UNIT 6

Trigonometry

Students' learning outcomes

At the end of the unit, the students will be able to:

- Identify angles in standard positions expressed in degrees and radian.
- Apply Pythagoras theorem and the sine, the cosine and tangent ratios for acute angles of a right angle.
- Solve real life trigonometric problems in 2-D involving angles of elevation and depression
- Prove the trigonometric identities and apply them to draw different trigonometric relations.
- Solve real life problems involving trigonometric identities.

EXAMPLE 1: Convert 73.12° To Degrees, Minutes, And Seconds.

SOLUTION:

Degrees: The whole number part is 73°.

Minutes: Take the decimal part (0.12) and multiply by 60: 0.12 × 60 = 7.20. The whole number part is 7, so it's 7 minutes.

Seconds: Now take the decimal part (0.2) and multiply by 60: 0.2 × 60 = 12. So, it's 12 seconds.

Final result: **73° 7′ 12″.**

Example 2: Convert 109.42° To Degrees, Minutes, And Seconds.

Solution:

Degrees: The whole number part is 109°.

Minutes: Take the decimal part (0.42) and multiply by 60: 0.42 × 60 = 25.2. The whole number part is 25, so it's 25 minutes.

Seconds: Now take the decimal part (0.2) and multiply by 60: 0.2 × 60 = 12. So, it's 12 seconds.

Final result: **109° 25' 12".**

Example 3 Convert 45° 45' 45" to decimal degrees.

Solution: Degrees: Keep 45.

Minutes to decimal: $\frac{45}{60} = 0.75$

Seconds to decimal: $\frac{45}{3600} = 0.0125$

Add them together: $45 + 0.75 + 0.0125 = 45.7625$

Final result: **45.7625°**

Example 4: Convert 94° 27' 54" to decimal degrees.

Solution: Degrees: Keep 94

Minutes to decimal: $\frac{27}{60} = 0.45$

Seconds to decimal: $\frac{54}{3600} = 0.015$

Add them together: $94 + 0.45 + 0.015 = 94{:}465$

Final result: **94.465°**

Conversion between degrees and radians

Radians to Degrees: $1 \text{ rad} = \frac{180°}{\pi} \text{ degrees}$

Degrees to Radians: $1° = \frac{\pi}{180°} \text{ rad}$

Example 5: Convert radians to degree

(i) $\frac{5\pi}{3}$ rad (ii) $\frac{7\pi}{6}$ rad

(iii) $\frac{11\pi}{6}$ (iv) 1.2 rad

Solution: (i) $\frac{5\pi}{3} \text{ rad} = \frac{5\pi}{3} \times \frac{180°}{\pi} = 300°$

$(1 \text{ rad} = \frac{180°}{\pi} \text{ degrees})$

(ii) $\frac{7\pi}{6} \text{ rad} = \frac{7\pi}{6} \times \frac{180°}{\pi} = 210°$

(iii) $\frac{11\pi}{6} \text{ rad} = \frac{11\pi}{6} \times \frac{180°}{\pi} = 330°$

(iv) $1.2 \text{ rad} = 1.2 \times \frac{180°}{\pi} = 68.75°$

$(\because \pi = 3.14159)$

Example 6: Convert degree to radian

(i) 15° (ii) 75°

(iii) 315° (iv) 15° 15′

(i) $15° = 15 \times \frac{\pi}{180} = \frac{\pi}{12} \text{ rad}$ or 0.262 rad

(ii) $75° = 75 \times \frac{\pi}{180} = \frac{5\pi}{12} \text{ rad}$ or 1.309 rad

(iii) $315° = 315 \times \frac{\pi}{180} = \frac{7\pi}{4} \text{ rad}$ or 5.498 rad

(iv) $15°\,15′ = 15° + \frac{15}{60} = 15.25° = 15.25 \times \frac{\pi}{180}$ or 0.266 rad

Turns	0 turn	$\frac{1}{12}$ turn	$\frac{1}{8}$ turn	$\frac{1}{6}$ turn	$\frac{1}{4}$ turn	$\frac{1}{2}$ turn	1 turn
Radians	0 rad	$\frac{\pi}{6}$ rad	$\frac{\pi}{4}$ rad	$\frac{\pi}{3}$ rad	$\frac{\pi}{2}$ rad	π rad	2π rad
Degrees	0°	30°	45°	60°	90°	180°	360°

Example 7: Find the arc length of a sector with radius r =10cm and central angle $\theta = 60°$.

Solution: Convert $\theta = 60°$ to radians: $\theta = 60° \times \frac{\pi}{180°} = \frac{\pi}{3}$ radians.

$$\ell = r\theta = 10 \times \frac{\pi}{3} = 10.47 \text{ cm}$$

The arc length is approximately 10.47 cm

Example 8: Find the area of a sector with radius $r = 8$ cm and central angle $\theta = 45°$.

Solution: Convert $\theta = 45°$ to radians: $\theta = 45° \times \frac{\pi}{180°} = \frac{\pi}{4}$ radians. (Quarter Angle)

$$A = \frac{1}{2}r^2\theta = \frac{1}{2} \times 8^2 \times \frac{\pi}{4} = 8\pi \text{ cm}^2 \; 25.12\text{cm}^2.$$

The area of the sector is approximately 25.12 cm^2.

Example 9: If arc length of a sector of radius 7cm is 11cm, find the angle subtended by the arc in radians and degrees.

Solution: $r = 5$ cm ; $\ell = 11$ cm, ; $\theta = ?$

$\therefore \quad l = r\theta$

$11 = 5\theta \quad \Rightarrow \quad \theta = \frac{11}{5} = 2.2$ rad

$\theta = 2.2 \times \frac{180}{\pi} = 126.1°$

Thus, the angle subtended by the arc in radians is 2.2 rad and degrees is 126.1°.

EXERCISE 6.1

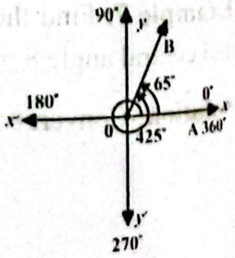
1. Find in which quadrant the following angles lie. Write a co-terminal angle for each:

(i) 65° **(ii)** 135° **(iii)** – 40°
(iv) 210° **(v)** – 150°

Solution:

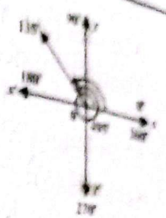
(i) 65°

- 65° lies between 0° and 90°.
  Quadrant: I
- Co-terminal angle: Add 360°:
  $65° + 360° = 425°$
  Co-terminal angle: 425°



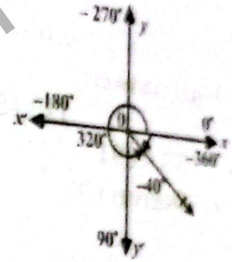
(ii) **135°**

- 135° lies between 90° and 180°.
  Quadrant: II
- Co-terminal angle: Add 360°:
  $135° + 360° = 495°$
  Co-terminal angle: **495°**



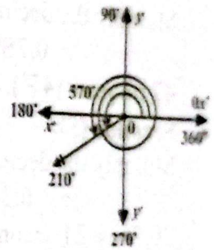
(iii) **–40°**

- –40° is a negative angle, measured clockwise. Adding 360° to bring it within 0° to 360°:
  $-40° + 360° = 320°$
- 320° lies between 270° and 360°.
  Quadrant: IV
  Co-terminal angle: 320°



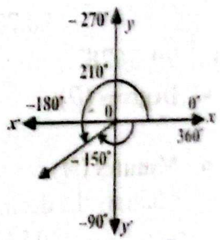
(iv) **210°**

- 210° lies between 180° and 270°.
  Quadrant: III
- Co-terminal angle: Add 360°:
  $210° + 360° = 570°$
  Co-terminal angle: **570°**



(v) **–150°**

- –150° is a negative angle.
  Adding 360°:
  $-150° + 360° = 210°$
- 210° lies between 180° and 270°.
  Quadrant: III
  Co-terminal angle: 210°



2. Convert the following into degrees, minutes and seconds:

(i) **123.456°** (ii) **58.7891°** (iii) **90.5678°**

Solutions:

(i) **123.456°**

- **Degrees (*D*):**
  $123.456° \rightarrow 123°$

- **Minutes (M):**
  Multiply the decimal part (0.456) by 60:
  $$0.456 \times 60 = 27.36$$
  27 minutes (27′), decimal part 0.36 remains.
- **Seconds (S):**
  Multiply the decimal part (0.36) by 60:
  $$0.36 \times 60 = 21.6$$
  $21.6 \approx 22$ seconds (22″).

Final Answer:
$$123.456° = 123°27'22''$$

(ii) **58.7891°**

- **Degrees (D):**
  $$58.7891° \rightarrow 58°$$
- **Minutes (M):**
  Multiply the decimal part (0.7891) by 60:
  $$0.7891 \times 60 = 47.346$$
  47 minutes (47′), decimal part 0.346 remains.
- **Seconds (S):**
  Multiply the decimal part (0.346) by 60:
  $$0.346 \times 60 = 20.76$$
  $20.76 \approx 21$ seconds (21″).

Final Answer:
$$58.7891° = 58°47'21''$$

(iii) **90.5678°**

- **Degrees (D):**
  $$90.5678° \rightarrow 90°$$
- **Minutes (M):**
  Multiply the decimal part (0.5678) by 60:
  $$0.5678 \times 60 = 34.068$$
  34 minutes (34′), decimal part 0.068 remains.
- **Seconds (S):**
  Multiply the decimal part (0.068) by 60:
  $$0.068 \times 60 = 4.08$$
  $4.08 \approx 4$ seconds (4″).

Final Answer:
$$90.5678° = 90°34'4.08''$$

3. **Convert the following into decimal degrees.**

(i) **65° 32' 15"** **(ii)** **42° 18' 45"**
(iii) **78° 45' 36"**

Solution: **Convert 65° 32' 15" to decimal degrees:**

1. **Degrees:** 65° remains as is.
2. **Minutes to decimal:**
$$\frac{32}{60} = 0.5333\ldots$$
3. **Seconds to decimal:**
$$\frac{15}{3600} = 0.004166\ldots$$
4. **Add them together:**
$$65 + 0.5333 + 0.004166 = 65.5375$$

Final Result: 65.5375°.

Convert 42° 18' 45" to decimal degrees:

1. **Degrees:** 42° remains as 42.
2. **Minutes to decimal degrees:**
$$\frac{18}{60} = 0.3$$
3. **Seconds to decimal degrees:**
$$\frac{45}{3600} = 0.0125$$
4. **Add them together:**
$$42 + 0.3 + 0.0125 = 42.3125$$

Final Result: 42.3125°.

Convert 78° 45' 36" to decimal degrees:

1. **Degrees:** 78° remains as 78°.
2. **Minutes to decimal degrees:**
$$\frac{45}{60} = 0.75$$
3. **Seconds to decimal degrees:**
$$\frac{36}{3600} = 0.01$$

4. **Add them together:**

$$78 + 0.75 + 0.01 = 78.76$$

Final Result: 78.76°.

4. **Convert the following into radian.**

(i) **36°** (ii) **22.5°** (iii) **67.5°**

Solution: **Convert 36° to radians:**

1. **Formula:**

$$\text{Radians} = \text{Degrees} \times \frac{\pi}{180}$$

2. **Calculation:**

$$36° \times \frac{\pi}{180} = \frac{\pi}{5} \text{ rad} \quad \text{or approximately } 0.628 \text{ rad.}$$

Final Result: $\frac{\pi}{5}$ rad or 0.629 rad.

Convert 22.5° to radians:

1. **Formula:**

$$\text{Radians} = \text{Degrees} \times \frac{\pi}{180}$$

2. **Calculation:**

$$22.5° \times \frac{\pi}{180} = \frac{\pi}{8} \text{ rad} \quad \text{or approximately } 0.3929 \text{ rad.}$$

Final Result:

$$\frac{\pi}{8} \text{ rad} \quad \text{or } 0.3929 \text{ rad.}$$

Convert 67.5° to radians:

1. **Formula:**

$$\text{Radians} = \text{Degrees} \times \frac{\pi}{180}$$

2. **Calculation:**

$$67.5° \times \frac{\pi}{180} = \frac{3\pi}{8} \text{ rad} \quad \text{or approximately } 1.179 \text{ rad.}$$

Final Result:

$$\frac{3\pi}{8} \text{ rad} \quad \text{or } 1.179 \text{ rad.}$$

5. **Convert the following into degrees.**

(i) $\frac{\pi}{16}$ **rad** (ii) $\frac{11\pi}{5}$ **rad**

(iii) $\frac{7\pi}{6}$ **rad**

Solution: **Convert $\frac{\pi}{16}$ rad to degrees:**

1. **Formula:**

$$\text{Degrees} = \text{Radians} \times \frac{180}{\pi}$$

2. **Calculation:**

$$\frac{\pi}{16} \times \frac{180}{\pi} = \frac{180}{16} = 11.25°$$

Final Result:

11.25°

Convert $\frac{11\pi}{5}$ rad to degrees:

1. **Formula:**

$$\text{Degrees} = \text{Radians} \times \frac{180}{\pi}$$

2. **Calculation:**

$$\frac{11\pi}{5} \times \frac{180}{\pi} = \frac{11 \times 180}{5} = 396°$$

Final Result:

396°

Convert $\frac{7\pi}{6}$ rad to degrees:

1. **Formula:**

$$\text{Degrees} = \text{Radians} \times \frac{180}{\pi}$$

2. **Calculation:**

$$\frac{7\pi}{6} \times \frac{180}{\pi} = \frac{7 \times 180}{6} = 210°$$

Final Result:

210°

6. **Find the arc length and area of a sector with:**

(i) **$r = 6$ cm and central angle $\theta = \frac{\pi}{3}$ radians.**

(ii) **$r = \frac{4.8}{\pi}$ cm and central angle $\theta = \frac{5\pi}{6}$ radians.**

Solution:

(i) Area of sector = A = ?

