# UNIT 7 Coordinate Geometry

## Students' learning outcomes

**At the end of the unit, the students will be able to:**

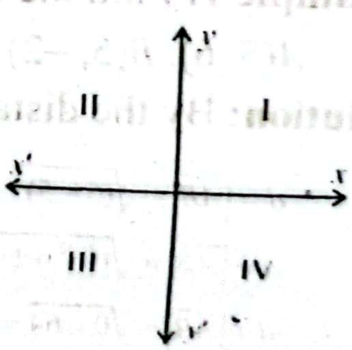
- Derive distance formula by locating the position of two points in coordinate plane
- Calculate the midpoint of a line segment
- Find the gradient of a straight line when coordinates of two points are given
- Find the equation of a straight line in the form $y = mx + c$
- Find the gradient of parallel and perpendicular lines
- Apply distance and midpoint formulas to solve real-life situations such as physical measurements or distances between locations.
- Apply concepts from coordinate Geometry to real world problems (such as, aviation and navigation, landscaping, map reading, longitude and latitude).
- Derive equation of a straight line in
  - slope- intercept form
  - point-slope form
  - two-point form
  - intercepts form
  - symmetric form
  - normal form.
- Show that a linear equation in two variables represents a straight line and reduce the general form of the equation of a straight line to the other standard forms.

**Quadrant I:** All points $(x, y)$ with $x > 0, y > 0$

**Quadrant II:** All points $(x, y)$ with $x < 0, y > 0$

**Quadrant III:** All points $(x, y)$ with $x < 0, y < 0$

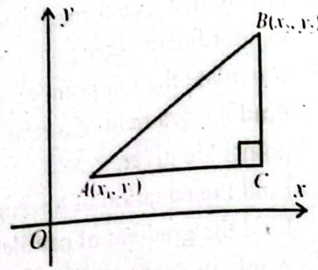
**Quadrant IV:** All points $(x, y)$ with $x > 0, y < 0$



## The Distance Formula

Let $A(x_1, y_1)$ and $B(x_2, y_2)$ be two points in the plane. To find the distance $d = |\overline{AB}|$, we draw a horizontal line from $A$ to a point $C$ lies directly below $B$, forming a right triangle $ABC$.

**Note:** $|\overline{AB}|$ stands for $m\overline{AB}$




So that $|\overline{AC}| = |x_2 - x_1|$ and

$|\overline{BC}| = |y_2 - y_1|$

By using Pythagoras Theorem, we have

$$d^2 = |\overline{AB}|^2 = |\overline{AC}|^2 + |\overline{BC}|^2$$
$$= (x_2 - x_1)^2 + (y_2 - y_1)^2$$

or $\boxed{d = |\overline{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$ ...(i)

The distance is always taken to be non-negative. It is not a directed distance from $A$ to $B$.

If A and B lie on a line parallel to one of the coordinate axes, then by the formula (i), the distance $\overline{AB}$ is absolute value of the directed distance $\overline{AB}$.

The formula (i) shows that any of two points can be taken as first point.

**Example 1:** Find the distance between the points:

(i) $A(5, 6), B(5, -2)$ (ii) $C(-4, -2), D(0, 9)$

**Solution:** By the distance formula, we have

(i) $d = |\overline{AB}| = \sqrt{(5-5)^2 + (-2-6)^2}$

$d = |\overline{AB}| = \sqrt{(0)^2 + (-8)^2}$

$d = |\overline{AB}| = \sqrt{0 + 64} = 8$

(ii) $d = |\overline{CD}| = \sqrt{(0-(-4))^2 + (9-(-2))^2}$

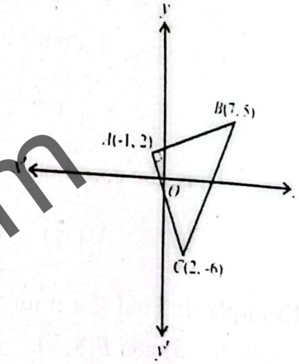
$d = |\overline{CD}| = \sqrt{(0+4)^2 + (9+2)^2}$

$d = |\overline{CD}| = \sqrt{4^2 + 11^2}$

$d = |\overline{CD}| = \sqrt{16 + 121} = \sqrt{137}$

**Example 2:** Show that the points $A(-1, 2)$, $B(7, 5)$ and $C(2, -6)$ are vertices of a right triangle.

**Solution:** Let $a$, $b$ and $c$ denote the lengths of the sides $\overline{BC}, \overline{CA}$ and $\overline{AB}$ respectively.




By using the distance formula, we have

$c = |\overline{AB}| = \sqrt{(7-(-1))^2 + (5-2)^2} = \sqrt{73}$

$a = |\overline{BC}| = \sqrt{(2-7)^2 + (-6-5)^2} = \sqrt{14}$

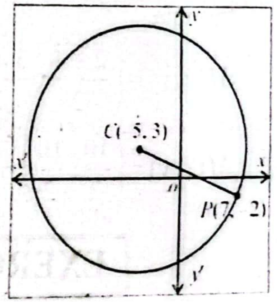
$b = |\overline{CA}| = \sqrt{(2-(-1))^2 + (-6-2)^2} = \sqrt{73}$

Clarity: $a^2 = b^2 + c^2$

Therefore, $ABC$ is a right triangle with right angle at $A$.

**Example 3:** The point $C(-5, 3)$ is the centre of a circle and $P(7, -2)$ lies on the circle. What is the radius of the circle?

**Solution:** The radius of the circle is the distance from the points $C$ to $P$. By the using distance formula, we have




Radius $= |\overline{CP}|$

$= \sqrt{(7-(-5))^2 + (-2-3)^2}$

$= \sqrt{144 + 25} = \sqrt{169}$

$= 13$ units

1. **x-Coordinate of the Midpoint**

The $x$-coordinate of the midpoint is the average of the $x$–coordinates of points $A$ and $B$.

i.e., $x = \dfrac{x_1 + x_2}{2}$

2. $y$-coordinate of the Midpoint

Similarly, the $y$-coordinate of the midpoint is the average of the $y$-coordinates of points $A$ and $B$.

i.e., $y = \frac{y_1 + y_2}{2}$

Thus, the coordinates of the midpoint $M(x, y)$ are:

$$M(x,y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

**Example 4:** Find the midpoint of the line segment joining the points $A(2, 3)$ and $B(8, 7)$.

**Solution:** Using the midpoint formula:

$$M(x,y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Substitute $x_1 = 2, y_1 = 3, x_2 = 8$ and $y_2 = 7$ into the midpoint formula

$$M(x,y) = \left(\frac{2+8}{2}, \frac{3+7}{2}\right)$$

$$M(x,y) = \left(\frac{10}{2}, \frac{10}{2}\right) = (5, 5)$$

## EXERCISE 7.1

1. **Describe the location in the plane of the point $P(x, y)$, for which**

(i) $x > 0$ (ii) $x > 0$ and $y > 0$

(iii) $x = 0$ (iv) $y = 0$

(v) $x > 0$ and $y \leq 0$ (vi) $y = 0, x = 0$

(vii) $x = y$ (viii) $x \geq 3$

(ix) $y > 0$

(x) $x$ and $y$ have opposite signs.

***Solution:*** Here is a description of the location of the point $P(x, y)$ in the plane for each condition:

(i) $x > 0$

The point lies in the **right half** of the plane, where $x$ is positive. This includes Quadrants I and IV but excludes the $y$-axis and points to the left of it.

(ii) $x > 0$ and $y > 0$

The point lies in **Quadrant I**, where both $x$ and $y$ are positive.

(iii) $x = 0$

The point lies on the $y$-**axis**, which is the vertical line where $x = 0$.

(iv) $y = 0$

The point lies on the $x$-**axis**, which is the horizontal line where $y = 0$.

(v) $x > 0$ and $y < 0$

The point lies in **Quadrant IV**, where $x$ is positive and $y$ is negative.

(vi) $y = 0$ and $x = 0$

The point lies at the **origin** (0,0), where both $x$ and $y$ are zero.

(vii) $x = y$

The point lies on the **line** $x = y$, which is a diagonal line passing through the origin at a 45° angle in Quadrants I and III.

(viii) $x \geq 3$

The point lies on or to the **right of the vertical line** $x = 3$. This includes all points where $x = 3$ or greater, regardless of $y$.

(ix) $y > 0$

The point lies in the **upper half** of the plane, where $y$ is positive. This includes Quadrants I and II but excludes the $x$-axis and points below it.

(x) $x$ and $y$ have opposite signs

The point lies in **Quadrants II and IV**:

- In Quadrant II, $x < 0$ and $y > 0$.
- In Quadrant IV, $x > 0$ and $y < 0$.

2. **Find the distance between the points:**

(i) $A(6, 7), B(0, -2)$ (ii) $C(-5, -2), D(3, 2)$

(iii) $L(0, 3), M(-2, -4)$ (iv) $P(-8, -7), Q(0, 0)$

***Solution:*** The formula to find the distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ in a 2D plane is:

$$\text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

(i) **Points $A(6,7)$ and $B(0,-2)$:**

$$\text{Distance} = \sqrt{(0-6)^2 + (-2-7)^2} = \sqrt{(-6)^2 + (-9)^2}$$

$$= \sqrt{36+81} = \sqrt{117} = 3\sqrt{13}.$$

(ii) **Points $C(-5,-2)$ and $D(3,2)$:**

$$\text{Distance} = \sqrt{(3-(-5))^2 + (2-(-2))^2}$$

$$= \sqrt{(3+5)^2 + (2+2)^2}$$

$$= \sqrt{8^2 + 4^2} = \sqrt{64+16} = \sqrt{80} = 4\sqrt{5}.$$

(iii) **Points $L(0,3)$ and $M(-2,-4)$:**

$$\text{Distance} = \sqrt{(-2-0)^2 + (-4-3)^2} = \sqrt{(-2)^2 + (-7)^2}$$

$$= \sqrt{4+49} = \sqrt{53}.$$

(iv) **Points $P(-8,-7)$ and $Q(0,0)$:**

$$\text{Distance} = \sqrt{(0-(-8))^2 + (0-(-7))^2} = \sqrt{(8)^2 + (7)^2}$$

$$= \sqrt{64+49} = \sqrt{113}.$$

3. **Find in each of the following:**

(i) **The distance between the two given pints.**

(a) $A(3, 1), B(-2, -4)$ (b) $A(-8, 3), B(2, -1)$

(c) $A\left(-\sqrt{5}, -\frac{1}{3}\right), B\left(-3\sqrt{5}, 5\right)$

**Sol.**

(a) **Points $A(3,1)$ and $B(-2,-4)$:**

- **Distance:**

$$\text{Distance} = \sqrt{(-2-3)^2 + (-4-1)^2} = \sqrt{(-5)^2 + (-5)^2}$$

$$= \sqrt{25+25} = \sqrt{50} = 5\sqrt{2}.$$

(b) **Points $A(-8,3)$ and $B(2,-1)$:**

- **Distance:**

$$\text{Distance} = \sqrt{(2-(-8))^2 + (-1-3)^2} = \sqrt{(10)^2 + (-4)^2}$$

$$= \sqrt{100+16} = \sqrt{116} = 2\sqrt{29}.$$

(c) **Points $A\left(-\sqrt{5}, -\frac{1}{3}\right)$ and $B(-3\sqrt{5}, 5)$:**

- **Distance:**

$$\text{Distance} = \sqrt{\left(-3\sqrt{5} - (-\sqrt{5})\right)^2 + \left(5 - \left(-\frac{1}{3}\right)\right)^2}.$$

Simplify each term:

$$x\text{-difference} = -3\sqrt{5} + \sqrt{5} = -2\sqrt{5},$$

$$y\text{-difference} = 5 + \frac{1}{3} = \frac{15}{3} + \frac{1}{3} = \frac{16}{3}.$$

Substitute:

$$\text{Distance} = \sqrt{(-2\sqrt{5})^2 + \left(\frac{16}{3}\right)^2} = \sqrt{4 \times 5 + \frac{256}{9}}$$

$$= \sqrt{20 + \frac{256}{9}}.$$

Convert to a common denominator:

$$\text{Distance} = \sqrt{\frac{180}{9} + \frac{256}{9}} = \sqrt{\frac{436}{9}} = \frac{\sqrt{436}}{3} = \frac{2\sqrt{109}}{3}.$$

**(ii) Midpoint of the line segment joining the two points:**

**(a)** $A(3, 1), B(-2, -4)$ **(b)** $A(-8, 3), B(2, -1)$

**(c)** $A\left(-\sqrt{5}, -\frac{1}{3}\right), B\left(-3\sqrt{5}, 5\right)$

**Sol: (a)** $A(3, 1), B(-2, -4)$

$$\text{Midpoint} = \left(\frac{3+(-2)}{2}, \frac{1+(-4)}{2}\right) = \left(\frac{1}{2}, -\frac{3}{2}\right).$$

**(b)** $A(-8, 3), B(2, -1)$

$$\text{Midpoint} = \left(\frac{-8+2}{2}, \frac{3+(-1)}{2}\right) = \left(\frac{-6}{2}, \frac{2}{2}\right) = (-3, 1).$$

**(c)** $A\left(-\sqrt{5}, -\frac{1}{3}\right), B\left(-3\sqrt{5}, 5\right)$

$$\text{Midpoint} = \left(\frac{-\sqrt{5} + (-3\sqrt{5})}{2}, \frac{-\frac{1}{3} + 5}{2}\right)$$

Simplify each term:

$$x\text{-coordinate} = \frac{-\sqrt{5} - 3\sqrt{5}}{2} = \frac{-4\sqrt{5}}{2} = -2\sqrt{5},$$

$$y\text{-coordinate} = \frac{-\frac{1}{3} + 5}{2} = \frac{-\frac{1}{3} + \frac{15}{3}}{2} = \frac{\frac{14}{3}}{2} = \frac{14}{6} = \frac{7}{3}.$$

So, the midpoint is:

$$\left(-2\sqrt{5}, \frac{7}{3}\right).$$

**4. Which of the following points are at a distance of 15 units from the origin?**

**(i)** $\left(\sqrt{176}, 7\right)$ **(ii)** $(10, -10)$

**(iii)** $(1, 15)$

***Solution:***

(i) $(\sqrt{176}, 7)$:

Substitute $x = \sqrt{176}$ and $y = 7$:

$$\text{Distance} = \sqrt{(\sqrt{176})^2 + 7^2} = \sqrt{176 + 49} = \sqrt{225} = 15 \text{ unit}$$

This point is at a distance of 15 units from the origin.

(ii) $(10, -10)$:

Substitute $x = 10$ and $y = -10$:

$$\text{Distance} = \sqrt{10^2 + (-10)^2} = \sqrt{100 + 100} = \sqrt{200} = 10\sqrt{2} \text{ unit}$$

Since $10\sqrt{2} \neq 15$, this point is not at a distance of 15 units from the origin.

(iii) $(1, 15)$:

Substitute $x = 1$ and $y = 15$:

$$\text{Distance} = \sqrt{1^2 + 15^2} = \sqrt{1 + 225} = \sqrt{226}.$$

Since $\sqrt{226} \neq 15$, this point is not at a distance of 15 units from the origin.

**5. Show that:**

**(i) the points $A(0, 2)$, $B(\sqrt{3}, 1)$ and $C(0, -2)$ are vertices of a right triangle.**

**(ii) the points $A(3, 1)$, $B(-2, -3)$ and $C(2, 2)$ are vertices of an isosceles triangle.**

**(iii) the points $A(5, 2)$, $B(-2, 3)$, $C(-3, -4)$ and $D(4, -5)$ are vertices of a parallelogram.**

***Solution:***

**(i) Show that A(0, 2), B($\sqrt{3}$, 1), and C(0, -2) form a right triangle**

- **Distance $AB$:**

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(\sqrt{3} - 0)^2 + (1 - 2)^2}$$
$$= \sqrt{3 + 1} = \sqrt{4} = 2.$$

**Distance $BC$:**

$$BC = \sqrt{(\sqrt{3} - 0)^2 + (1 - (-2))^2} = \sqrt{3 + 9} = \sqrt{12} = 2\sqrt{3}.$$

- **Distance $AC$:**

$$AC = \sqrt{(0 - 0)^2 + ((-2) - 2)^2} = \sqrt{0 + 16} = \sqrt{16} = 4.$$

Check the Pythagorean theorem:

$$AB^2 + BC^2 = AC^2.$$

Substitute the values:

$$(2)^2 + (2\sqrt{3})^2 = (4)^2.$$
$$4 + 12 = 16.$$
$$16 = 16.$$

Since the Pythagorean theorem holds, the points $A$, $B$, and $C$ form a right triangle.

**(ii) Show that A(3, 1), B(-2, -3), and C(2, 2) form an isosceles triangle**

To check if the triangle is isosceles, we calculate the distances between all pairs of points:

- **Distance $AB$:**

$$AB = \sqrt{((-2) - 3)^2 + ((-3) - 1)^2} = \sqrt{(-5)^2 + (-4)^2}$$
$$= \sqrt{25 + 16} = \sqrt{41}.$$

**Distance $BC$:**

$$BC = \sqrt{(2 - (-2))^2 + (2 - (-3))^2} = \sqrt{(4)^2 + (5)^2}$$
$$= \sqrt{16 + 25} = \sqrt{41}.$$

- **Distance $AC$:**

$$AC = \sqrt{(2 - 3)^2 + (2 - 1)^2} = \sqrt{(-1)^2 + (1)^2} = \sqrt{1 + 1}$$
$$= \sqrt{2}.$$

Since $AB = BC = \sqrt{41}$, two sides of the triangle are equal, and the triangle is isosceles.

**(iii) Show that A(5, 2), B(-2, 3), C(-3, -4), and D(4, -5) form a parallelogram**

To verify that the points form a parallelogram, we check if the opposite sides are equal in length. Calculate the distances:

- **Distance $AB$:**

$$AB = \sqrt{\left((-2)-5\right)^2 + (3-2)^2} = \sqrt{(-7)^2 + (1)^2} = \sqrt{49+1}$$
$$= \sqrt{50}.$$

**Distance $CD$:**

$$CD = \sqrt{\left(4-(-3)\right)^2 + \left((-5)-(-4)\right)^2} = \sqrt{(7)^2 + (-1)^2}$$
$$= \sqrt{49+1} = \sqrt{50}.$$

- **Distance $BC$:**

$$BC = \sqrt{\left((-3)-(-2)\right)^2 + \left((-4)-3\right)^2} = \sqrt{(-1)^2 + (-7)^2}$$
$$= \sqrt{1+49} = \sqrt{50}.$$

- **Distance $AD$:**

$$AD = \sqrt{(4-5)^2 + \left((-5)-2\right)^2} = \sqrt{(-1)^2 + (-7)^2}$$
$$= \sqrt{1+49} = \sqrt{50}.$$

Since $AB = CD$ and $BC = AD$, the opposite sides are equal in length. Thus, the points $A$, $B$, $C$, and $D$ form a parallelogram.

6. **Find $h$ such that the points $A(\sqrt{3}, -1)$, $B(0, 2)$ and $C(h, -2)$ are vertices of a right triangle with right angle at the vertex $A$.**

***Solution:*** To find $h$ such that the points $A(\sqrt{3}, -1)$, $B(0,2)$, and $C(h, -2)$ form a right triangle with a right angle at $A$, we need to verify that the dot product of the vectors $\vec{AB}$ and $\vec{AC}$ is zero. This condition ensures the angle at $A$ is 90°.

Step 1: Vectors $\vec{AB}$ and $\vec{AC}$

- **Vector $\vec{AB}$:**

$$\vec{AB} = (x_B - x_A, y_B - y_A) = \left(0-\sqrt{3}, 2-(-1)\right) = (-\sqrt{3}, 3).$$

2. **Vector $\vec{AC}$:**

$$\vec{AC} = (x_C - x_A, y_C - y_A) = \left(h-\sqrt{3}, -2-(-1)\right)$$
$$= (h-\sqrt{3}, -1).$$

Step 2: Dot Product of $\vec{AB}$ and $\vec{AC}$

The dot product of $\vec{AB}$ and $\vec{AC}$ is:

$$\vec{AB} \cdot \vec{AC} = (x_1 \cdot x_2) + (y_1 \cdot y_2).$$

Substitute the components:

$$\vec{AB} \cdot \vec{AC} = (-\sqrt{3})(h-\sqrt{3}) + (3)(-1).$$

Simplify: $\vec{AB} \cdot \vec{AC} = -\sqrt{3}h + 3 + (-3).$

$$\vec{AB} \cdot \vec{AC} = -\sqrt{3}h$$

Step 3: Set the Dot Product to Zero

For $A$ to be the right angle:

$$-\sqrt{3}h = 0.$$

Solve for $h$: $-\sqrt{3}h = 0.$

$$h = 0.$$

**7. Find $h$ such that $A(-1, h)$, $B(3, 2)$ and $C(7, 3)$ are collinear.**

**Sol.** To determine the value of $h$ such that the points $A(-1, h)$, $B(3,2)$, and $C(7,3)$ are collinear, we use the condition for collinearity: the area of the triangle formed by the points must be zero.

**Formula for the Area of a Triangle:**

The area of a triangle formed by three points $A(x_1, y_1)$, $B(x_2, y_2)$, and $C(x_3, y_3)$ is:

$$\text{Area} = \frac{1}{2} \mid x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \mid$$

If the points are collinear, the area is 0.

**Substituting the Coordinates:**

Substitute $A(-1, h)$, $B(3,2)$, and $C(7,3)$ into the formula:

$$\text{Area} = \frac{1}{2} \mid -1(2-3) + 3(3-h) + 7(h-2) \mid = 0$$

Simplify the terms:

$$\text{Area} = \frac{1}{2} \mid -1(-1) + 3(3-h) + 7(h-2) \mid = 0$$

$$\text{Area} = \frac{1}{2} \mid 1 + 9 - 3h + 7h - 14 \mid = 0$$

$$\text{Area} = \frac{1}{2} \mid -4 + 4h \mid = 0$$

$$\text{Area} = \frac{1}{2} \mid 4h - 4 \mid = 0$$

**Solving for $h$:**

$$\mid 4h - 4 \mid = 0$$

$$4h - 4 = 0$$

$$4h = 4$$

$$h = 1$$

**Final Answer:**

The value of $h$ is:



**8. The points $A(-5, -2)$ and $B(5, -4)$ are ends of a diameter of a circle. Find the centre and radius of the circle.**

**Sol.** To find the center and radius of the circle where $A(-5, -2)$ and $B(5, -4)$ are the endpoints of the diameter, we proceed as follows:

**Step 1: Centre of the Circle**

The center of the circle is the midpoint of the diameter. Using the **midpoint formula**, the midpoint of the line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$ is:

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Substitute $A(-5, -2)$ and $B(5, -4)$:

$$\text{Centre} = \left(\frac{-5+5}{2}, \frac{-2+(-4)}{2}\right)$$

$$\text{Centre} = \left(\frac{0}{2}, \frac{-6}{2}\right)$$

$$\text{Centre} = (0, -3)$$

**Step 2: Radius of the Circle**

The radius is half the length of the diameter. The length of the diameter is the distance between $A$ and $B$, calculated using the **distance formula**: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Substitute $A(-5, -2)$ and $B(5, -4)$:

$$d = \sqrt{(5 - (-5))^2 + (-4 - (-2))^2}$$

$$d = \sqrt{(5+5)^2 + (-4+2)^2}$$

$$d = \sqrt{10^2 + (-2)^2}$$

$$d = \sqrt{100 + 4} = \sqrt{104} = 2\sqrt{26}$$

The radius is half of the diameter:

$$\text{Radius} = \frac{d}{2} = \frac{\sqrt{104}}{2} = \sqrt{26}$$

**Final Answer:** • **Centre: $(0, -3)$** • **Radius: $\sqrt{26}$**

9. **Find $h$ such that the points $A(h,1)$, $B(2,7)$ and $C(-6,-7)$ are vertices of a right triangle with right angle at the vertex $A$.**

**Sol.** **Step 1: Slopes of the sides**

SLOPE OF $AB$:

$$m_{AB} = \frac{y_B - y_A}{x_B - x_A} = \frac{7-1}{2-h} = \frac{6}{2-h}$$

SLOPE OF $AC$:

$$m_{AC} = \frac{y_C - y_A}{x_C - x_A} = \frac{-7-1}{-6-h} = \frac{-8}{-6-h} = \frac{8}{6+h}$$

**Step 2: Perpendicularity condition**

For $AB \perp AC$, the product of their slopes must satisfy:

$$m_{AB} \cdot m_{AC} = -1$$

Substitute the slopes:

$$\frac{6}{2-h} \cdot \frac{8}{6+h} = -1$$

**Step 3: Solve for $h$**

Simplify the equation:

$$\frac{6 \cdot 8}{(2-h)(6+h)} = -1$$

$$\frac{48}{(2-h)(6+h)} = -1$$

Multiply through by $(2-h)(6+h)$ (assuming $h \neq 2$ and $h \neq -6$):

$$48 = -(2-h)(6+h)$$

Expand the right-hand side:

$$48 = h^2 + 4h - 12$$

$$h^2 + 4h - 60 = 0$$

$$(h+10)(h-6) = 0$$

$$h = -10 \text{ or } h = 6$$

10. **A quadrilateral has the points $A(9,3)$, $B(-7,7)$, $C(-3,-7)$ and $D(5,-5)$ as its vertices. Find the midpoints of its sides. Show that the figure formed by joining the midpoints consecutively is a parallelogram.**

**Sol.** Let's solve it step by step without using code.

Step 1: Calculate Midpoints of Sides

The formula for the midpoint of a line segment joining two points $(x_1, y_1)$ and $(x_2, y_2)$ is:

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

- **Midpoint of $AB$:**

$$M_1 = \left(\frac{9+(-7)}{2}, \frac{3+7}{2}\right) = \left(\frac{2}{2}, \frac{10}{2}\right) = (1,5)$$

3. **Midpoint of $BC$:**

$$M_2 = \left(\frac{-7+(-3)}{2}, \frac{7+(-7)}{2}\right) = \left(\frac{-10}{2}, \frac{0}{2}\right) = (-5,0)$$

- **Midpoint of $CD$:**

$$M_3 = \left(\frac{-3+5}{2}, \frac{-7+(-5)}{2}\right) = \left(\frac{2}{2}, \frac{-12}{2}\right) = (1,-6)$$

- **Midpoint of $DA$:**

$$M_4 = \left(\frac{5+9}{2}, \frac{-5+3}{2}\right) = \left(\frac{14}{2}, \frac{-2}{2}\right) = (7,-1)$$

Step 2: Verify Opposite Sides Are Equal and Parallel

To confirm the figure is a parallelogram, we need to check:

- **Opposite sides are equal in length.**

- **Opposite sides are parallel (equal slopes).**

**Distance Between Opposite Sides**

The distance between two points $(x_1, y_1)$ and $(x_2, y_2)$ is:

$$\text{Distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

- **Distance $M_1M_3$:**

$$M_1M_3 = \sqrt{(1-1)^2 + (5-(-6))^2} = \sqrt{0^2 + (11)^2} = 11$$

4. **Distance $M_2M_4$:**

$$M_2M_4 = \sqrt{(-5-7)^2 + (0-(-1))^2} = \sqrt{(-12)^2 + (1)^2}$$
$$= \sqrt{144+1} = \sqrt{145}$$

**Slopes of Opposite Sides**

The slope between two points $(x_1, y_1)$ and $(x_2, y_2)$ is:

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

- **Slope $M_1M_2$:**

$$\text{Slope of } M_1M_2 = \frac{0-5}{-5-1} = \frac{-5}{-6} = \frac{5}{6}$$

5. **Slope $M_3M_4$:**

$$\text{Slope of } M_3M_4 = \frac{-1-(-6)}{7-1} = \frac{5}{6}$$

- **Slope $M_1M_4$:**

$$\text{Slope of } M_1M_4 = \frac{-1-5}{7-1} = \frac{-6}{6} = -1$$

- **Slope $M_2M_3$:**

$$\text{Slope of } M_2M_3 = \frac{-6-0}{1-(-5)} = \frac{-6}{6} = -1$$

Step 3: Conclusion

- Opposite sides ($M_1M_3$ and $M_2M_4$) are **equal in length**.
- Opposite sides ($M_1M_2$ and $M_3M_4$, as well as $M_1M_4$ and $M_2M_3$) have **equal slopes**, confirming they are parallel.

Thus, the figure formed by joining the midpoints is a **parallelogram.**

**Example 5:** Show that the points $A(-3, 6)$, $B(3, 2)$ and $C(6, 0)$ are collinear.

**Solution:** We know that the points $A$, $B$ and $C$ are collinear if the line $AB$ and $BC$ have the same slopes.

Here Slope of $AB = \frac{2-6}{3-(-3)} = \frac{-4}{3+3} = \frac{-4}{6} = \frac{-2}{3}$ and slope of

$BC = \frac{0-2}{6-3} = \frac{-2}{3}$

$\because$ Slope of $AB$ = Slope of $BC$

Thus $A$, $B$ and $C$ are collinear.

**Example 6:** Show that the triangle with vertices $A(1, 1)$, $B(4, 5)$ and $C(12, -1)$ is a right triangle.

**Solution:** Slope of $AB = m_1 = \frac{5-1}{4-1} = \frac{4}{3}$ and Slope of

$BC = m_2 = \frac{-1-5}{12-4} = \frac{-6}{8} = \frac{-3}{4}$

Since $m_1 . m_2 = \left(\frac{4}{3}\right)\left(-\frac{3}{4}\right) = -1$, therefore, $\overline{AB} \perp \overline{BC}$

So $\Delta ABC$ is a right triangle.

**Example 7:** Find an equation of the straight line if

(a) its slope is 2 and $y$-intercept is 5

(b) it is perpendicular to a line with slope -6 and its $y$-intercept is $\frac{4}{3}$

**Solution:** (a) The slope and $y$-intercept of the line are respectively:

$$m = 2 \text{ and } c = 5$$

Thus $y = 2x + 5$ (Slope-intercept form: $y = mx + c$) is the required equation.

(b) The slope of the given line is

$$m_1 = -6$$

The slope of the required line is: $m_2 = -\frac{1}{m_1} = \frac{1}{6}$

The slope and $y$-intercept of the required line are respectively:

$$m_2 = \frac{1}{6} \text{ and } c = \frac{4}{3}$$

Thus, $y = -\frac{1}{6}x + \frac{4}{3}$ or $6y = x + 8$ is the required equation.

**Example 8:** Write down an equation of the straight line passing through (5, 1) and parallel to a line passing through the points (0,–1), (7, –15).

**Solution:** Let $m$ be the slope of the required straight line, then

$$m = \frac{-15-(-1)}{7-0_1} \quad (\because \text{Slopes of parallel lines are equal})$$

$$= -2$$

As the point (5, 1) lies on the required line having slope –2 so, by point-slope form of equation of the straight line, we have

$$y - (1) = -2(x - 5)$$

or $\quad y = -2x + 11$

or $\quad 2x + y - 11 = 0$

is an equation of the required line.

**Example 9:** Find an equation of line through the points (–2,1) and (6,–4).

**Solution:** Using two-points form of the equation of straight line, the required equation is:

$$y - 1 = \frac{-4-1}{6-(-2)}[x - (-2)] \text{ or}$$

$$y - 1 = \frac{-5}{8}(x + 2) \text{ or } 5x + 8y + 2 = 0$$

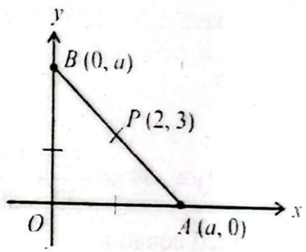
**Example 10:** Write down an equation of the line which cuts the $x$–axis at (2, 0) and $y$–axis at (0, –4).

**Solution:** As 2 and –4 are respectively $x$ and $y$–intercepts of the required line, so by two-intercepts form of equation of a straight line, we have

$$\frac{x}{2} + \frac{y}{-4} = 1 \quad \text{or} \quad 2x - y - 4 = 0$$

Which is the required equation.

**Example 11:** Find an equation of the line through the point $P(2, 3)$ which forms an isosceles triangle with the coordinate axes in the first quadrant.




**Solution:** Let $OAB$ be an isosceles triangle so that the line $AB$ passes through $A(a, 0)$ and $B(0, a)$, where $a$ is some positive real number.

Slope of $AB = \frac{a-0}{0-a} = -1$. But $AB$ passes through $P(2, 3)$.

Equation of the line through $P(2, 3)$ with slope –1 is

$$y - 3 = -1(x - 2) \quad \text{or} \quad x + y - 5 = 0$$

**Example 12:** The length of perpendicular from the origin to a line is 5 units and the inclination of this perpendicular is 120°. Find the slope and $y$-intercept of the line.

**Solution:** Here $p = 5$, $\alpha = 120°$.

Equation of the line in normal form is

$$x\cos 120° + y \sin 120° = 5$$

$$\Rightarrow \quad -\frac{1}{2}x + \frac{\sqrt{3}}{2}y = 5$$

$$\Rightarrow \quad x - \sqrt{3}y + 10 = 0 \quad \ldots(i)$$

To find the slope of the line, we re-write (i) as: $y = \frac{x}{\sqrt{3}} + \frac{10}{\sqrt{3}}$

which is slope-intercept form of the equation.

Here $\quad m = \frac{1}{\sqrt{3}}$ and $c = \frac{10}{\sqrt{3}}$

**Example 13:** Transform the equation $5x - 12y + 39 = 0$ into

(i) Slope intercept form (ii) Two-intercept form

(iii) Normal form (iv) Point-slope form

(v) Two-point form (vi) Symmetric form

**Solution:**

(i) We have $12y = 5x + 39$ or $y = \frac{5}{12}x + \frac{39}{12}, m = \frac{5}{12}$, y-intercept $c = \frac{39}{12}$.

(ii) $5x - 12y = -39$ or $\frac{5x}{-39} + \frac{12y}{39} = 1$ *or* $\frac{x}{-39/5} + \frac{y}{39/12} = 1$ is the required equation.

(iii) $5x - 12y = -39$. Divide both sides by $\pm\sqrt{5^2 + 12^2} = \pm 13$. Since R.H.S is to be positive, we have to take negative sign.

Hence $\frac{5x}{-13} + \frac{12y}{13} = 3$ is the normal form of the equation.

(iv) A point on the line is $\left(\frac{-39}{5}, 0\right)$ and its slope is $\frac{5}{12}$.

Equation can be written as: $y - 0 = \frac{5}{12}\left(x + \frac{39}{5}\right)$

(v) Another point on the line is $\left(0, \frac{39}{12}\right)$. Line through $\left(\frac{-39}{5}, 0\right)$ and $\left(0, \frac{39}{12}\right)$ is $\frac{y - 0}{0 - \frac{39}{12}} = \frac{x + \frac{39}{5}}{\frac{-39}{5} - 0}$

(vi) We have $\tan\alpha = \frac{5}{12} = m$, so $\sin\alpha = \frac{5}{13}$, $\cos\alpha = \frac{12}{13}$. A point of the line is $\left(\frac{-39}{5}, 0\right)$

Equation of the line in symmetric form is

$$\frac{x + \frac{39}{5}}{\frac{12}{13}} = \frac{y - 0}{\frac{5}{13}} = r\,(\text{say})$$

## EXERCISE 7.2

**1. Find the slope and inclination of the line joining the points:**

**(i) (−2, 4) ; (5, 11) (ii) (3, −2) ; (2, 7)**

**(iii) (4, 6) ; (4, 8)**

***Solution:***

(i) **Points $(-2, 4)$ and $(5, 11)$**

FIND THE SLOPE:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{11 - 4}{5 - (-2)} = \frac{7}{7} = 1$$

FIND THE INCLINATION:

$$\theta = \tan^{-1}(m) = \tan^{-1}(1) = 45°$$

**Sketch of the Lines:**

- This is a line with a slope of 1, which means it rises 1 unit for every 1 unit it moves to the right. This line will have an inclination of 45°, making it a diagonal line at a 45° angle to the x-axis.

(ii) **Points $(3, -2)$ and $(2, 7)$**

FIND THE SLOPE:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - (-2)}{2 - 3} = \frac{9}{-1} = -9$$

FIND THE INCLINATION:

$$\theta = \tan^{-1}(m) = \tan^{-1}(-9)$$

Calculating the angle using a calculator:

$$\theta \approx -83.6^\circ$$

Since inclination is measured from the positive x-axis, we add 180° (to convert to a positive angle):

$$\theta \approx 96.4^\circ$$

**Sketch of lines:**

This is a line with a negative slope $(-9)$, which means it falls steeply as we move to the right. The inclination is 96.4°, so it is slightly more than vertical.

**(iii) Points $(4,6)$ and $(4,8)$**

FIND THE SLOPE:

Since the x-coordinates are the same $(x_1 = x_2 = 4)$, the line is vertical. The slope of a vertical line is undefined.

FIND THE INCLINATION:

The inclination of a vertical line is always 90°.

**Sketch of lines:**

This is a vertical line since the x-coordinates are the same. Its inclination is 90°.

**2. By means of slopes, show that the following points lie on the same line:**

(i) $A(-1,-3)$ ; $B(1,5)$ ; $C(2,9)$

(ii) $P(4,-5)$ ; $Q(7,5)$ ; $R(10,15)$

(iii) $L(-4,6)$ ; $M(3,8)$ ; $N(10,10)$

(iv) $X(a,2b)$; $Y(c,a+b)$; $Z(2c-a,2a)$

**Sol. (i) Points: $A(-1,-3), B(1,5), C(2,9)$**

*Slope of AB:*

$$\text{Slope of } AB = \frac{5-(-3)}{1-(-1)} = \frac{8}{2} = 4$$

*Slope of BC:*

$$\text{Slope of } BC = \frac{9-5}{2-1} = \frac{4}{1} = 4$$

Since Slope of $AB$ = Slope of $BC = 4$, the points are collinear.

**(ii) Points: $P(4,-5), Q(7,5), R(10,15)$**

*Slope of PQ:*

$$\text{Slope of } PQ = \frac{5-(-5)}{7-4} = \frac{10}{3}$$

*Slope of QR:*

$$\text{Slope of } QR = \frac{15-5}{10-7} = \frac{10}{3}$$

Since Slope of $PQ$ = Slope of $QR = \frac{10}{3}$, the points are collinear.

**(iii) Points: $L(-4,6), M(3,8), N(10,10)$**

*Slope of LM:*

$$\text{Slope of } LM = \frac{8-6}{3-(-4)} = \frac{2}{7}$$

*Slope of MN:*

$$\text{Slope of } MN = \frac{10-8}{10-3} = \frac{2}{7}$$

Since Slope of $LM$ = Slope of $MN = \frac{2}{7}$, the points are collinear.

**(iv) Points: $X(a,2b), Y(c,a+b), Z(2c-a,2a)$**

*Slope of XY:*

$$\text{Slope of } XY = \frac{(a+b)-2b}{c-a} = \frac{a-b}{c-a}$$

*Slope of YZ:*

$$\text{Slope of } YZ = \frac{2a-(a+b)}{(2c-a)-c} = \frac{a-b}{c-a}$$

Since Slope of $XY$ = Slope of $YZ = \frac{a-b}{c-a}$, the points are collinear.

**3. Find $k$ so that the line joining $A(7, 3)$; $B(k, -6)$ and the line joining $C(-4, 5)$ ; $D(-6, 4)$ are:**

**(i) parallel (ii) perpendicular.**

**Sol.** To find $k$ such that the line joining points $A(7,3)$ and $B(k,-6)$ is parallel or perpendicular to the line joining points $C(-4,5)$ and $D(-6,4)$, we will use the concepts of **slope**.

**Step 1: Find the slope of the line joining points $C(-4,5)$ and $D(-6,4)$**

The slope $m_{CD}$ of the line joining points $C(x_1, y_1)$ and $D(x_2, y_2)$ is given by:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Substitute the coordinates of $C(-4,5)$ and $D(-6,4)$:

$$m_{CD} = \frac{4-5}{-6-(-4)} = \frac{-1}{-2} = \frac{1}{2}$$

**(i) To make the lines parallel**

For two lines to be **parallel**, their slopes must be equal. The slope of the line joining $A(7,3)$ and $B(k,-6)$ is:

$$m_{AB} = \frac{-6-3}{k-7} = \frac{-9}{k-7}$$

For the lines to be parallel, we set $m_{AB} = m_{CD}$:

$$\frac{-9}{k-7} = \frac{1}{2}$$

Now, solve for $k$:

$$-9 \times 2 = 1 \times (k-7)$$

$$-18 = k - 7$$

$$k = -18 + 7 = -11$$

So, for the lines to be parallel, $k = -11$.

**(ii) To make the lines perpendicular**

For two lines to be **perpendicular**, the product of their slopes must be $-1$. Therefore, we require:

$$m_{AB} \times m_{CD} = -1$$

Substitute $m_{CD} = \frac{1}{2}$ and $m_{AB} = \frac{-9}{k-7}$.

$$\frac{-9}{k-7} \times \frac{1}{2} = -1$$

Now, solve for $k$:

$$\frac{-9}{2(k-7)} = -1$$

Multiply both sides by $2(k-7)$ to eliminate the denominator:

$$-9 = -2(k-7)$$

Simplify:

$$-9 = -2k + 14$$

Solve for $k$:

$$-9 - 14 = -2k$$

$$-23 = -2k$$

$$k = \frac{23}{2}$$

So, for the lines to be perpendicular, $k = \frac{23}{2}$.

**4. Using slopes, show that the triangle with its vertices $A(6, 1)$, $B(2, 7)$ and $C(-6, -7)$ is a right triangle.**

**Sol.** To show that the triangle with vertices $A(6,1)$, $B(2,7)$, and $C(-6,-7)$ is a right triangle using slopes, we need to check if two of the sides of the triangle are **perpendicular**. