)$ and $(9,0)$.

Step 3: Plot the Constraints and Identify the Feasible Region

We need to plot the inequalities on the graph:

- For $3x + 5y \geq 15$, the feasible region is **above** the line $3x + 5y = 15$.
- For $x + 6y \geq 9$, the feasible region is **above** the line $x + 6y = 9$.
- For $x \geq 0$, the feasible region is in the **right half-plane**
- For $y \geq 0$, the feasible region is in the **upper half-plane**

The feasible region is the intersection of these regions.

Step 4: Find the Corner Points of the Feasible Region

We need to determine the points where the boundaries of the feasible region intersect. These are the **corner points**.

INTERSECTION OF $3x + 5y = 15$ AND $x + 6y = 9$:

Solve the system of equations:

- $3x + 5y = 15$
- $x + 6y = 9$

From the second equation, solve for x:

$$x = 9 - 6y$$

Substitute $x = 9 - 6y$ into the first equation:

$$3(9 - 6y) + 5y = 15$$
$$27 - 18y + 5y = 15$$
$$27 - 13y = 15$$
$$-13y = -12 \Rightarrow y = \frac{12}{13}$$

Substitute $y = \frac{12}{13}$ into $x = 9 - 6y$:

$$x = 9 - 6\left(\frac{12}{13}\right) = 9 - \frac{72}{13} = \frac{117}{13} - \frac{72}{13} = \frac{45}{13}$$

So, the intersection point is $\left(\frac{45}{13}, \frac{12}{13}\right)$.

INTERSECTION OF $x + 6y = 9$ AND THE X-AXIS (I.E., $y = 0$):

Substitute $y = 0$ into $x + 6y = 9$:

$$x = 9$$

So, the intersection point is $(9,0)$.

INTERSECTION OF $3x + 5y = 15$ AND THE Y-AXIS (i.e., $x = 0$):

Substitute $x = 0$ into $3x + 5y = 15$:

$$5y = 15 \Rightarrow y = 3$$

So, the intersection point is (0,3).

Step 5: Evaluate the Objective Function at the Corner Points

The corner points of the feasible region are:

- (9,0) • (0,3)
- $\left(\frac{45}{13}, \frac{12}{13}\right)$

Evaluate the objective function $z = 3x + y$ at these points:

- At (0,3):

$$z = 3(0) + 3 = 3$$

- At (9,0):

$$z = 3(9) + 0 = 27$$

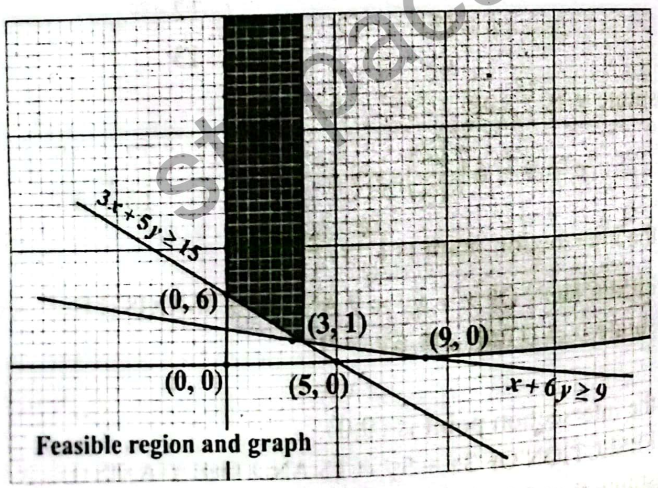
- At $\left(\frac{45}{13}, \frac{12}{13}\right)$:

$$z = 3\left(\frac{45}{13}\right) + \frac{12}{13} = \frac{135}{13} + \frac{12}{13} = \frac{147}{13} \approx 11.31$$

Step 6: Conclusion

The **maximum value** of $z = 3x + y$ is $\boxed{27}$, and it occurs at the point (9,0).

The **minimum value** of $z = 3x + y$ is $\boxed{3}$, and it occurs at the point (0,3).



Feasible region and graph

REVIEW EXERCISE 5

1. Four options are given against each statement. Encircle the correct one.

i. In the following, linear equation is:
   a) $5x > 7$ b) $4x - 2 < 1$
   c) $2x + 1 = 1$ d) $4 = 1 + 3$

ii. Solution of $5x - 10 = 10$ is:
   a) 0 b) 50
   c) 4 d) −4

iii- If $7x + 4 < 6x + 6$, then x belongs to the interval
   a) $(2, \infty)$ b) $[2, \infty)$
   c) $(-\infty, 2)$ d) $(-\infty, 2]$

iv- A vertical line divides the plane into
   a) left half plane b) right half plane
   c) full plane d) two half plane

v- The linear equation formed out of the linear inequality is called
   a) Linear equation b) Associated equation
   c) Quadratic equal d) None of these

vi- $3x + 4 < 0$ is:
   a) Equation b) Inequality
   c) Not inequality d) identity

vii- Corner point is also called:
   a) Code b) Vertex
   c) Curve d) Region

viii- (0,0) is solution of inequality:
   a) $4x + 5y > 8$ b) $3x + y > 6$
   c) $-2x + 3y < 0$ d) $x + y > 4$

ix- The solution region restricted to the first quadrant is called:
   a) Objective region b) Feasible region

c) Solution region d) Constraints region

x- A function that is to be maximized or minimized is called:

a) Solution function b) Objective function

c) Feasible function d) None of these

Answers:

(i)	c	(ii)	c	(iii)	c	(iv)	d	(v)	b
(vi)	b	(vii)	b	(viii)	c	(ix)	b	(x)	b

2. Solve and represent their solutions on real line.

(i) $\frac{x+5}{3}=1-x$ (ii) $\frac{2x+1}{3}+\frac{1}{2}=1-\frac{x-1}{3}$

(iii) $3x+7<16$ (iv) $5(x-3)\geq 15x-(10x+4)$

Solution:

(i) $\frac{x+5}{3}=1-x$

$$x+5=3(1-x)$$
$$x+5=3-3x$$
$$x+3x=3-5$$
$$4x=-2$$
$$x=\frac{-2}{4}=-\frac{1}{2}$$

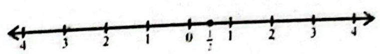
Solution: $x=-\frac{1}{2}$

$x=-\frac{1}{2}$ 

(ii) $\frac{2x+1}{3}+\frac{1}{2}=1-\frac{x-1}{3}$

$$6\left(\frac{2x+1}{3}\right)+6\left(\frac{1}{2}\right)=6\left(1-\frac{x-1}{3}\right)$$
$$2(2x+1)+3=6-2(x-1)$$
$$4x+2+3=6-2x+2$$
$$4x+5=8-2x$$
$$4x+2x=8-5$$
$$6x=3$$
$$x=\frac{3}{6}=\frac{1}{2}$$

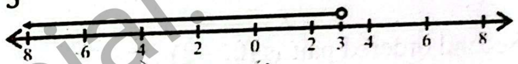
Solution: $x=\frac{1}{2}$

$x=\frac{1}{2}$ 

(iii) $3x+7<16$

$$3x<16-7$$
$$3x<9$$
$$x<3$$

Solution: $x<3$

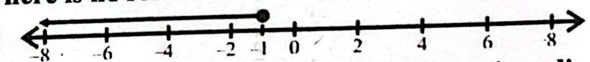
$x<3$ 

(iv) $5(x-3)\geq 26x-(10x+4)$

$$5x(x-3)\geq 26x-(10x+4)$$
$$5x-15\geq 26x-10x-4$$
$$5x-15\geq 16x-4$$
$$-15+4\geq 16x-5x$$
$$-11\geq 11x$$
$$\frac{-11}{11}\geq x$$
$$\boxed{x\leq -1}$$

The given inequality is $(-\infty,-1)$ or $\infty<x\leq -1$

Solution: There is no solution.

$x\leq -1$ 

3. Find the solution region of the following linear equalities:

(i) $3x-4y\leq 12$; $3x+2y\geq 3$

(ii) $2x+y\leq 4$; $x+2y\leq 6$

Solution:

(i) $3x-4y\leq 12$ and $3x+2y\geq 3$

Sol. Associated equations are

$$3x-4y=12 \quad \ldots(i)$$

and $3x + 2y = 3$ ...(ii)

***x*-intercept**

Put $y = 0$

$$3x - 4(0) = 12$$
$$3x = 12$$
$$x = 4$$

First ordered pair is (4, 0)

***y*-intercept**

Put $x = 0$

$$3(0) - 4y = 12$$
$$-4y = 12$$
$$y = -3$$

Second ordered pair is (0, – 3)

Take $x = y = 0$ (origin test)

$$3(0) - 4(0) < 12$$
$$0 < 12$$

Which is true. So shading lies towards origin side.

***x*-intercept**

Put $y = 0$

$$3x + 2(0) = 3$$
$$3x = 3$$
$$x = 1$$

First ordered pair is (1, 0)

***y*-intercept**

Put $x = 0$

$$3(0) + 2y = 3$$
$$2y = 3$$
$$y = \frac{3}{2}$$
$$y = 1.5$$

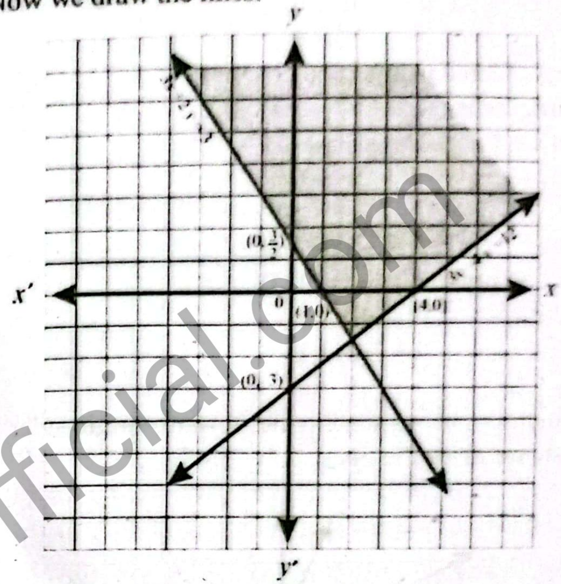
Second ordered pair is (0, 1.5)

Take $x = y = 0$ (origin test)

$$3(0) + 2(0) < 3$$
$$0 < 3$$

Which is false, so shading lies away from origin side.

Now we draw the lines.



(ii) $2x + y \le 4$; $x + 2y \le 6$

Sol. Associated equations are

$2x + y = 4$ ...(i)

and $x + 2y = 6$ ...(ii)

***x*-intercept**

Put $y = 0$

$$2x = 4$$
$$x = 2$$

First ordered pair is (2, 0)

***y*-intercept**

Put $x = 0$

$$2(0) + y = 4$$
$$y = 4$$

Second ordered pair is (0, 4)

Put $x = y = 0$ (origin test)

$$2(0) - 0 < 4$$
$$0 < 4$$

Which is true. So shading lies towards origin side.

***x*-intercept**

Put $y = 0$

$$x + 0 = 6$$
$$x = 6$$

First ordered pair is (6, 0)

y-intercept

Put $x = 0$

$$0 + 2y = 6$$
$$2y = 6$$
$$y = 3$$

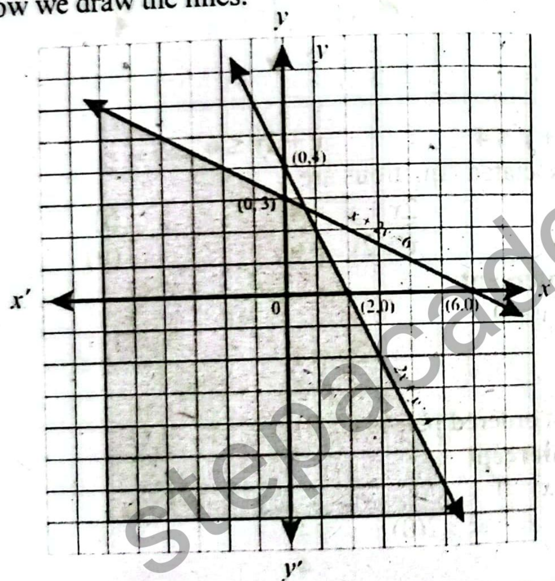
Second ordered pair is (0, 3)

Take $x = y = 0$ (origin test)

$$0 + 2(0) < 6$$
$$0 < 6$$

Which is true, so shading lies towards origin side.

Now we draw the lines.



4. Find the maximum value of $g(x, y) = x + 4y$ subject to constraints

$x + y \leq 4, x \geq 0$ and $y \geq 0$.

Solution: To find the maximum value of the function $g(x,y) = x + 4y$ subject to the constraints $x + y \leq 4$, $x \geq 0$, and $y \geq 0$, we can follow these steps:

Step 1: Identify the Feasible Region

The feasible region is determined by the system of inequalities:

- $x + y \leq 4$
- $x \geq 0$
- $y \geq 0$

This defines a triangular region in the first quadrant, bounded by:

- The line $x + y = 4$, which intersects the axes at $(4,0)$ and $(0,4)$.
- The coordinate axes (where $x = 0$ and $y = 0$).

Step 2: Find the Vertices of the Feasible Region

The vertices of the feasible region occur at the intersections of the boundary lines:

- $(0,0)$ from the intersection of $x = 0$ and $y = 0$.
- $(4,0)$ from the intersection of $x + y = 4$ and $y = 0$.
- $(0,4)$ from the intersection of $x + y = 4$ and $x = 0$.

Thus, the vertices of the feasible region are $(0,0)$, $(4,0)$, and $(0,4)$.

Step 3: Evaluate $g(x,y)$ at the Vertices

Now, we evaluate the objective function $g(x,y) = x + 4y$ at each of the vertices.

- At $(0,0)$: $g(0,0) = 0 + 4(0) = 0$
- At $(4,0)$: $g(4,0) = 4 + 4(0) = 4$
- At $(0,4)$: $g(0,4) = 0 + 4(4) = 16$

Step 4: Find the Maximum Value

The maximum value of $g(x,y)$ occurs at the vertex $(0,4)$, where $g(0,4) = 16$.

Conclusion:

The maximum value of $g(x,y) = x + 4y$ subject to the given constraints is $\boxed{16}$.

5. **Find the minimum value of $f(x,y) = 3x + 5y$ subject to constraints**

$$x + 3y \geq 3, \quad x + y \geq 2, \quad x \geq 0, \quad y \geq 0.$$

Sol. Associated equations are

$$x + 3y = 3 \quad \ldots(i)$$

and

$$x + y = 2 \quad \ldots(ii)$$

***x*-intercept**

Put $y = 0$

$$x + 3(0) = 3$$

$$x = 3$$

First ordered pair is (3, 0)

***y*-intercept**

Put $x = 0$

$$0 + 3y = 3$$

$$3y = 3$$

$$y = \frac{3}{3}$$

$$y = 1$$

Second ordered pair is (0, 1)

Take $x = y = 0$ (origin test)

$$0 + 3(0) < 3$$

$$0 < 3$$

Which is false, so shading lies away from origin side.

***x*-intercept**

Put $y = 0$

$$x + 0 = 2$$

$$x = 2$$

First ordered pair is (2, 0)

***y*-intercept**

Put $x = 0$

$$0 + y = 2$$

$$y = 2$$

Second ordered pair is (0, 2)

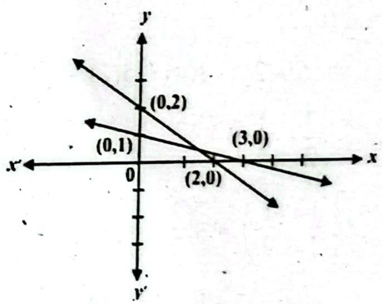
Take $x = y = 0$ (origin test)

$$0 + 0 > 2$$

$$0 > 2$$

Which is false, so shading lies towards origin side.

Now we draw the lines.



Corner points are (0, 2), $A(x, y)$(3, 0)

We find corner point A. We solve (i) and (ii)

$$x + 3y = 3$$

$$\underline{x + y = 2}$$

$$2y = 1$$

$$y = \frac{1}{2}$$

Put $x = \frac{1}{2}$ in (i) for x:

$$x + 3\left(\frac{1}{2}\right) = 3$$

$$x + \frac{3}{2} = 3$$

$$x = 3 - \frac{3}{2}$$

$$x = \frac{3}{2}$$

Corner point $A\left(\frac{3}{2}, \frac{1}{2}\right)$

Now $f(x, y) = 3x + 5y$

Corner point (0, 2)

$$f(0, 2) = 3(0) + 5(2) = 10$$

Corner point $\left(\frac{3}{2}, \frac{1}{2}\right)$

$$f\left(\frac{3}{2}, \frac{1}{2}\right) = 3\left(\frac{3}{2}\right) + 5\left(\frac{1}{2}\right)$$

$$= \frac{9}{2} + \frac{5}{2} = \frac{14}{2} = 7$$

Corner point (3, 0)

$$f(3, 0) = 3(3) + 5(0)$$

$$= 9$$

Minimum value of function is 7 at corner point $\left(\frac{3}{2}, \frac{1}{2}\right)$.