Arc length = ℓ = ?

$$r = 6cm \text{ and } \theta = \frac{\pi}{3} rad$$

Arc length = $\ell = r\theta$

$$\ell = 6 \times \frac{\pi}{3} cm$$

$$\ell = 6.28 cm$$

Area of sector = $A = \frac{1}{2}r^2\theta$

$$= \frac{1}{2} \times (6)^2 \times \frac{\pi}{3}$$

$$= \frac{1}{6} \times 36 \times \pi$$

$$= 6\pi cm^2$$

$$= 18.85 cm^2$$

(ii) **$r = \frac{4.8}{\pi}$ cm and central angle $\theta = \frac{5\pi}{6}$ radians.**

Sol:

$$r = \frac{4.8}{\pi} cm \text{ and } \theta = \frac{5\pi}{6} rad.$$

Arc length = $\ell = r\theta$

$$\ell = \frac{4.8}{\pi} \times \frac{5\pi}{6}$$

$$= 4 cm$$

Area of sector = $A = \frac{1}{2}r^2\theta$

$$= \frac{1}{2}\left(\frac{4.8}{\pi}\right)^2 \frac{5\pi}{6}$$

$$= \frac{1}{12} \times \frac{23.04}{\pi^2} \times 5\pi$$

$$= \frac{1}{12} \times \frac{115.2}{\pi}$$

$$= 3.056 cm^2$$

7. **If the central angle of a sector is 60° and the radius of the circle is 12 cm, find the area of the sector and the percentage of the total area of the circle it represents.**

Solution: **Step 1: Formula for the area of a sector**

The area of a sector is given by:

$$\text{Area of sector} = \frac{\text{Central angle}}{360°} \times \pi r^2$$

Step 2: Calculate the area of the sector

Substitute the given values:

- Central angle = 60°
- Radius (r) = 12 cm

$$\text{Area of sector} = \frac{60}{360} \times \pi(12)^2$$

Simplify the fraction:

$$\frac{60}{360} = \frac{1}{6}$$

Now calculate:

$$\text{Area of sector} = \frac{1}{6} \times \pi \times 144 = \frac{144\pi}{6} = 24\pi \text{ cm}^2$$

Approximating $\pi \approx 3.1416$:

$$\text{Area of sector} \approx 24 \times 3.1416 = 75.3984 \text{ cm}^2$$

Step 3: Find the total area of the circle

The area of the full circle is given by:

Total area of the circle $= \pi r^2$

Substitute $r = 12$:

Total area of the circle $= \pi(12)^2 = 144\pi \text{ cm}^2$

Approximating $\pi \approx 3.1416$:

Total area of the circle $\approx 144 \times 3.1416 = 452.3904 \text{ cm}^2$

Step 4: Calculate the percentage of the circle represented by the sector

The percentage is given by:

$$\text{Percentage} = \frac{\text{Area of sector}}{\text{Total area of the circle}} \times 100$$

Substitute the values:

$$\text{Percentage} = \frac{24\pi}{144\pi} \times 100 = \frac{24}{144} \times 100 = 16.67\%$$

8. Find the percentage of the area of sector subtending an angle $\frac{\pi}{8}$ radians.

Solution: Let's solve the problem step by step in detail to calculate the percentage of the area of a sector subtending an angle $\frac{\pi}{8}$ radians.

Step 1: Understand the relationship between the angle and the sector area

The area of a sector is proportional to the angle it subtends at the center of a circle. The proportion is given by:

$$\text{Area of sector} = \left(\frac{\text{Angle subtended by the sector}}{\text{Angle for the full circle}}\right) \times \text{Total area of the circle}$$

The percentage of the circle's area covered by the sector is therefore:

$$\text{Percentage of area} = \left(\frac{\text{Angle subtended by the sector}}{\text{Angle for the full circle}}\right) \times 100$$

Step 2: Define the full circle angle and substitute values

The angle for a full circle is 2π radians. The angle subtended by the sector is $\frac{\pi}{8}$ radians. Substituting these values into the formula:

$$\text{Percentage of area} = \left(\frac{\frac{\pi}{8}}{2\pi}\right) \times 100$$

Step 3: Simplify the fraction

Simplify $\frac{\frac{\pi}{8}}{2\pi}$:

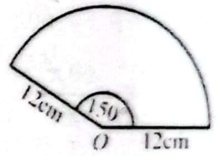
$$\frac{\frac{\pi}{8}}{2\pi} = \frac{1}{8}\cdot\frac{1}{2} = \frac{1}{16}.$$

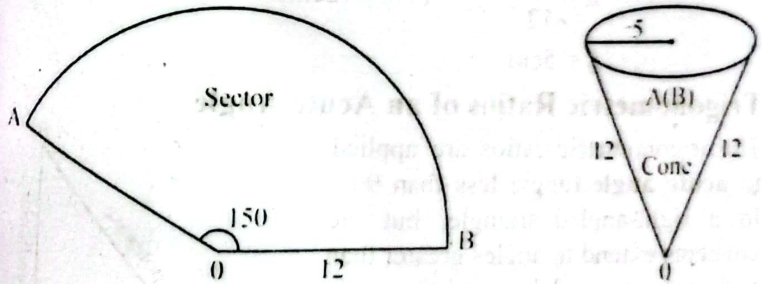
Step 4: Multiply by 100 to get the percentage

Now calculate the percentage:

$$\text{Percentage of area} = \frac{1}{16} \times 100 = 6.25\%$$

9. A circular sector of radius r =12 has an angle of 150°. This sector is cut out and then bent to form a cone. What is the slant height and the radius of the base of this cone?





Radius of sector = r = 12cm

Angle of sector = 150°

$= 150 \times \frac{\pi}{180} rad$

$= \frac{5\pi}{6} rad$

i) The radius of sector is slant height of cone, so

Slant height of cone = $\ell = 12$cm

ii) We know that

The area of sector = curved surface area of cone ….(i)

Area of sector $\frac{1}{2}r^2\theta$

$=\frac{1}{2}(12)^2\frac{5\pi}{6}$

$=\frac{1}{2}\times 144\times\frac{5\pi}{6}$

$=60\pi\ \text{cm}^2$

Let radius of cone = R = ?

Curved surface area of cone = $\pi R\ell$

$\because$ using eq. (i)

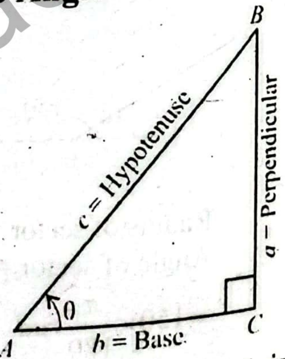
$\Rightarrow \pi R\ell = 60\pi$

$R=\frac{60\not\pi}{\not\pi\ell}$

$R=\frac{60}{12}$ $(\because \ell = 12\text{cm})$

$R = 5$cm

Trigonometric Ratios of an Acute Angle



The trigonometric ratios are applied to **acute angle** (angle less than 90°) in a right-angled triangle, but the concepts extend to angles greater than 90° and are widely used in many areas of mathematics and science.

Let us consider a right-angled triangle *ABC* with respect to an angle θ (theta) = $m\angle CAB$ with $m\angle ACB = 90°$ in anti-clockwise direction from *AC* to *CB*. In the triangle *ABC*, the side *BC* is called perpendicular, which is opposite to an angle 'θ'.

The side *AC* is called the base and the side *AB* is called the hypotenuse. Let $m\overline{BC} = a$, $m\overline{AC} = b$ and $m\overline{AB} = c$.

For this right angled triangle *ABC*, the trigonometric ratios of an angle "θ" are defined as:

$\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{a}{c}$; $\text{cosec}\ \theta = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{c}{a}$

$\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{b}{c}$; $\sec\theta = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{c}{b}$

$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{a}{b}$; $\cot\theta = \frac{\text{Base}}{\text{Perpendicular}} = \frac{b}{a}$

The six trigonometric ratios described with reference to a right-angled triangle ACB are: sine (sin), cosine(cos), tangent(tan), cosecant (cosec or esc), secant (sec) and cotangent (cot).

Note:

(i) $\text{cosec}\,\theta = \frac{1}{\sin\theta}$

(ii) $\sec\theta = \frac{1}{\cos\theta}$

(iii) $\cot\theta = \frac{1}{\tan\theta}$

We note that: $\tan\theta = \frac{a}{b}$

$= \frac{a/c}{b/c}$ (Dividing by *c*)

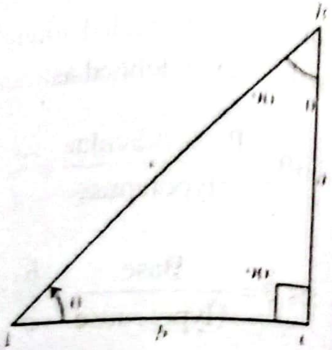
$\tan\theta = \frac{\sin\theta}{\cos\theta}$

Similarly, $\cot\theta = \frac{\cos\theta}{\sin\theta}$

Trigonometric Ratios of Complementary Angles

We consider a right-angled triangle *ACB*, in which $m\angle A = \theta$, $m\angle C = 90°$ then, $m\angle B = 90° - \theta$. Using the trigonometric ratios of $\angle B$, we get

$$\sin B = \sin(90^\circ - \theta) = \frac{m\overline{AC}}{m\overline{AB}} = \frac{b}{c} \quad \ldots\text{(i)}$$



Using ratios of $\angle A$, we get

$$\cos\theta = \frac{m\overline{AC}}{m\overline{AB}} = \frac{b}{c} \quad \ldots\text{(ii)}$$

Form (i) and (ii), we get,

$$\boxed{\sin(90^\circ - \theta) = \cos\theta}$$

Similarly, we have

$\cos(90^\circ - \theta) = \sin\theta$;	$\tan(90^\circ - \theta) = \cot\theta$
$\cot(90^\circ - \theta) = \tan\theta$;	$\sec(90^\circ - \theta) = \text{cosec}\,\theta$
$\text{cosec}(90^\circ - \theta) = \sec\theta$	

EXERCISE 6.2

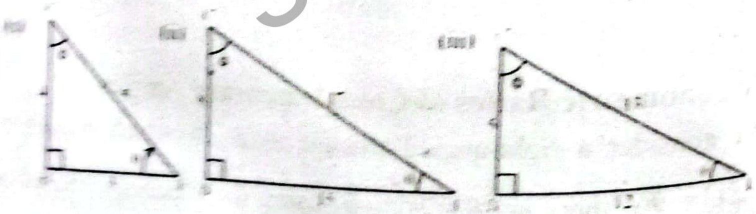
1. For each of the following right-angled triangles, find the trigonometric ratios

(i) $\sin\theta$ (ii) $\cos\theta$ (iii) $\tan\theta$

(iv) $\sec\theta$ (v) $\text{cosec}\,\theta$ (vi) $\cot\phi$

(vii) $\tan\phi$ (viii) $\text{cosec}\,\phi$ (ix) $\sec\phi$

(x) $\cos\phi$

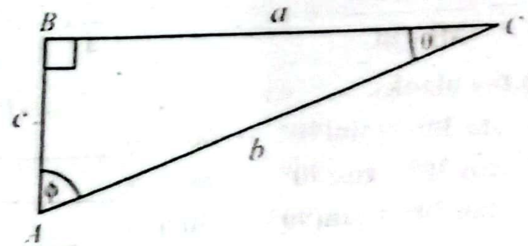


θ	Diagram (i)	Diagram (ii)	Diagram (iii)
$\sin\theta$	$\frac{4}{5}$	$\frac{8}{17}$	$\frac{5}{13}$
$\cos\theta$	$\frac{3}{5}$	$\frac{15}{17}$	$\frac{12}{13}$
$\tan\theta$	$\frac{4}{3}$	$\frac{8}{15}$	$\frac{5}{12}$
$\sec\theta$	$\frac{5}{3}$	$\frac{17}{15}$	$\frac{13}{12}$
$\text{cosec}\,\theta$	$\frac{5}{4}$	$\frac{17}{8}$	$\frac{13}{5}$
$\cot\theta$	$\frac{4}{3}$	$\frac{8}{15}$	$\frac{5}{12}$
$\tan\phi$	$\frac{3}{4}$	$\frac{15}{8}$	$\frac{12}{5}$
$\text{cosec}\,\phi$	$\frac{5}{3}$	$\frac{17}{15}$	$\frac{13}{12}$
$\sec\phi$	$\frac{5}{4}$	$\frac{17}{8}$	$\frac{13}{5}$
$\cos\phi$	$\frac{4}{5}$	$\frac{8}{17}$	$\frac{5}{13}$

2. For the following right-angled triangles ABC find the trigonometric ratios for which $m\angle A = \phi$ and $m\angle C = \theta$

(i) $\sin\theta$ (ii) $\cos\theta$ (iii) $\tan\theta$ (iv) $\sin\phi$

(v) $\cos\phi$ (vi) $\tan\phi$



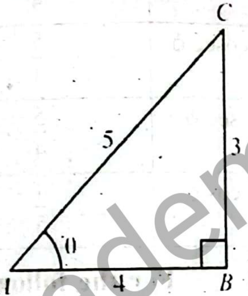
	θ	Diagram (a)	Diagram (b)
(i)	sin m∠θ	$\frac{c}{b}$	$\frac{AB}{AC}$
(ii)	cos m θ	$\frac{a}{b}$	$\frac{BC}{AC}$
(iii)	tan m∠θ	$\frac{c}{a}$	$\frac{AB}{BC}$
(iv)	sin ϕ	$\frac{a}{b}$	$\frac{BC}{AC}$
(v)	cos ϕ	$\frac{c}{b}$	$\frac{AB}{AC}$
(vi)	tan ϕ	$\frac{a}{c}$	$\frac{BC}{AB}$

3. **Considering the adjoining triangle *ABC*, verify that:**

(i) $\sin\theta \ \text{cosec}\,\theta = 1$

(ii) $\cos\theta \ \sec\theta = 1$

(iii) $\tan\theta \ \cot\theta = 1$



Solution:

$\sin\theta \,.\, \text{cosec}\,\theta = \frac{BC}{AC}\cdot\frac{AC}{BC} = 1$

$\sin\theta \,.\, \cos\theta = \left(\frac{3}{3}\times\frac{5}{5} = 1\right)$

$\cos\theta \,.\, \sec\theta = \frac{AB}{AC}\cdot\frac{AC}{AB} = 1 \qquad \left(\frac{4}{5}\times\frac{5}{4} = 1\right)$

$\tan\theta \,.\, \cot\theta = \frac{BC}{AB}\cdot\frac{AB}{BC} = 1 \qquad \left(\frac{3}{4}\times\frac{4}{3} = 1\right)$

4. **Fill in the blanks.**

(i) $\sin 30° = \sin(90° - 60°) =$ ______

(ii) $\cos 30° = \cos(90° - 60°) =$ ______

(iii) $\tan 30° = \tan(90° - 30°) =$ ______

(iv) $\tan 60° = \tan(90° - 60°) =$ ______

(v) $\sin 60° = \sin(90° - 30°) =$ ______

(vi) $\cos 60° = \cos(90° - 30°) =$ ______

(vii) $\sin 45° = \sin(90° - 45°) =$ ______

(viii) $\tan 45° = \tan(90° - 45°) =$ ______

(ix) $\cos 45° = \cos(90° - 45°) =$ ______

Solution: Let's solve these blanks step by step using the complementary angle identities:

$$\sin(90° - \theta) = \cos\theta, \quad \cos(90° - \theta) = \sin\theta,$$
$$\tan(90° - \theta) = \cot\theta, \quad \cot(90° - \theta) = \tan\theta.$$

(i) $\sin 30° = \sin(90° - 60°) = \cos 60°$

Answer: 60°

(ii) $\cos 30° = \cos(90° - 60°) = \sin 60°$

Answer: 60°

(iii) $\tan 30° = \tan(90° - 30°) = \cot 60°$

Answer: 60°

(iv) $\tan 60° = \tan(90° - 60°) = \cot 30°$

Answer: 30°

(v) $\sin 60° = \sin(90° - 30°) = \cos 30°$

Answer: 30°

(vi) $\cos 60° = \cos(90° - 60°) = \sin 30°$

Answer: 60°

(vii) $\sin 45° = \sin(90° - 45°) = \cos 45°$

Answer: 45°

(viii) $\tan 45° = \tan(90° - 45°) = \cot 45°$

Answer: 45°

(ix) $\cos 45° = \cos(90° - 45°) = \sin 45°$

Answer: 45°

5. **In a right angled triangle *ABC*, $m\angle B = 90°$ and *C* is an acute angle of 60°. Also $\sin m\angle A = \frac{a}{b}$, then find the *x* following trigonometric ratios.**

(i) $\frac{m\overline{BC}}{m\overline{AB}}$ (ii) $\cos 60°$

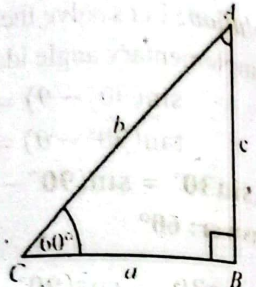
(iii) $\tan 60^\circ$ (iv) $\operatorname{cosec} 60^\circ$

(v) $\cot 60^\circ$ (vi) $\sin 30^\circ$

(vii) $\cos 30^\circ$ (viii) $\tan 30^\circ$

(ix) $\sec 30^\circ$ (x) $\cot 30^\circ$

Solution:



(i) $\frac{m\,BC}{m\,AB} = \frac{a}{c} = \tan 30^\circ = \cot 60^\circ$

$\therefore \quad c = \frac{a}{\tan 30} = \frac{a}{\cot 60}$

(ii) $\cos 60 = \frac{BC}{AC} = \frac{a}{b}$ (iii) $\tan 60^\circ = \frac{AB}{BC} = \frac{c}{a}$

(iv) $\operatorname{cosec} 60^\circ = \frac{b}{c} = \frac{AC}{AB}$ (v) $\cot 60^\circ = \frac{a}{c} = \frac{BC}{AB}$

(vi) $\sin 30^\circ = \frac{a}{b} = \frac{BC}{AC}$ (vii) $\cos 30^\circ = \frac{AB}{AC} = \frac{c}{b}$

(viii) $\tan 30^\circ = \frac{BC}{AB} = \frac{a}{c}$ (ix) $\sec 30^\circ = \frac{AC}{AB} = \frac{b}{c}$

(x) $\cot 30^\circ = \frac{c}{a} = \frac{AB}{BC}$

Example 10: Show that $(\sec^2\theta - 1)\cos^2\theta = \sin^2\theta$

Solution: L.H.S $= (\sec^2\theta - 1)\cos^2\theta$

$= \tan^2\theta \cdot \cos^2\theta \qquad (1 + \tan^2\theta = \sec^2\theta)$

L.H.S. $= \frac{\sin^2\theta}{\cos^2\theta} \cdot \cos^2\theta \qquad \left(\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right)$

$\sin^2\theta = \text{R.H.S}$

Hence, $(\sec^2\theta - 1)\cos^2\theta = \sin^2\theta$

Example 11: Show that $\tan\theta + \cot\theta = \sec\theta \operatorname{cosec}\theta$

Solution: L.H.S $= \tan\theta + \cot\theta$

$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$

$= \frac{\sin\theta \cdot \sin\theta + \cos\theta \cdot \cos\theta}{\cos\theta \cdot \sin\theta}$

$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta \cos\theta}$

$= \frac{1}{\sin\theta\cos\theta} \qquad (\because \sin^2\theta + \cos^2\theta = 1)$

$= \frac{1}{\cos\theta} \cdot \frac{1}{\sin\theta}$

$= \sec\theta \cdot \operatorname{cosec}\theta = \text{R.H.S.}$

Hence, $\tan\theta + \cot\theta = \sec\theta \operatorname{cosec}\theta$

Example 12: Show that $\frac{1}{\operatorname{cosec}\theta - \cot\theta} - \frac{1}{\sin\theta} = \frac{1}{\sin\theta} - \frac{1}{\operatorname{cosec}\theta + \cot\theta}$

Solution:

L.H.S $= \frac{1}{\operatorname{cosec}\theta - \cot\theta} - \frac{1}{\sin\theta}$

$= \frac{1}{\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}} - \frac{1}{\sin\theta}$

$= \frac{\sin\theta}{1 - \cos\theta} - \frac{1}{\sin\theta}$

$= \frac{\sin\theta(1 + \cos\theta)}{(1 - \cos\theta)(1 + \cos\theta)} - \frac{1}{\sin\theta}$

$= \frac{\sin\theta(1 + \cos\theta)}{1 - \cos^2\theta} - \frac{1}{\sin\theta}$

$= \frac{\sin\theta(1 + \cos\theta)}{\sin^2\theta} - \frac{1}{\sin\theta}$

$= \frac{1 + \cos\theta}{\sin\theta} - \frac{1}{\sin\theta}$

$= \frac{1 + \cos\theta - 1}{\sin\theta}$

R.H.S $= \frac{1}{\sin\theta} - \frac{1}{\operatorname{cosec}\theta + \cot\theta}$

$= \frac{1}{\sin\theta} - \frac{1}{\frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}}$

$= \frac{1}{\sin\theta} - \frac{\sin\theta}{1 + \cos\theta}$

$= \frac{1}{\sin\theta} - \frac{\sin\theta(1 - \cos\theta)}{(1 + \cos\theta)(1 - \cos\theta)}$

$= \frac{1}{\sin\theta} - \frac{\sin\theta(1 - \cos\theta)}{1 - \cos^2\theta}$

$= \frac{1}{\sin\theta} - \frac{\sin\theta(1 - \cos\theta)}{\sin^2\theta}$

$= \frac{1}{\sin\theta} - \frac{1 - \cos\theta}{\sin\theta}$

$= \frac{1 - 1 + \cos\theta}{\sin\theta}$

$$= \frac{\cos\theta}{\sin\theta} = \cot\theta$$

$$= \frac{\cos\theta}{\sin\theta} = \cot\theta$$

Hence, L.H.S = R.H.S

Example 13: Show that $\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta$

Solution: L.H.S $= \sin^6\theta + \cos^6\theta$

$= (\sin^2\theta)^3 + (\cos^2\theta)^3$

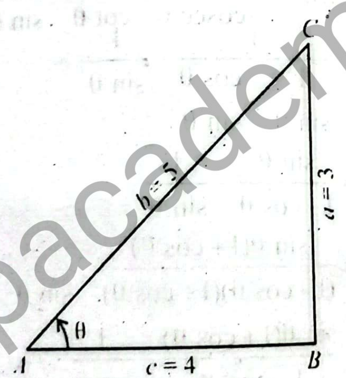
$= (\sin^2\theta + \cos^2\theta)(\sin^4\theta + \cos^4\theta - \sin^2\theta\cos^2\theta)$

$= (\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta - \sin^2\theta\cos^2\theta$

$= 1 - 3\sin^2\theta\cos^2\theta$ = R.H.S

Hence, $\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta$

Example 14: If $\tan\theta = \frac{3}{4}$ find the remaining trigonometric ratios, when θ lies in first quadrant.



Solution: Given: $\tan\theta = \frac{3}{4} = \frac{a}{c}$

Where, $a = 3, c = 4$

By Pythagoras theorem, we have

$b^2 = a^2 + c^2 = (3)^2 + (4)^2 = 9 + 16$

$= 25 \therefore b = 5$

Therefore, $\sin\theta = \frac{a}{b} = \frac{3}{5}$; $\text{cosec}\,\theta = \frac{b}{a} = \frac{5}{3}$

$\cos\theta = \frac{c}{b} = \frac{4}{5}$; $\sec\theta = \frac{b}{c} = \frac{5}{4}$

$\cot\theta = \frac{c}{a} = \frac{4}{3}$

EXERCISE 6.3

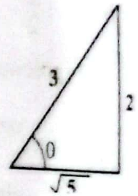
1. If θ lies in first quadrant, find the remaining trigonometric ratios of θ.

Sol. (i) $\sin\theta = \frac{2}{3}$

$\sin\theta = \frac{2}{3} = \frac{a}{b}$

$a = 2, b = 3, c = ?$

Using Pythagoras theorem



$b^2 = a^2 + c^2$

$3^2 = 2^2 + c^2$

$9 - 4 = c^2$

$c^2 = 5$

$c = \sqrt{5}$

$\cos\theta = \frac{c}{b} = \frac{\sqrt{5}}{3}$

$\tan\theta = \frac{a}{c} = \frac{2}{\sqrt{5}}$

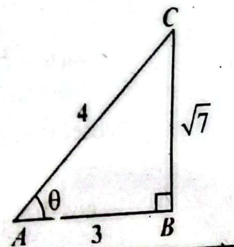
$\cot\theta = \frac{c}{a} = \frac{\sqrt{5}}{2}$

$\text{cosec}\,\theta = \frac{b}{a} = \frac{3}{2}$

$\sec\theta = \frac{b}{c} = \frac{3}{\sqrt{5}}$

(ii) $\cos\theta = \frac{3}{4}$

Sol. $(\overline{AB})^2 + (\overline{BC})^2 = (\overline{AC})^2$



$3 + (\overline{BC})^2 = 4^2$

$(\overline{BC})^2 = 16 - 9$

$(\overline{BC})^2 = 7$

$\overline{BC} = \sqrt{7}$

$\sin\theta = \dfrac{\sqrt{7}}{4}$, $\operatorname{cosec}\theta = \dfrac{4}{\sqrt{7}}$

$\tan\theta = \dfrac{\sqrt{7}}{3}$, $\cot\theta = \dfrac{3}{\sqrt{7}}$, $\sec\theta = \dfrac{4}{3}$

(iii) $\tan\theta = \dfrac{1}{2}$

Sol. Hypotenuse $= \sqrt{2^2 + 1^2}$

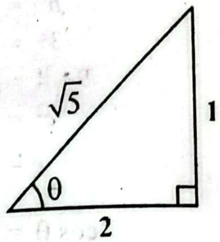
$\cot\theta = \dfrac{2}{1} = 2$

$\sin\theta = \dfrac{1}{\sqrt{5}}$

$\operatorname{cosec}\theta = \dfrac{1}{\frac{1}{\sqrt{5}}} = \sqrt{5}$

$\cos\theta = \dfrac{2}{\sqrt{5}}$

$\sec\theta = \dfrac{\sqrt{5}}{2}$



(iv) $\sec\theta = 3$

Sol. Hypotenuse $= \sqrt{3^2 - 1^2}$

$= \sqrt{8} = 2\sqrt{2}$

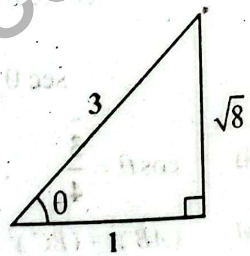
$\tan\theta = \dfrac{\sqrt{8}}{1} = \dfrac{2\sqrt{2}}{1} = 2\sqrt{2}$

$\cot\theta = \dfrac{1}{2\sqrt{2}}$

$\sin\theta = \dfrac{\sqrt{8}}{3} = \dfrac{2\sqrt{2}}{3}$

$\operatorname{cosec}\theta = \dfrac{3}{2\sqrt{2}}$

$\cos\theta = \dfrac{1}{3}$



(v) $\cot\theta = \sqrt{\dfrac{3}{2}}$

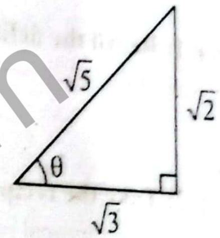
Sol. $\tan\theta = \sqrt{\dfrac{2}{3}}$

$\sin\theta = \dfrac{\sqrt{3}}{\sqrt{5}} = \sqrt{\dfrac{2}{5}}$

$\operatorname{cosec}\theta = \sqrt{\dfrac{5}{2}}$

$\cos\theta = \dfrac{\sqrt{2}}{\sqrt{5}} = \sqrt{\dfrac{3}{5}}$

$\sec\theta = \sqrt{\dfrac{5}{3}}$



Prove the following trigonometric identities:

2. $(\sin\theta + \cos\theta)^2 = 1 + 2\sin\theta\cos\theta$

To prove the identity:

$$(\sin\theta + \cos\theta)^2 = 1 + 2\sin\theta\cos\theta$$

Step 1: Expand the left-hand side ($(\sin\theta + \cos\theta)^2$)

$$(\sin\theta + \cos\theta)^2 = (\sin\theta)^2 + 2\sin\theta\cos\theta + (\cos\theta)^2$$

Step 2: Use the Pythagorean identity

From the Pythagorean identity:

$$\sin^2\theta + \cos^2\theta = 1.$$

Substitute this into the expanded expression:

$$(\sin\theta + \cos\theta)^2 = 1 + 2\sin\theta\cos\theta.$$

Step 3: Compare both sides

The left-hand side $(\sin\theta + \cos\theta)^2$ simplifies to the right-hand side $1 + 2\sin\theta\cos\theta$.

Conclusion

$$(\sin\theta + \cos\theta)^2 = 1 + 2\sin\theta\cos\theta \quad \text{(Proved).}$$

3. $\frac{\cos\theta}{\sin\theta} = \frac{1}{\tan\theta}$

To prove the identity:

$$\frac{\cos\theta}{\sin\theta} = \frac{1}{\tan\theta}$$

Step 1: Recall the definition of $\tan\theta$

By definition:

$$\tan\theta = \frac{\sin\theta}{\cos\theta}.$$

Step 2: Take the reciprocal of $\tan\theta$

Taking the reciprocal of both sides of the equation for $\tan\theta$:

$$\frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}.$$

Step 3: Compare the left-hand side with the right-hand side

The left-hand side of the equation is $\frac{\cos\theta}{\sin\theta}$, which is exactly the right-hand side of the identity we are trying to prove.

Conclusion

Thus, we have shown that:

$$\frac{\cos\theta}{\sin\theta} = \frac{1}{\tan\theta}.$$

The identity is **proved.**

4. $\frac{\sin\theta}{\operatorname{cosec}\theta} + \frac{\cos\theta}{\sec\theta} = 1$

To prove the identity:

$$\frac{\sin\theta}{\operatorname{cosec}\theta} + \frac{\cos\theta}{\sec\theta} = 1$$

Step 1: Recall the definitions of $\csc\theta$ and $\sec\theta$

We know the following trigonometric identities:

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta}, \quad \sec\theta = \frac{1}{\cos\theta}.$$

Step 2: Substitute the definitions of cosec θ and $\sec\theta$

Substitute the definitions of $\operatorname{cosec}\theta$ and $\sec\theta$ into the left-hand side of the equation:

$$\frac{\sin\theta}{\operatorname{cosec}\theta} + \frac{\cos\theta}{\sec\theta} = \frac{\sin\theta}{\frac{1}{\sin\theta}} + \frac{\cos\theta}{\frac{1}{\cos\theta}}.$$

Step 3: Simplify the fractions

Simplify each term:

$$\frac{\sin\theta}{\frac{1}{\sin\theta}} = \sin^2\theta, \quad \frac{\cos\theta}{\frac{1}{\cos\theta}} = \cos^2\theta.$$

So the left-hand side becomes:

$$\sin^2\theta + \cos^2\theta.$$

Step 4: Use the Pythagorean identity

From the Pythagorean identity:

$$\sin^2\theta + \cos^2\theta = 1.$$

Thus, the left-hand side simplifies to: 1.

Conclusion

We have shown that:

$$\frac{\sin\theta}{\operatorname{cosec}\theta} + \frac{\cos\theta}{\sec\theta} = 1.$$

The identity is **proved.**

5. **$\cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1$**

Solution: **Prove: $\cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1$**

PROOF:

We will use the **double angle identity** for cosine:

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta.$$

From the above identity, we know that:

$$\cos^2\theta - \sin^2\theta = \cos(2\theta).$$

Now, recall the identity for $\cos(2\theta)$ in terms of $\cos^2\theta$:

$$\cos(2\theta) = 2\cos^2\theta - 1.$$

Thus, we can replace $\cos(2\theta)$ in the first equation:

$$\cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1.$$

Therefore, the identity is **proved.**

6. $\cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta$

Solution: **Prove:** $\cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta$

PROOF:

We already know that:

$$\cos^2\theta - \sin^2\theta = \cos(2\theta).$$

Also, recall the identity for $\cos(2\theta)$ in terms of $\sin^2\theta$:

$$\cos(2\theta) = 1 - 2\sin^2\theta.$$

Thus, we can rewrite the equation as:

$$\cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta.$$

Therefore, the identity is **proved.**

7. $\dfrac{1-\sin\theta}{\cos\theta} = \dfrac{\cos\theta}{1+\sin\theta}$

Solution: $\dfrac{1-\sin\theta}{\cos\theta} = \dfrac{\cos\theta}{1+\sin\theta}$

Proof:

Multiply both sides:

$$(1-\sin\theta)(1+\sin\theta) = \cos^2\theta$$

Using the identity:

$$1 - \sin^2\theta = \cos^2\theta$$

Dividing both sides by $\cos\theta(1+\sin\theta)$:

$$\frac{(1-\sin\theta)}{\cos\theta} = \frac{\cos\theta}{1+\sin\theta}$$

Thus, the identity is **proved.**

8. $(\sec\theta - \tan\theta)^2 = \dfrac{1-\sin\theta}{1+\sin\theta}$

Solution: $(\sec\theta - \tan\theta)^2 = \dfrac{1-\sin\theta}{1+\sin\theta}$

Proof:

Expanding the left-hand side:

$$(\sec\theta - \tan\theta)^2 = \sec^2\theta - 2\sec\theta\tan\theta + \tan^2\theta$$

Using identities:

$$\sec^2\theta - \tan^2\theta = 1$$

$$-2\sec\theta\tan\theta = \frac{-2(1+\sin\theta)}{\cos^2\theta}$$

After simplifications, we get:

$$(\sec\theta - \tan\theta)^2 = \frac{1-\sin\theta}{1+\sin\theta}$$

Thus, the identity is **proved.**

9. $(\tan\theta + \cot\theta)^2 = \sec^2\theta\,\mathrm{cosec}^2\theta$

Solution: $(\tan\theta + \cot\theta)^2 = \sec^2\theta\,\mathrm{cosec}^2\theta$

Proof:

Using the identities:

$$\tan\theta = \frac{\sin\theta}{\cos\theta},\quad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta} = \frac{1}{\sin\theta\cos\theta}$$

Squaring both sides:

$$(\tan\theta + \cot\theta)^2 = \frac{1}{\sin^2\theta\cos^2\theta}$$

$$= \sec^2\theta\,\mathrm{cosec}^2\theta$$

Thus, the identity is **proved.**

10. $\dfrac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \tan\theta + \sec\theta$

Solution: $\dfrac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \tan\theta + \sec\theta$

Proof:

Expressing $\tan\theta$ and $\sec\theta$ in terms of sine and cosine:

$$\tan\theta = \frac{\sin\theta}{\cos\theta},\quad \sec\theta = \frac{1}{\cos\theta}$$

Rewriting the left-hand side and simplifying gives:

$$\frac{\frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta} - 1}{\frac{\sin\theta}{\cos\theta} - \frac{1}{\cos\theta} + 1}$$

Factorizing and simplifying results in:

$\tan\theta + \sec\theta$

Thus, the identity is **proved.**

11. $\sin^3\theta - \cos^3\theta = (\sin\theta - \cos\theta)(1 + \sin\theta\cos\theta)$

Solution: $\sin^3\theta - \cos^3\theta = (\sin\theta - \cos\theta)(1 + \sin\theta\cos\theta)$

Proof:

Using the identity:

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Here, $a = \sin\theta$ and $b = \cos\theta$, so:

$$\sin^3\theta - \cos^3\theta = (\sin\theta - \cos\theta)(\sin^2\theta + \sin\theta\cos\theta + \cos^2\theta)$$

Since $\sin^2\theta + \cos^2\theta = 1$, we get:

$$(\sin\theta - \cos\theta)(1 + \sin\theta\cos\theta)$$

Thus, the identity is **proved.**

12. $\sin^6\theta - \cos^6\theta = (\sin^2\theta - \cos^2\theta)(1 - \sin^2\theta\cos^2\theta)$

Solution: $\sin^6\theta - \cos^6\theta = (\sin^2\theta - \cos^2\theta)(1 - \sin^2\theta\cos^2\theta)$

Proof:

Using the identity:

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

We factorize:

$$\sin^6\theta - \cos^6\theta = (\sin^2\theta - \cos^2\theta)(\sin^4\theta + \sin^2\theta\cos^2\theta + \cos^4\theta)$$

Since:

$$\sin^4\theta + \cos^4\theta = 1 - 2\sin^2\theta\cos^2\theta$$

Substituting:

$$(\sin^2\theta - \cos^2\theta)(1 - \sin^2\theta\cos^2\theta)$$

Thus, the identity is **proved**.

Trigonometric ratios of 45° $\left(\frac{\pi}{4}\right)$

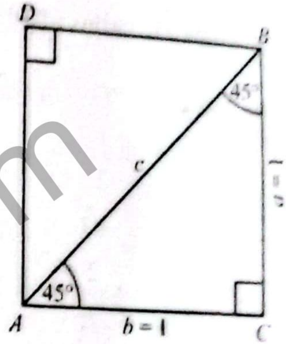
Consider a square *ACBD* of side length 1 unit.

We know that the diagonal bisect the angles.

So in the triangle *ABC*

$m\angle A = m\angle B = 45°$ and $m\angle C = 90°$.

Using Pythagoras theorem ΔABC,



$$c^2 = a^2 + b^2$$
$$c^2 = 1 + 1$$
$$c^2 = 2 \quad \Rightarrow \quad c = \sqrt{2}$$

The trigonometric ratio are:

$\sin 45° = \frac{a}{c} = \frac{1}{\sqrt{2}}$; $\operatorname{cosec} 45° = \frac{c}{a} = \sqrt{2}$

$\cos 45° = \frac{b}{c} = \frac{1}{\sqrt{2}}$; $\sec 45° = \frac{a}{b} = \sqrt{2}$

$\tan 45° = \frac{a}{b} = 1$; $\cot 45° = \frac{b}{a} = 1$

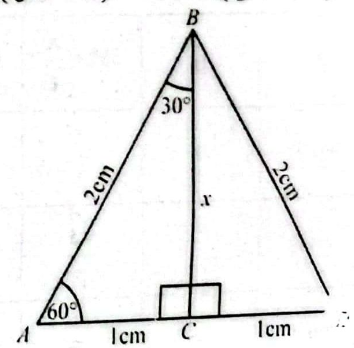
Trigonometric Ratios of 30° $\left(\frac{\pi}{6}\text{radian}\right)$ and 60° $\left(\frac{\pi}{3}\text{ radian}\right)$:

Consider an equilateral triangle *ABD* of side 2 units.

Draw a perpendicular bisector $\overline{BC}$ on $\overline{AD}$. The point *C* is the midpoint of $\overline{AD}$.

So, $m\overline{AC} = m\overline{CD}$ in which $m\angle BAC = 60°$, $m\angle ABC = 30°$, $m\angle ACB = 90°$.

Let $m\overline{BC} = x$ units.



Using Pythagoras theorem in the ΔABC.

$$2^2 = 1^2 + x^2$$

$x^2 = 4 - 1 \Rightarrow x^2 = 3 \Rightarrow x = \sqrt{3}$, ($m\overline{BC} = \sqrt{3}$ units)

Trigonometric ratios of 30° $\left(\frac{\pi}{6}\text{radian}\right)$:

In the triangle, ABC with $m\angle ABC = 30°$.

$\sin 30° = \frac{1}{2}$; $\text{cosec } 30° = 2$

$\cos 30° = \frac{\sqrt{3}}{2}$; $\sin 30° = \frac{2}{\sqrt{3}}$

$\tan 30° = \sqrt{3}$; $\cot 30° = \frac{1}{\sqrt{3}}$

Trigonometric Ratios of 60° $\left(\frac{\pi}{3}\text{radian}\right)$:

In right angle triangle ABC, with $m\angle A = 60°$.

$\sin 60° = \frac{\sqrt{3}}{2}$; $\cos 60° = \frac{1}{2}$; $\tan 60° = \sqrt{3}$

$\text{cosec } 60° = \frac{2}{\sqrt{3}}$; $\sec 60° = 2$; $\cot 60° = \frac{1}{\sqrt{3}}$

These results in the form of a table can be written as:

θ	0	$30° = \frac{\pi}{6}$	$45° = \frac{\pi}{4}$	$60° = \frac{\pi}{3}$	$90° = \frac{\pi}{2}$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞

EXERCISE 6.4

1. Find the value of the following trigonometric ratios without using the calculator.

(i) $\sin 30°$ **(ii)** $\cos 30°$

(iii) $\tan \frac{\pi}{6}$ **(iv)** $\tan 60°$

(v) $\sec 60°$ **(vi)** $\cos \frac{\pi}{3}$

(vii) $\cot 60°$ **(viii)** $\sin 60°$

(ix) $\sec 30°$ **(x)** $\text{cosec } 30°$

(xi) $\sin 45°$ **(xii)** $\cos \frac{\pi}{4}$

Solution:

(i) sin30°:

$$\sin 30° = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{1}{2}.$$

Answer: $\frac{1}{2}$

(ii) cos30°:

$$\cos 30° = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{\sqrt{3}}{2}.$$

(iii) $\tan \frac{\pi}{6}$

$$\tan 30° = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{1}{\sqrt{3}}$$

(iv) tan60°:

$$\tan 60° = \frac{\text{Opposite}}{\text{Adjacent}} = \sqrt{3}.$$

Answer: $\sqrt{3}$

(v) sec60°:

$$\sec 60° = \frac{1}{\cos 60°} = \frac{1}{\frac{1}{2}} = 2.$$

(vi) $\cos\frac{\pi}{3}$

$$\cos 60^\circ = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{1}{2}.$$

(vii) $\cot 60^\circ$

$$\cot 60^\circ = \frac{\text{Adjacent}}{\text{Opposite}} = \frac{1}{\sqrt{3}}$$

(viii) $\sin 60^\circ$:

$$\sin 60^\circ = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{\sqrt{3}}{2}.$$

(ix) $\sec 30^\circ$:

$$\sec 30^\circ = \frac{1}{\cos 30^\circ} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}$$

(x) $\csc 30^\circ$:

$$\csc 30^\circ = \frac{1}{\sin 30^\circ} = \frac{1}{\frac{1}{2}} = 2.$$

(xi) $\sin 45^\circ$:

$$\sin 45^\circ = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{1}{\sqrt{2}}$$

(xii) $\cos\frac{\pi}{4}$

$$\cos 45^\circ = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{1}{\sqrt{2}}$$

2. Evaluate:

(i) $2\sin 60^\circ \cos 60^\circ$

Solution: $2\sin 60^\circ\cos 60^\circ$

Using the known values for sine and cosine at 60°:

$$\sin 60^\circ = \frac{\sqrt{3}}{2}, \quad \cos 60^\circ = \frac{1}{2}.$$

Now, substitute these values into the expression:

$$2\sin 60^\circ\cos 60^\circ = 2\times\frac{\sqrt{3}}{2}\times\frac{1}{2} = \frac{\sqrt{3}}{2}.$$

Thus:

$$2\sin 60^\circ\cos 60^\circ = \frac{\sqrt{3}}{2}.$$

(ii) $2\cos\frac{\pi}{3}\sin\frac{\pi}{3}$

Solution: $2\cos\frac{\pi}{3}\sin\frac{\pi}{3}$

$$= 2\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$= \frac{\sqrt{3}}{2}$$

(iii) $2\sin 45^\circ + 2\cos 45^\circ$

Solution: $2\sin 45^\circ + 2\cos 45^\circ$

Using the known values for sine and cosine at 45°:

$$\sin 45^\circ = \frac{\sqrt{2}}{2}, \quad \cos 45^\circ = \frac{\sqrt{2}}{2}.$$

Now, substitute these values into the expression:

$$2\sin 45^\circ + 2\cos 45^\circ = 2\times\frac{\sqrt{2}}{2} + 2\times\frac{\sqrt{2}}{2} = \sqrt{2}+\sqrt{2} = 2\sqrt{2}.$$

Thus:

$$2\sin 45^\circ + 2\cos 45^\circ = 2\sqrt{2}.$$

(iv) $\sin 60^\circ\cos 30^\circ + \cos 60^\circ\sin 30^\circ$

Solution: $\sin 60^\circ\cos 30^\circ + \cos 60^\circ\sin 30^\circ$

Using the known values for sine and cosine:

$$\sin 60^\circ = \frac{\sqrt{3}}{2}, \quad \cos 30^\circ = \frac{\sqrt{3}}{2}, \quad \cos 60^\circ = \frac{1}{2}, \quad \sin 30^\circ = \frac{1}{2}.$$

Now, substitute these values into the expression:

$$\sin 60^\circ\cos 30^\circ + \cos 60^\circ\sin 30^\circ = \frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{2} + \frac{1}{2}\times\frac{1}{2} = \frac{3}{4}+\frac{1}{4}$$

$$= 1.$$

Thus:

$$\sin 60^\circ\cos 30^\circ + \cos 60^\circ\sin 30^\circ = 1.$$

(v) $\cos 60^\circ\cos 30^\circ - \sin 60^\circ\sin 30^\circ$

Solution: $\cos 60^\circ\cos 30^\circ - \sin 60^\circ\sin 30^\circ$

This is a standard identity for $\cos(A+B)$, so:

$$\cos 60^\circ \cos 30^\circ - \sin 60^\circ \sin 30^\circ = \cos(60^\circ + 30^\circ) = \cos 90^\circ$$
$$= 0.$$

Thus:
$$\cos 60^\circ \cos 30^\circ - \sin 60^\circ \sin 30^\circ = 0.$$

(vi) $\sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ$

Solution: **$\sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ$**

This is a standard identity for $\sin(A - B)$, so:
$$\sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ = \sin(60^\circ - 30^\circ) = \sin 30^\circ = \frac{1}{2}.$$

Thus:
$$\sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ = \frac{1}{2}.$$

(vii) $\cos 60^\circ \cos 30^\circ + \sin 60^\circ \sin 30^\circ$

Solution: **$\cos 60^\circ \cos 30^\circ + \sin 60^\circ \sin 30^\circ$**

This is a standard identity for $\cos(A - B)$, so:
$$\cos 60^\circ \cos 30^\circ + \sin 60^\circ \sin 30^\circ = \cos(60^\circ - 30^\circ) = \cos 30^\circ$$
$$= \frac{\sqrt{3}}{2}.$$

Thus: $\cos 60^\circ \cos 30^\circ + \sin 60^\circ \sin 30^\circ = \frac{\sqrt{3}}{2}.$

(viii) $\tan\frac{\pi}{6}\cot\frac{\pi}{6}+1$

Solution: $\tan\frac{\pi}{6}\cot\frac{\pi}{6}+1$
$$= \left(\frac{1}{\sqrt{3}}\right)(\sqrt{3})+1$$
$$= 1 + 1 = 2$$

3. If $\sin\frac{\pi}{4}$ and $\cos\frac{\pi}{4}$ equal to $\frac{1}{\sqrt{2}}$ each, then find the value of the followings:

(i) $2\sin 45^\circ - 2\cos 45^\circ$

Sol. $= 2(\sin 45^\circ - \cos 45^\circ)$
$$= 2\left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right) = 2\left(\frac{1-1}{\sqrt{2}}\right) = 2\times\frac{0}{\sqrt{2}} = 0$$

(ii) $3\cos 45^\circ + 4\sin 45^\circ$
$$= 3\left(\frac{1}{\sqrt{2}}\right)+4\left(\frac{1}{\sqrt{2}}\right)$$
$$= \frac{3}{\sqrt{2}}+\frac{4}{\sqrt{2}} = \frac{3+4}{\sqrt{2}} = \frac{7}{\sqrt{2}}$$

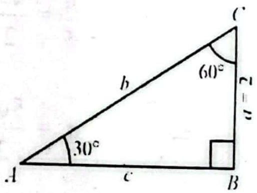
(iii) $5\cos 45^\circ - 3\sin 45^\circ$
$$= 5\left(\frac{1}{\sqrt{2}}\right)-3\left(\frac{1}{\sqrt{2}}\right)$$
$$= \frac{5}{\sqrt{2}}-\frac{3}{\sqrt{2}} = \frac{5-3}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$$

Case I: When measures of one side and one angle are given.

Example 15: Solve triangle ABC, in which $m\angle B = 90^\circ$, $m\angle A\ 30^\circ$, $a = 2$

Solution:

We are required to find b, c and $m\angle A$.



Now $m\angle C = m\angle B - m\angle A$
$$= 90^\circ - 30^\circ$$
$$= 60^\circ \quad \ldots(i)$$

and $\frac{a}{b} = \sin 30^\circ$

$\Rightarrow \frac{2}{b} = \sin 30^\circ \quad (\because a = 2)$

$\Rightarrow \frac{2}{b} = \frac{1}{2} \quad \left(\because \sin 30^\circ = \frac{1}{2}\right)$

$\Rightarrow b = 4 \quad \ldots(ii)$

and $\frac{a}{c} = \tan 30^\circ$

$\Rightarrow \frac{2}{c} = \frac{1}{\sqrt{3}} \quad \left(\because a = 2, \tan 30^\circ = \frac{1}{\sqrt{3}}\right)$

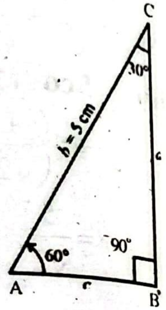
thus $c = 2\sqrt{3} \quad \ldots(iii)$

(i), (ii) and (iii) are the required results.

Case II: When measure of the hypotenuse and an angle are given.

Example 16: Solve triangle ABC, when $m\angle A = 60°$, $b = 5cm$, $m\angle B = 90°$

Solution: We are required to find a, c and $m\angle C$



$m\angle A = 60°$

$m\angle B = 90°$

$m\angle C = m\angle B - m\angle A$

$= 90° - 60°$

$= 30°$ ...(i)

Now $\frac{a}{b} = \sin 60°$

$\frac{a}{5} = \frac{\sqrt{3}}{2}$ $\left(\because b = 5, \sin 60° = \frac{\sqrt{3}}{2}\right)$

$\Rightarrow a = \frac{5\sqrt{3}}{2}$

$\Rightarrow a = 4.33$ cm ...(ii)

and $\frac{c}{b} = \cos 60°$

$\frac{c}{5} = \frac{1}{2}$ $\left(\because b = 5, \cos 60° = \frac{1}{2}\right)$

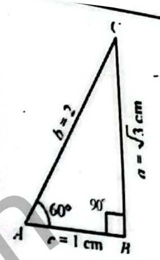
$\Rightarrow c = \frac{5}{2}$

$\Rightarrow c = 2.5$ cm ...(iii)

(i), (ii) and (iii) are the required results.

Case III: When measure of two sides are given.

Example 17: Solve triangle ABC, when $a = \sqrt{3}$ cm, $c = 1$cm and $m\angle B = 90°$



Solution: We are required to find b, $m\angle A$, $m\angle C$.

By Pythagoras theorem, we have

$b^2 = c^2 + a^2$

or $b^2 = (1)^2 + (\sqrt{2})^2$

or $b^2 = 1 + 2$

or $b^2 = 3$

or $b = \sqrt{3}$ ...(i)

Now $\sin m\angle A = \frac{a}{b} = \frac{\sqrt{2}}{\sqrt{3}} \Rightarrow m\angle A = \sin^{-1}\sqrt{\frac{2}{3}} = 54.7°$

$\Rightarrow m\angle A = 54.7°$ ...(ii)

and $m\angle C = m\angle B - m\angle A$

$= 90° - 54.7°$

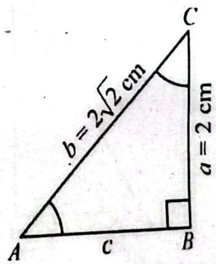
$= 35.3°$ ...(iii)

(i), (ii) and (iii) are the required results.

Case IV: When measure of one side and hypotenuse are given.

Example 18: Solve tringle ABC, when a = 2cm, $b = 2\sqrt{2}$ cm and $m\angle B = 90°$

Solution: We are required to find $m\angle A$, $m\angle C$ and 'c'.



By Pythagoras theorem, we have

$b^2 = a^2 + c^2$ or $c^2 = b^2 - a^2$

$c^2 = (2\sqrt{2})^2 - (2)^2$

$c^2 = 8 - 4$

$c^2 = 4$

or $c = 2$ ...(i)

Now $\frac{c}{b} = \cos m\angle A$

or $\frac{c}{b} = \cos m\angle A = \frac{1}{\sqrt{2}}$

or $\frac{2}{2\sqrt{2}} = \cos m\angle A = \frac{1}{\sqrt{2}}$ $\therefore$ $m\angle A = 45''$

$\Rightarrow m\angle A = 45^\circ$

Thus, $m\angle A = m\angle B - m\angle C$ ...(ii)

$= 90^\circ - 45^\circ$

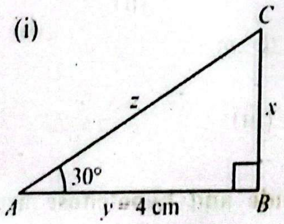
$= 45^\circ$ ...(iii)

Hence (i), (ii) and (iii) are the required results.

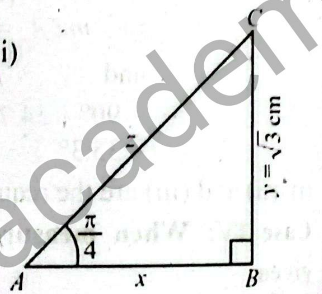
EXERCISE 6.5

1. Find the values of x, y and z from the following right angled triangles.

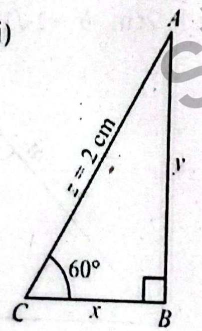
(i)



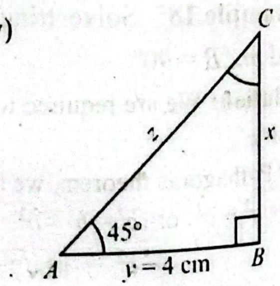
(ii)



(iii)



(iv)



Solution:

(i) $y = 4\text{cm}, x = ?, Z = ?$

Using $\tan 30^\circ = \frac{x}{y}$

$\frac{1}{\sqrt{3}} = \frac{x}{y}$

$\sqrt{3}\, x = 4$

$x = \frac{4}{\sqrt{3}}$ cm

Using Pythagoras theorem to find z.

$z^2 = x^2 + y^2$

$z^2 = \left(\frac{4}{\sqrt{3}}\right)^2 + (4)^2$

$z^2 = \frac{16}{3} + 16$

$z^2 = \frac{16+48}{3} = \frac{64}{3}$

$z^2 = \sqrt{\frac{64}{3}}$

$z^2 = \frac{8}{\sqrt{3}} cm$

(ii) $x = ?, y = \sqrt{3}$ cm; $z = ?$

This is an isosceles triangle

$m\angle A = \frac{\pi}{4}, m\angle C = \frac{\pi}{4}$

Using $\tan\frac{\pi}{4} = \frac{y}{x}$

$1 = \frac{\sqrt{3}}{x}$

$x = \sqrt{3}$ cm

Using Pythagoras theorem to find z.

$z^2 = x^2 + y^2$

$$z^2 = \left(\sqrt{3}\right)^2 + \left(\sqrt{3}\right)^2$$
$$z^2 = 3 + 3$$
$$z^2 = 6$$
$$z^2 = \sqrt{6}\ cm$$

(iii) $x = ?, y = ?, z = 2cm$

$m\angle C = 60^\circ$

Using $\sin 60^\circ = \frac{y}{z}$

$$\frac{\sqrt{3}}{2} = \frac{y}{2}$$
$$2y = 2\sqrt{3}$$
$$y = \sqrt{3}\,cm$$

Using $\tan 60^\circ = \frac{y}{z}$

$$\sqrt{3} = \frac{\sqrt{3}}{x}$$
$$\sqrt{3}x = \sqrt{3}$$
$$x = \frac{\sqrt{3}}{\sqrt{3}}$$
$$x = 1cm$$

(iv) $y = 4cm, z = ?, x = ?$

Using $\tan 45^\circ = \frac{x}{y}$

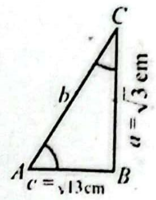
$$1 = \frac{x}{4}$$
$$x = 4cm$$

Using $\cos 45^\circ = \frac{y}{z}$

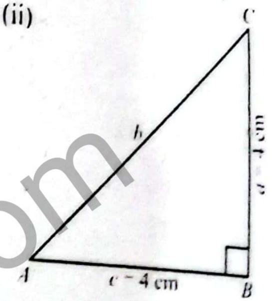
$$\frac{1}{\sqrt{2}} = \frac{4}{z}$$
$$z = 4\sqrt{2}\,cm$$

2. Find the unknown side and angles of the following triangles.

(i)



(ii)



Sol. (i) $c = \sqrt{13}cm, a = \sqrt{3}cm, m\angle A = ?, m\angle C = ?$

Using Pythagoras theorem

$$b^2 = a^2 + c^2$$
$$= \left(\sqrt{3}\right)^2 + \left(\sqrt{13}\right)^2$$
$$= 3 + 13$$
$$b^2 = 16$$
$$b = 4\text{cm}$$

Using $\tan(m\angle A) = \frac{\sqrt{3}}{\sqrt{13}}$

$$m\angle A = \tan^{-1}\left(\frac{\sqrt{3}}{\sqrt{13}}\right)$$
$$m\angle A = 26.66''$$

So $m\angle C = 90'' - 26.66''$

$$m\angle C = 63.34''$$

(ii) $c = 4cm, a = 4cm, b = ?, m\angle A = ?, m\angle C = ?$

Using Pythagoras theorem

$$b^2 = a^2 + c^2$$
$$= 4^2 + 4^2$$
$$= 16 + 16$$
$$b^2 = 32$$
$$b^2 = 16 \times 2$$

$$z^2 = (\sqrt{3})^2 + (\sqrt{3})^2$$
$$z^2 = 3 + 3$$
$$z^2 = 6$$
$$z^2 = \sqrt{6}\ cm$$

(iii) $x = ?, y = ?, z = 2cm$

$m\angle C = 60^o$

Using $\sin 60^o = \frac{y}{z}$

$$\frac{\sqrt{3}}{2} = \frac{y}{2}$$
$$2y = 2\sqrt{3}$$
$$y = \sqrt{3}\,cm$$

Using $\tan 60^o = \frac{y}{z}$

$$\sqrt{3} = \frac{\sqrt{3}}{x}$$
$$\sqrt{3}x = \sqrt{3}$$
$$x = \frac{\sqrt{3}}{\sqrt{3}}$$
$$x = 1cm$$

(iv) $y = 4cm, z = ?, x = ?$

Using $\tan 45^o = \frac{x}{y}$

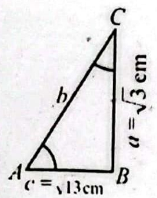
$$1 = \frac{x}{4}$$
$$x = 4cm$$

Using $\cos 45^o = \frac{y}{z}$

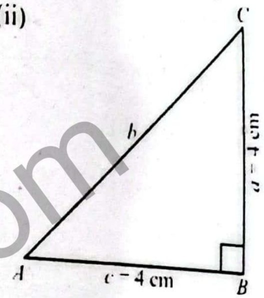
$$\frac{1}{\sqrt{2}} = \frac{4}{z}$$
$$z = 4\sqrt{2}\,cm$$

2. Find the unknown side and angles of the following triangles.

(i)



(ii)



Sol. (i) $c = \sqrt{13}cm,\ a = \sqrt{3}cm,\ m\angle A = ?,\ m\angle C = ?$

Using Pythagoras theorem

$$b^2 = a^2 + c^2$$
$$= (\sqrt{3})^2 + (\sqrt{13})^2$$
$$= 3 + 13$$
$$b^2 = 16$$
$$b = 4\text{cm}$$

Using $\tan(m\angle A) = \frac{\sqrt{3}}{\sqrt{13}}$

$$m\angle A = \tan^{-1}\left(\frac{\sqrt{3}}{\sqrt{13}}\right)$$
$$m\angle A = 26.66^o$$

So $m\angle C = 90^o - 26.66^o$

$$m\angle C = 63.34^o$$

(ii) $c = 4cm,\ a = 4cm, b = ?,\ m\angle A = ?, m\angle C = ?$

Using Pythagoras theorem

$$b^2 = a^2 + c^2$$
$$= 4^2 + 4^2$$
$$= 16 + 16$$
$$b^2 = 32$$
$$b^2 = 16 \times 2$$

$b = \sqrt{16\times2}$

$b = 4\sqrt{2}cm$

Using $\tan(m\angle A) = \frac{4}{4}$.

$\tan(m\angle A) = 1$

$m\angle A = \tan^{-1}(1)$

$m\angle A = 45^o$

So, $m\angle C = 90^o - 45^o$

$m\angle C = 45^o$

3. Each side of a square field is 60*m* long. Find the lengths of the diagonals of the field.

Solve the following triangles when $m\angle B = 90°$:

Sol. To find the length of the diagonal of a square, we can use the Pythagorean theorem. In a square, the diagonal divides the square into two right-angled triangles, where the sides of the square are the legs of the triangle, and the diagonal is the hypotenuse.

The side of the square = 60 m.

The formula for the length of the diagonal *d* of a square is:

$$d = \sqrt{s^2 + s^2}$$

Since both sides are equal, we have:

$$d = \sqrt{2s^2}$$

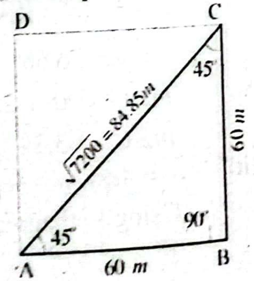
Substituting the value of s = 60 m:

$$d = \sqrt{2\times60^2}$$

$$d = \sqrt{2\times3600}$$

$$d = \sqrt{7200}$$

$$d \approx 84.85\, m = 60\sqrt{2}m$$



Thus, the length of the diagonal of the square field is approximately 84.85 m.

Solve the following right angled triangles when:

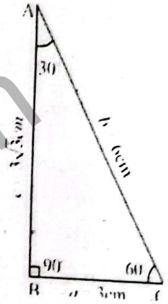
4. $m\angle C = 60°$, $c = 3\sqrt{3}$cm

Sol. $m\angle B = 90^o, a = ?$

$m\angle A = ?, b = ?$

$m\angle A = 90^o - 60^o$

$m\angle A = 30^o$



Using $\tan 30^o = \frac{a}{3\sqrt{3}}$

$\frac{1}{\sqrt{3}} = \frac{a}{3\sqrt{3}}$

$a\sqrt{3} = 3\sqrt{3}$

$a = \frac{3\sqrt{3}}{\sqrt{3}}$

$a = 3cm$

$\frac{3\sqrt{3}}{x} = \tan 60$

$x = \frac{3\sqrt{3}}{\tan 60} = \frac{3\sqrt{3}}{\sqrt{3}} = 3$

$b^2 = 3\sqrt{3} + (3)^2 = 27 + 9 = 36$

$\therefore\ b = 6\ cm$

5. $m\angle C = 45°$, $a = 8$ **cm**

Sol. $\angle C = 45, a = 8$ cm,

$b^2 = a^2 + c^2$

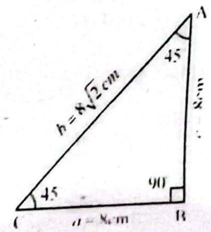
$= 64 + 64 = 128$

$b = \sqrt{128} = \sqrt{64\times2}$

$b = 8\sqrt{2}$ cm

$\angle A = 90^o - 45^o = 45^o$

$c = 8$ cm



6. **$a = 12cm, c = 6$ cm**

Sol. $a = 12cm, c = 6cm, \angle B = 90^\circ$

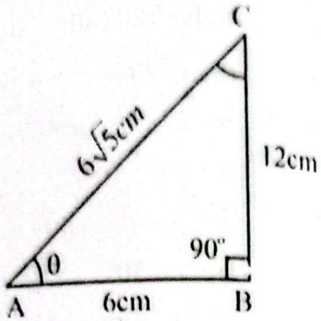
$b^2 = a^2 + c^2$

$b^2 = (12)^2 + (6)^2$

$b^2 = 144 + 36 = 180$

$b = 6\sqrt{5}$ cm

$\cos C = \frac{12}{180} = 0.894$



$m\angle C = 26.6^\circ$

$m\angle C = 26.6^\circ, m\angle A = 63.4^\circ$

7. **$m\angle A = 60^\circ$, c = 4cm**

Sol. $m\angle A = 60^\circ \therefore \angle C = 90^\circ - 60^\circ = 30^\circ$

$\frac{a}{4} = \tan 60$

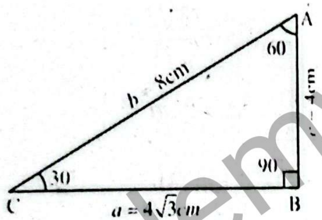
$a = 4\sqrt{3}$

$b^2 = a^2 + c^2$

$= (4\sqrt{3})^2 + (4)^2$

$= 48 + 16 = 64$

$b = 8$ cm



8. **$m\angle A = 60^\circ$, $c = 4$cm**

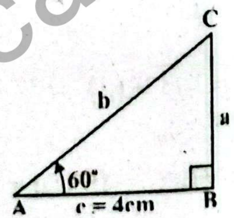
Sol. $m\angle A = 60^\circ$, $c = 4$cm .

$m\angle B = 90^\circ$, b = ?, c = ?

$m\angle C = 90^\circ - 60^\circ$

$m\angle C = 30^\circ$

$c = 4cm$



Using $\tan 60^\circ = \frac{a}{4}$

$\sqrt{3} = \frac{a}{4}$

$a = 4\sqrt{3}cm$

Using Pythagoras theorem,

$b^2 = a^2 + c^2$

$= \left(4\sqrt{3}\right)^2 + (4)^2$

$= 16 \times 3 + 16$

$b^2 = 48 + 16$

$b^2 = 64$

$b = \sqrt{64}$

$b = 8cm$

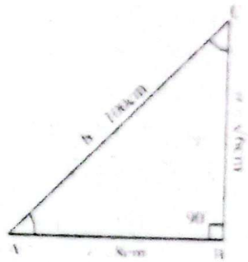
9. **$b = 10$cm, $a = 6$ cm**

Sol. According to fig.

$a^2 + c^2 = b^2$

or $c^2 = b^2 - a^2 = 100 - 36 = 64$

$c = \sqrt{64} = 8$



If $\theta = \angle A$ then $\sin\theta = \frac{6}{10} = \frac{3}{5}$

$m\angle A = \sin^{-1}\left(\frac{3}{5}\right)$

$m\angle A = 36.869$

$m\angle A = 36.9^\circ$

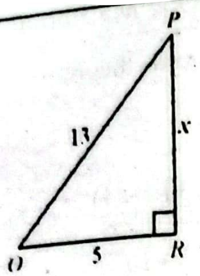
Now, $m\angle A + m\angle C = 90^\circ$

$m\angle C = 90^\circ - m\angle A$

$m\angle C = 90^\circ - 36.9^\circ$

$\boxed{m\angle C = 53.1^\circ}$

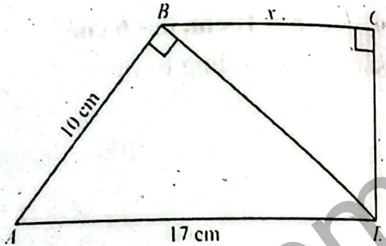
10. Let Q and R be the two points on the same bank of a canal. The point P is placed on the other bank straight to point R. Find the width of the canal and the angle PQR.



Solution: $PQ = 13km$; $QR = 5km$

$$x^2 = (13)^2 - (5)^2$$
$$= 169 - 25$$
$$x^2 = 144$$
$$x = 12km$$

11. **Calculate the length x adjoining figure.**



Sol. From the fig. ABD is a right angled triangle

(By Pythagoras Theorem)

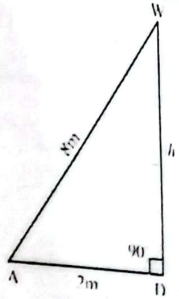
$\therefore\ AD^2 = AB^2 + BD^2$

$$(BD)^2 = (AD)^2 - (AB)^2$$
$$= (17)^2 - (10)^2$$
$$BD^2 = 289 - 100 = 189 \quad \ldots (A)$$

ΔBDC is right angled triangle

$$x^2 + (CD)^2 = (BD)^2$$
$$x^2 = (BD)^2 - (CD)^2$$
$$x^2 = 189 - (8)^2$$
$$x^2 = 189 - 64$$
$$= 125$$
$$x = 5\sqrt{5}$$

12. **If the ladder is placed along the wall such that the foot of the ladder is 2m away from the wall. If the length of the ladder is 8m, find the height of the wall?**



Solution:

The ladder acts as the hypotenuse of the triangle, the distance from the foot of the ladder to the wall is one of the legs, and the height of the wall is the other leg.

Let:

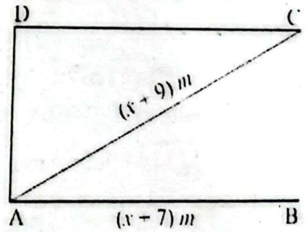
- h be the height of the wall (the vertical leg),
- 2 m be the distance from the foot of the ladder to the wall (the horizontal leg),
- 8 m be the length of the ladder (the hypotenuse).

Using the Pythagorean theorem:

$$h^2 + 2^2 = 8^2$$
$$h^2 + 4 = 64$$
$$h^2 = 64 - 4 = 60$$
$$h = \sqrt{60} \approx 7.75\ \text{m}$$

So, the height of the wall is approximately 7.75 m.

13. The diagonal of a rectangular field $ABCD$ is $(x+9)m$ and the sides are $(x+7)m$ and x m. Find the value of x.



Sol.
$$(x+9)^2 = (x+7)^2 + x^2$$
$$x^2 + 81 + 18x = x^2 + 49 + 14x + x^2$$
$$x^2 + 18x + 81 = 2x^2 + 14x + 49$$
$$2x^2 + 14x - 18x - x^2 + 49x - 81 = 0$$
$$x^2 - 4x - 32 = 0$$
$$x(x-8) + 4(x-8) = (x+4)(x-8) = 0$$
$$\therefore \quad x = 8m \qquad (\because x \text{ is not } -4.)$$

14. Calculate the value of 'x' in each case.

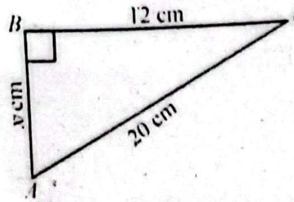
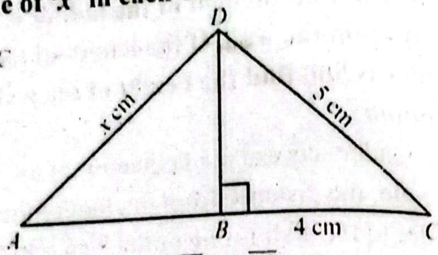


Fig (a)



$m\overline{AB} = m\overline{BC}$

Fig (b)

Calculate the value of x.

Sol. In fig (a)

$$AC^2 = (AB)^2 + (BC)^2$$
$$(AB)^2 = (AC)^2 - (BC)^2$$
$$= (20)^2 - (12)^2$$
$$= 400 - 144$$
$$(AB)^2 = 256$$
$$x^2 = 256$$
$$x = \sqrt{256} = 16cm$$

Fig (b) In ΔBDC

$$(BD)^2 = (DC)^2 - (BC)^2 = 25 - 16 = 9$$

In ΔABD

$$(AD)^2 = (AB)^2 - (BD)^2$$
$$= (4cm)^2 + 9 = 16 + 9 = 25$$
$$AD = \sqrt{25} = 5cm \text{ Ans.}$$

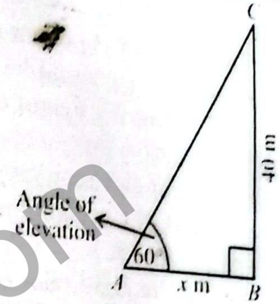
Example 19: The angle of elevation of the top of a pole 40m high is 60° when seen from a point on the ground level. Find the distance of the point from the foot of the pole.

Solution: In the triangle ABC, we have

$$m\overline{BC} = 40 \text{ m}$$
$$m\angle A = 60°$$

Let $m\overline{AB} = x$. (the point B is the foot of the pole BC)

In right angled triangle ABC,

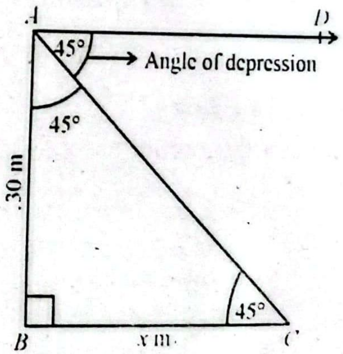


$$\tan 60° = \frac{m\overline{BC}}{m\overline{AB}}$$
$$\sqrt{3} = \frac{40}{x}$$
$$\Rightarrow \quad x = \frac{40}{\sqrt{3}}$$
$$\Rightarrow \quad x = 23.09 \text{ m}$$

Hence, distance of the point from the foot of the pole = $23.09m$

Example 20: From the top of a lookout tower, the angle of depression of a building on the ground level is 45°. How far is a man on the ground form the tower, if the height of the lower is $30m$.

Solution: In the triangle ABC, AB is the tower and point C is the position of man. We have



$$m\overline{AB} = 30 \text{ m}$$
$$m\angle CAD = m\angle C = 45°$$
$$m\overline{BC} = x\text{m} = ?$$

Let x be right angled triangle ABC,

$$\tan 45° = \frac{m\overline{AB}}{m\overline{BC}}$$
$$\Rightarrow \quad 1 = \frac{30}{x}$$
$$\Rightarrow \quad x = 30 \text{ m}$$

Hence, man is 30 m far from the tower.

EXERCISE 6.6

1. **The angle of elevation of the top of a flag post from a point on the ground level 40m away from the flag post is 60°. Find the height of the post.**

Sol. We can solve this problem using trigonometry. We are given the angle of elevation (60°) and the distance from the flag post (40 meters). We need to find the height of the flag post, which we can label as h.

Step 1: Set up the right triangle

In this case, we have a right triangle, where:

- The angle of elevation is 60°.
- The base of the triangle (horizontal distance from the point of observation to the base of the flag post) is 40 meters.
- The height of the flag post is h, which is the vertical side of the triangle.

We can use the tangent function, as it relates the opposite side (height h) to the adjacent side (distance of 40 meters):

$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$$

Substituting the known values:

$$\tan(60°) = \frac{h}{40}$$

Step 2: Solve for *h*

We know that $\tan(60°) = \sqrt{3}$,

so:

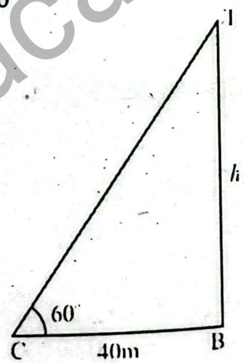
$$\sqrt{3} = \frac{h}{40}$$

Now, solve for h:

$h = 40 \times \sqrt{3}$

$h \approx 40 \times 1.732$

$h \approx 69.28\,\text{m}$

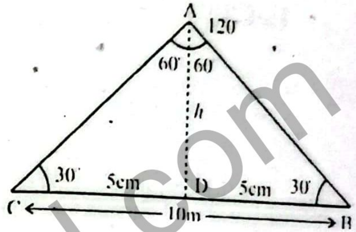


Final Answer:

The height of the flag post is approximately **69.28 meters.**

2. **An isosceles triangle has a vertical angle of 120° and a base 10cm long. Find the length of its altitude.**

Sol. To solve this problem, we'll first break down the isosceles triangle's geometry and use trigonometry to find the length of the altitude.



Given:

- The vertical angle $\angle A = 120°$.
- The length of the base $BC = 10\,\text{cm}$.
- The triangle is isosceles, meaning the two legs AB and AC are equal in length.
- We need to find the length of the altitude from vertex A to base BC.

Step 1: Understanding the Triangle

Let's label the vertices of the triangle as A, B, and C, with the base BC being 10 cm.

Since the triangle is isosceles, the altitude from vertex A to the base BC will bisect BC into two equal segments. So, the two halves of the base will each have a length of $\frac{10}{2} = 5\,\text{cm}$.

This forms two right-angled triangles, where:

- The vertical altitude from A to BC is the height h.
- The base of each right triangle is 5 cm.
- The angle at A in each of the right triangles is 60°, since the vertical angle is 120° and the two base angles are equal, so each is $\frac{180°-120}{2} = 30°$.

Step 2: Use Trigonometry to Find the Altitude

In one of the right triangles, we can use the tangent function, since it relates the opposite side (altitude h) and the adjacent side (half the base, which is 5 cm).

$$\tan(30^\circ) = \frac{h}{5}$$

We know that $\tan(30^\circ) = \frac{1}{\sqrt{3}}$ so:

$$\therefore \frac{h}{5} = \frac{1}{\sqrt{3}}$$

Solving for h:

$$h = \frac{5}{\sqrt{3}}$$

$$h = 2.89$$

Step 3: Diagram of the Triangle

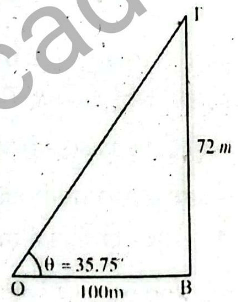
Here's a simple representation of the isosceles triangle:

Final Answer:

- The length of the altitude is approximately **2.89 cm**.

3. A tree is 72m high. Find the angle of elevation of its top from a point 100m away on the ground level.

Sol. To solve this, we can use trigonometry, specifically the tangent function, since we are given the height of the tree and the distance from the point of observation.



Given:

- Height of the tree $h = 72$ m.
- Distance from the point of observation to the base of the tree $d = 100$ m.

We need to find the angle of elevation θ.

Step 1: Using the Tangent Function

In a right-angled triangle, the tangent of the angle of elevation is given by:

$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$$

Here, the opposite side is the height of the tree $h = 72$ m and the adjacent side is the horizontal distance from the observer to the tree $d = 100$ m.

Thus, we have:

$$\tan(\theta) = \frac{72}{100}$$

$$\tan(\theta) = 0.72$$

Step 2: Finding the Angle θ

To find the angle, take the inverse tangent (also called arctangent):

$$\theta = \tan^{-1}(0.72)$$

Using a calculator:

$$\theta \approx 35.75^\circ$$

Final Answer:

The angle of elevation is approximately 35.75°.

4. A ladder makes an angle of 60° with the ground and reaches a height of 10m along the wall. Find the length of the ladder.

Sol. To find the length of the ladder, we can use trigonometry. The situation forms a right triangle, where:

- The height of the ladder along the wall is the opposite side to the angle.
- The length of the ladder is the hypotenuse.
- The angle between the ladder and the ground is 60°.

Given:

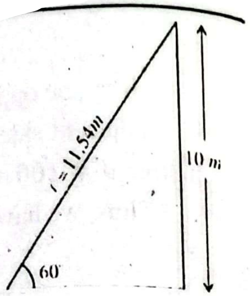
- The angle of elevation of the ladder, $\theta = 60^\circ$.
- The height of the ladder along the wall (opposite side), $h = 10$ m.

We need to find the length of the ladder, which is the hypotenuse L.

Step 1: Using the Sine Function

The sine of the angle of elevation is related to the opposite side and the hypotenuse by the formula:

$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}}$$



Substitute the known values:

$$\sin(60°) = \frac{10}{L}$$

Step 2: Solve for L

We know that $\sin(60°) = \frac{\sqrt{3}}{2}$, so:

$$\frac{\sqrt{3}}{2} = \frac{10}{L}$$

Multiply both sides by L to solve for L:

$$L = \frac{10}{\frac{\sqrt{3}}{2}} = 10 \times \frac{2}{\sqrt{3}} = \frac{20}{\sqrt{3}}$$

To rationalize the denominator:

$$L = \frac{20}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{20\sqrt{3}}{3}$$

Thus, the length of the ladder is:

$L \approx 11.54$ m (after simplifying the expression).

Final Answer:

The length of the ladder is approximately **11.54 meters.**

5. **A light house tower is 150m high from the sea level. The angle of depression from the top of the tower to a ship is 60°. Find the distance between the ship and the tower.**

Sol. To solve this problem, we can use trigonometry. The situation forms a right triangle, where:

- The height of the lighthouse is the vertical side of the triangle (opposite side to the angle).
- The distance from the ship to the base of the lighthouse is the horizontal side (adjacent side to the angle).
- The angle of depression is 60° from the top of the tower to the ship.

Given:

- Height of the lighthouse, $h = 150$ m.
- Angle of depression from the top of the tower to the ship, $\theta = 60°$.

We need to find the distance between the ship and the tower, which is the horizontal side of the triangle.

Step 1: Using the Tangent Function

The tangent of the angle of depression is related to the opposite side and the adjacent side by the formula:

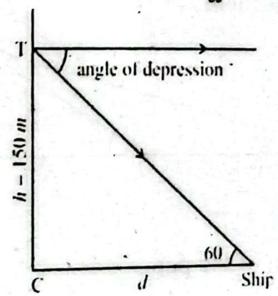
$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$$

In this case:

- The opposite side is the height of the lighthouse, $h = 150$ m,
- The adjacent side is the distance between the ship and the base of the tower (let this be d).

Thus, we can write:

$$\tan(60°) = \frac{150}{d}$$



Step 2: Solve for d

We know that $\tan(60°) = \sqrt{3}$, so:

$$\sqrt{3} = \frac{150}{d}$$

Rearrange to solve for d:

$$d = \frac{150}{\sqrt{3}} = \frac{150 \times \sqrt{3}}{3} = 50\sqrt{3}$$

Now, calculate d:

$$d \approx 50 \times 1.732 = 86.6\,\text{m}$$

Final Answer:

The distance between the ship and the tower is approximately **86.6 meters.**

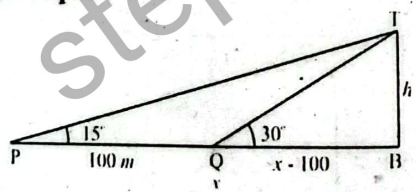
6. **Measure of an angle of elevation of the top of a pole is 15° from a point on the ground, in walking 100m towards the pole the measure of angle is found to be 30°. Find the height of the pole.**

Sol. We can solve this problem using trigonometry. Here's a step-by-step solution:

Given:

- Angle of elevation from the first point (at distance x from the pole): 15°
- Angle of elevation from the second point (100 meters closer to the pole): 30°
- The horizontal distance between the two points is 100 meters.

Step 1: Set up the problem



Let:

- h be the height of the pole.
- x be the distance from the first point (where the angle of elevation is 15°) to the base of the pole.
- The second point is 100 meters closer, so the distance from the second point to the pole is $x - 100$.

Step 2: Use trigonometry

We can use the tangent of the angles of elevation for both points to set up two equations.

- From the first point, the angle of elevation is 15°, so:

$$\tan(15°) = \frac{h}{x}$$

2. From the second point, the angle of elevation is 30°, so:

$$\tan(30°) = \frac{h}{x - 100}$$

Step 3: Substitute the values of the tangents

We know the following trigonometric values:

- $\tan(15°) \approx 0.2679$
- $\tan(30°) = \frac{1}{\sqrt{3}} \approx 0.5774$

Substitute these into the equations:

- $0.2679 = \frac{h}{x}$

$$h = 0.2679 \cdot x$$

2. $0.5774 = \frac{h}{x-100}$

$$h = 0.5774 \cdot (x - 100)$$

Step 4: Set up a system of equations

Now, equate the two expressions for h:

$$0.2679 \cdot x = 0.5774 \cdot (x - 100)$$

Step 5: Solve for x

Expand both sides:

$$0.2679 \cdot x = 0.5774 \cdot x - 57.74$$

Move all terms involving x to one side:

$$0.5774 \cdot x - 0.2679 \cdot x = 57.74$$

$$0.3095 \cdot x = 57.74$$

Now solve for x:

$$x = \frac{57.74}{0.3095} \approx 186.6\,\text{m}$$

Step 6: Find the height h

Substitute $x = 186.3$ into the first equation for h:

$$h = 0.2679 \times 186.3 \approx 50\,\text{m}$$

Final Answer:

The height of the pole is approximately **49.9 meters.**

7. Find the measure of an angle of elevation of the Sun, if a tower 300m high casts a shadow 450mm long.

Sol. To find the angle of elevation of the Sun, we can use basic trigonometry. The situation described forms a right triangle, where:

- The height of the tower is 300 meters.
- The length of the shadow is 450 meters.
- The angle of elevation is the angle between the ground and the line of sight from the top of the tower to the Sun.

Step 1: Use the tangent function

In a right triangle, the tangent of the angle of elevation (θ) is given by:

$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$$

Here:

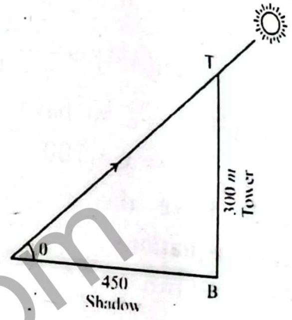
- The opposite side is the height of the tower, which is 300 meters.
- The adjacent side is the length of the shadow, which is 450 meters.

Thus:

$$\tan(\theta) = \frac{300}{450} = \frac{2}{3}$$

Step 2: Find the angle

Now, to find the angle of elevation (θ), take the inverse tangent (or arctan) of $\frac{2}{3}$:



$$\theta = \tan^{-1}\left(\frac{2}{3}\right)$$

Using a calculator:

$$\theta \approx 33.69°$$

Final Answer:

The angle of elevation of the Sun is approximately **33.69°**.

8. Measure of angle of elevation of the top of a cliff is 25°, on walking 100 metres towards the cliff, measure of angle of elevation of the top is 45°. Find the height of the cliff.

Sol. To find the height of the cliff, we can solve this problem using trigonometry. The situation involves two points from which we are observing the top of the cliff:

- Initially, the angle of elevation from the first point is 25°.
- After walking 100 meters towards the cliff, the angle of elevation increases to 45°.

Let's define the following variables:

- h = height of the cliff (which we need to find)
- x = horizontal distance from the first point to the base of the cliff.
- The second point is 100 meters closer to the cliff, so the horizontal distance from the second point to the base of the cliff is $x - 100$.

Step 1: Use the tangent function for both situations

For the first point, we know:

$$\tan(25°) = \frac{h}{x} \qquad \text{.... (i)}$$

So,

$$h = x \cdot \tan(25°) \qquad \text{(Equation 1)}$$

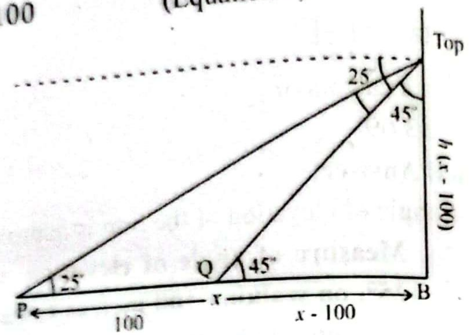
For the second point, after walking 100 meters closer to the cliff:

$$\tan(45^\circ) = \frac{h}{x - 100} \quad \dots (ii)$$

Since $\tan(45^\circ) = 1$, we have:

$$h = x - 100 \quad \text{(Equation 2)}$$

Step 2: Solve the system of equations



Now we have two equations:

- $h = x \cdot \tan(25^\circ)$
- $h = x - 100$

Set the two expressions for h equal to each other:

$$x \cdot \tan(25^\circ) = x - 100$$

Now, solve for x:

$$x \cdot \tan(25^\circ) - x = -100$$

Factor out x:

$$x(\tan(25^\circ) - 1) = -100$$

Now, solve for x:

$$x = \frac{-100}{\tan(25^\circ) - 1}$$

Using a calculator:

$$\tan(25^\circ) \approx 0.4663$$

Thus,

$$x = \frac{-100}{0.4663 - 1} = \frac{-100}{-0.5337} \approx 187.3 \text{ meters}$$

Step 3: Find the height of the cliff

Now that we have the value of x, substitute it into Equation 2 to find the height of the cliff:

$$h = x - 100 = 187.3 - 100 = 87.4 \text{ meters}$$

Final Answer:

The height of the cliff is approximately **87.4 meters.**

9. From the top of a hill 300m high, the measure of the angle of depression of a point on the nearer shore of the river is 70° and measure of the angle of depression of a point, directly across the river is 50°. Find the width of the river How far is the river from the foot of the hill?

Sol. This is a trigonometric problem involving two angles of depression from the top of a hill to two points: one on the nearer shore of the river and one directly across the river. Let's solve this step by step.

Given:

- Height of the hill, $h = 300$ m
- Angle of depression to the nearer shore, $\theta_1 = 70^\circ$
- Angle of depression to the point directly across the river, $\theta_2 = 50^\circ$

We are asked to find:

- The **width of the river.**
- The **distance from the river to the foot of the hill.**

Step 1: Define variables and use trigonometry

Let's define the following variables:

- Let d_1 be the horizontal distance from the foot of the hill to the nearer shore of the river.
- Let d_2 be the horizontal distance from the foot of the hill to the point directly across the river.
- The width of the river, W, will be the difference between d_2 and d_1:

$$W = d_2 - d_1$$

Step 2: Use the tangent function for both angles of depression

The tangent of an angle of depression relates the height of the hill to the horizontal distance. Specifically, for any angle of depression θ, we can use the formula:

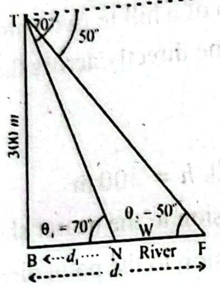
$$\tan(\theta) = \frac{h}{d}$$

FOR THE NEARER SHORE (DISTANCE d_1):

$$\tan(70°) = \frac{300}{d_1}$$

Solving for d_1:

$$d_1 = \frac{300}{\tan(70°)}$$



Using $\tan(70°) \approx 2.747$:

$$d_1 = \frac{300}{2.747} \approx 109.2 \text{ m}$$

FOR THE POINT DIRECTLY ACROSS THE RIVER (DISTANCE d_2):

$$\tan(50°) = \frac{300}{d_2}$$

Solving for d_2:

$$d_2 = \frac{300}{\tan(50°)}$$

Using $\tan(50°) \approx 1.1918$:

$$d_2 = \frac{300}{1.1918} \approx 251.7 \text{ m}$$

Step 3: Calculate the width of the river

Now that we have both distances, the width of the river is:

$$W = d_2 - d_1 = 251.7 - 109.2 = 142.3 \text{ m}$$

Step 4: Distance from the river to the foot of the hill

The distance from the foot of the hill to the river is just the value of d_2, which is **251.7 meters.**

Final Answers:

- The width of the river is approximately **142.3 meters.**
- The distance from the river to the foot of the hill is approximately **251.7 meters.**

10. A kite has 120m of string attached to it when at an elevation of 50°. How far is it above the hand holding it? (Assume that the string is tight.)

Sol: To solve this problem, we can use trigonometry. Given that the string forms an angle of 50° with the horizontal ground, we can model the situation as a right triangle. The string represents the hypotenuse, and we need to find the vertical distance (height) from the hand to the kite.

Given:

- The length of the string (hypotenuse) = 120 m
- The angle of elevation = 50°

We need to find the vertical height of the kite, which is the opposite side of the right triangle formed by the string and the ground.

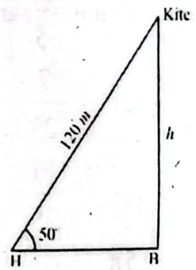
Step 1: Use the sine function

The sine of an angle in a right triangle is defined as the ratio of the opposite side to the hypotenuse. Thus:

$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}}$$

In our case: $\sin(50°) = \frac{h}{120}$

where h is the height of the kite above the hand.



Step 2: Solve for h

We can rearrange the equation to solve for h:

$$h = 120 \times \sin(50°)$$

Using $\sin(50°) \approx 0.766$: $h = 120 \times 0.766 \approx 91.92 \text{ m}$

Final Answer:

The kite is approximately **91.92 meters** above the hand holding it.

REVIEW EXERCISE 6

1. Choose the correct option.

(i) The value of $\tan^{-1} 2$ in radians is:

(a) $\frac{\pi}{2}$ (b) $\frac{3\pi}{2}$

(c) $0.4636\,\pi$ (d) 0.4636

(ii) In a right triangle, the hypotenuse is 13 units and one of the angles is $\theta = 30°$. The length of the opposite side?

(a) 6.5 units (b) 7.5 units

(c) 6 units (d) 5 units

(iii) A person standing 50m away from a building sees the top of the building at an angle of elevation of 45°. Height of the building is:

(a) 50 m (b) 25 m

(c) 35 m (d) 70 m

(iv) $\sec^2\theta - \tan^2\theta =$ ________.

(a) $\sin^2\theta$ (b) 1

(c) $\cos^2\theta$ (d) $\cot^2\theta$

(v) If $\sin\theta = \frac{3}{5}$, and θ is an acute angle, $\cos\theta =$ ________.

(a) $\frac{7}{25}$ (b) $\frac{24}{25}$

(c) $\frac{16}{25}$ (d) $\frac{4}{25}$

(vi) $\frac{5\pi}{24}$ rad = ________ degrees.

(a) 30° (b) 37.5°

(c) 45° (d) 52.5°

(vii) $292.5° =$ ________ rad.

(a) $\frac{17\pi}{6}$ (b) $\frac{17\pi}{4}$

(c) $1.6\,\pi$ (d) $1.625\,\pi$

(viii) Which of the following is a valid identity?

(a) $\cos\left(\frac{\pi}{2}-\theta\right) = \sin\theta$ (b) $\cos\left(\frac{\pi}{2}-\theta\right) = \cos\theta$

(c) $\cos\left(\frac{\pi}{2}-\theta\right) = \sec\theta$ (d) $\cos\left(\frac{\pi}{2}-\theta\right) = \operatorname{cosec}\theta$

ix. $\sin 60° =$ ________.

(a) 1 (b) $\frac{1}{2}$

(c) $\sqrt{(3)^2}$ (d) $\frac{\sqrt{3}}{2}$

x. $\cos^2 100\pi + \sin^2 100\pi =$ ________.

(a) 1 (b) 2

(c) 3 (d) 4

Answers:

(i)	c	(ii)	a	(iii)	a	(iv)	b	(v)	d
(vi)	b	(vii)	d	(viii)	a	(ix)	d	(x)	a

2. Convert the given angles from:

(a) degrees to radians giving answer in terms of π.

(i) 255° (ii) 75° 45′ (iii) 142.5°

Sol. To convert angles from degrees to radians, we use the following formula:

$$\text{Angle in radians} = \text{Angle in degrees} \times \frac{\pi}{180}$$

Let's convert the given angles:

(i) **255° to radians:**

$$255° \times \frac{\pi}{180} = \frac{255\pi}{180} = \frac{17\pi}{12}$$

So, $255° = \frac{17\pi}{12}$ radians.

(ii) 75° 45' to radians:

First, convert the angle in degrees and minutes to just degrees:

$$75°45' = 75° + \frac{45'}{60} = 75° + 0.75° = 75.75°$$

Now, convert 75.75° to radians:

$$75.75° \times \frac{\pi}{180} = \frac{75.75\pi}{180} = \frac{303\pi}{720} = \frac{101\pi}{240}.$$

So, $75°45' = \frac{101\pi}{240}$ radians.

(iii) 142.5° to radians:

$$142.5° \times \frac{\pi}{180} = \frac{142.5\pi}{180} = \frac{285\pi}{360} = \frac{19\pi}{24}.$$

So, $142.5° = \frac{19\pi}{24}$ radians.

(b) radians to degrees giving answer in degrees and minutes.

(i) $\frac{17\pi}{24}$ (ii) $\frac{7\pi}{12}$ (iii) $\frac{11\pi}{16}$

To convert from radians to degrees, we use the following formula:

$$\text{Angle in degrees} = \text{Angle in radians} \times \frac{180}{\pi}$$

Let's convert each of the given angles.

(i) $\frac{17\pi}{24}$ radians to degrees:

$$\frac{17\pi}{24} \times \frac{180}{\pi} = \frac{17 \times 180}{24} = \frac{3060}{24} = 127.5°$$

Now, convert the decimal part (0.5) into minutes:

$$0.5° \times 60 = 30'$$

So, $\frac{17\pi}{24}$ radians $= 127°30'$.

(ii) $\frac{7\pi}{12}$ radians to degrees:

$$\frac{7\pi}{12} \times \frac{180}{\pi} = \frac{7 \times 180}{12} = \frac{1260}{12} = 105°$$

There is no decimal part, so the angle is simply 105°.

(iii) $\frac{11\pi}{16}$ radians to degrees:

$$\frac{11\pi}{16} \times \frac{180}{\pi} = \frac{11 \times 180}{16} = \frac{1980}{16} = 123°45'$$

So, $\frac{11\pi}{16}$ radians $= 123°45'$.

3. Prove the following trigonometric identities:

(i) $\frac{\sin\theta}{1-\cos\theta} = \frac{1+\cos\theta}{\sin\theta}$

(ii) $\sin\theta\,(\operatorname{cosec}\theta - \sin\theta) = \frac{1}{\sec^2\theta}$

(iii) $\frac{\operatorname{cosec}\theta - \sec\theta}{\operatorname{cosec}\theta + \sec\theta} = \frac{1-\cos\theta}{1+\tan\theta}$

(iv) $\tan\theta + \cot\theta = \frac{1}{\sin\theta\cos\theta}$

(v) $\frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} + \frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \frac{2}{1 - 2\sin^2\theta}$

(vi) $\frac{1+\cos\theta}{1-\cos\theta} = (\operatorname{cosec}\theta + \cot\theta)^2$

Let's prove these trigonometric identities step by step.

(i) Prove

$$\frac{\sin\theta}{1-\cos\theta} = \frac{1+\cos\theta}{\sin\theta}$$

Proof:

- Start with the LHS:

$$\frac{\sin\theta}{1-\cos\theta}$$

- Multiply numerator and denominator by $1+\cos\theta$ (rationalizing the denominator):

$$\frac{\sin\theta}{1-\cos\theta}\cdot\frac{1+\cos\theta}{1+\cos\theta}=\frac{\sin\theta(1+\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}$$

- Simplify the denominator using the identity $(1-\cos\theta)(1+\cos\theta)=1-\cos^2\theta=\sin^2\theta$:

$$\frac{\sin\theta(1+\cos\theta)}{\sin^2\theta}$$

- Simplify the fraction:

$$\frac{\sin\theta(1+\cos\theta)}{\sin^2\theta}=\frac{1+\cos\theta}{\sin\theta}$$

Thus,

$$\frac{\sin\theta}{1-\cos\theta}=\frac{1+\cos\theta}{\sin\theta}$$

(ii) Prove

$$\sin\theta(\operatorname{cosec}\theta-\sin\theta)=\frac{1}{\sec^2\theta}$$

Proof:

- Start with the LHS:

$$\sin\theta(\operatorname{cosec}\theta-\sin\theta)$$

- Substitute $\operatorname{cosec}\theta=\frac{1}{\sin\theta}$:

$$\sin\theta\left(\frac{1}{\sin\theta}-\sin\theta\right)$$

- Simplify:

$$\sin\theta\left(\frac{1-\sin^2\theta}{\sin\theta}\right)$$

- Simplify further:

$$\frac{\sin\theta(1-\sin^2\theta)}{\sin\theta}=1-\sin^2\theta$$

- Use the identity $1-\sin^2\theta=\cos^2\theta$:

$$\cos^2\theta$$

- Substitute $\cos^2\theta=\frac{1}{\sec^2\theta}$:

$$\frac{1}{\sec^2\theta}$$

Thus,

$$\sin\theta(\operatorname{cosec}\theta-\sin\theta)=\frac{1}{\sec^2\theta}$$

(iii) Prove

$$\frac{\operatorname{cosec}\theta-\sec\theta}{\operatorname{cosec}\theta+\sec\theta}=\frac{1-\cos\theta}{1+\tan\theta}$$

Proof:

- Start with the LHS:

$$\frac{\operatorname{cosec}\theta-\sec\theta}{\operatorname{cosec}\theta+\sec\theta}$$

- Substitute $\operatorname{cosec}\theta=\frac{1}{\sin\theta}$ and $\sec\theta=\frac{1}{\cos\theta}$:

$$\frac{\frac{1}{\sin\theta}-\frac{1}{\cos\theta}}{\frac{1}{\sin\theta}+\frac{1}{\cos\theta}}$$

- Simplify the numerator and denominator:

Numerator: $\frac{1}{\sin\theta}-\frac{1}{\cos\theta}=\frac{\cos\theta-\sin\theta}{\sin\theta\cos\theta}$

Denominator: $\frac{1}{\sin\theta}+\frac{1}{\cos\theta}=\frac{\cos\theta+\sin\theta}{\sin\theta\cos\theta}$

- Combine:

$$\frac{\frac{\cos\theta-\sin\theta}{\sin\theta\cos\theta}}{\frac{\cos\theta+\sin\theta}{\sin\theta\cos\theta}}=\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}$$

- Multiply numerator and denominator by -1 to match the RHS:

$$\frac{\sin\theta-\cos\theta}{\sin\theta+\cos\theta}$$

Thus,

$$\frac{\operatorname{cosec}\theta-\sec\theta}{\operatorname{cosec}\theta+\sec\theta}=\frac{1-\cos\theta}{1+\tan\theta}$$

(iv) **Prove**

$$\tan\theta + \cot\theta = \frac{1}{\sin\theta\cos\theta}$$

Proof:

- Start with the LHS:

$$\tan\theta + \cot\theta$$

- Substitute $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\cot\theta = \frac{\cos\theta}{\sin\theta}$:

$$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

- Take the LCM:

$$\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$$

- Use the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$:

$$\frac{1}{\sin\theta\cos\theta}$$

Thus,

$$\tan\theta + \cot\theta = \frac{1}{\sin\theta\cos\theta}$$

(v) **Prove**

$$\frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} + \frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \frac{2}{1 - 2\sin^2\theta}$$

Proof:

- Start with the LHS:

$$\frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} + \frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}$$

- Take the LCM:

$$\frac{(\cos\theta + \sin\theta)^2 + (\cos\theta - \sin\theta)^2}{(\cos\theta - \sin\theta)(\cos\theta + \sin\theta)}$$

- Expand the numerator using $(a \pm b)^2 = a^2 \pm 2ab + b^2$:

$$(\cos^2\theta + 2\cos\theta\sin\theta + \sin^2\theta) + (\cos^2\theta - 2\cos\theta\sin\theta + \sin^2\theta)$$

- Combine like terms:

$$2\cos^2\theta + 2\sin^2\theta = 2(\cos^2\theta + \sin^2\theta)$$

- Use the identity $\cos^2\theta + \sin^2\theta = 1$:

$$2(1) = 2$$

- Simplify the denominator:

$$(\cos\theta - \sin\theta)(\cos\theta + \sin\theta) = \cos^2\theta - \sin^2\theta$$

- The LHS becomes:

$$\frac{2}{\cos^2\theta - \sin^2\theta}$$

- Use the identity $\cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta$:

$$\frac{2}{1 - 2\sin^2\theta}$$

Thus,

$$\frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} + \frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \frac{2}{1 - 2\sin^2\theta}$$

(vi) **Prove**

$$\frac{1 + \cos\theta}{1 - \cos\theta} = (\operatorname{cosec}\theta + \cot\theta)^2$$

Proof:

- Start with the RHS:

$$(\operatorname{cosec}\theta + \cot\theta)^2$$

- Substitute $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$ and $\cot\theta = \frac{\cos\theta}{\sin\theta}$:

$$\left(\frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}\right)^2$$

- Combine terms:

$$\left(\frac{1 + \cos\theta}{\sin\theta}\right)^2$$

- Expand:

$$\frac{(1 + \cos\theta)^2}{\sin^2\theta}$$

- Simplify the LHS:

$$\frac{1 + \cos\theta}{1 - \cos\theta}$$

- Multiply numerator and denominator by $1 + \cos\theta$ (rationalizing the denominator):

$$\frac{(1+\cos\theta)^2}{1-\cos^2\theta}$$

- Use the identity $1 - \cos^2\theta = \sin^2\theta$:

$$\frac{(1+\cos\theta)^2}{\sin^2\theta}$$

Thus,

$$\frac{1+\cos\theta}{1-\cos\theta} = (\operatorname{cosec}\theta + \cot\theta)^2$$

4. **If $\tan\theta = \frac{3}{\sqrt{2}}$ then find the remaining trigonometric ratios when θ lies in first quadrant.**

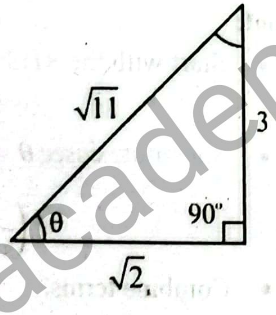
Sol. Hypotenuse $= \sqrt{3^2 + (\sqrt{2})^2}$

$= \sqrt{9+2} = \sqrt{11}$

$\sin\theta = \frac{3}{\sqrt{11}}, \operatorname{cosec}\theta = \frac{\sqrt{11}}{3},$

$\cos\theta = \frac{\sqrt{2}}{\sqrt{11}} = \sqrt{\frac{2}{11}}$

$\sec\theta = \sqrt{\frac{11}{2}}, \cot\theta = \frac{\sqrt{2}}{3}$



All trigonometric ratios have positive sign. (θ lies in 1st quadrant.)

5. **From a point on the ground, the angle of elevation to the top of a 30-meter-high building is 28°. How far is the point from the base of the building?**

Sol. To solve this problem, we can use trigonometry. The given information is:

- The height of the building is $h = 30$ m.
- The angle of elevation to the top of the building is $\theta = 28°$.

We need to find the distance from the point on the ground to the base of the building. Let's call this distance d.

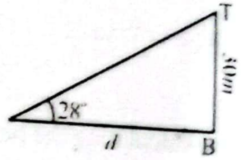
Step 1: Use the tangent function.

The tangent function relates the angle of elevation to the opposite side (height of the building) and the adjacent side (distance from the point to the base of the building) in a right triangle:

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

In this case:

$$\tan 28° = \frac{30}{d}$$



Step 2: Solve for d.

Rearrange the equation to solve for d:

$$d = \frac{30}{\tan 28°}$$

Now, let's calculate the value of d:

$$\tan 28° \approx 0.5317$$

So, $$d = \frac{30}{0.5317} \approx 56.42 \text{ m}$$

Step 3: Conclusion: The point is approximately 56.42 meters away from the base of the building.

6. **A ladder leaning against a wall forms an angle of 65° with the ground. If the ladder is 10 meters long, how high does it reach on the wall?**

Solution : Step-by-Step:

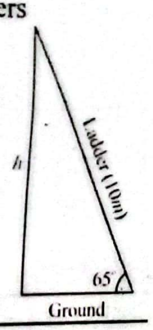
Given:

- Length of the ladder (hypotenuse) = 10 meters
- Angle with the ground = 65°

We need to find the height the ladder reaches on the wall, which corresponds to the **opposite** side of the right triangle formed by the ladder.

We can use the sine function from trigonometry to solve this:

$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}}$$



Where: • $\theta = 65°$

- The opposite side is the height we are looking for
- The hypotenuse is the length of the ladder, which is 10 meters

Rearranging the formula to solve for the opposite side (height):

$$\text{opposite} = \sin(\theta) \times \text{hypotenuse}$$

Substituting the known values:

$$\text{opposite} = \sin(65°) \times 10$$

Using a calculator for $\sin(65°)$, we get approximately:

$$\sin(65°) \approx 0.9063$$

Now, calculating the height:

$$\text{opposite} = 0.9063 \times 10 = 9.063 \text{ meters}$$

Conclusion: The height the ladder reaches on the wall is approximately 9.06 meters.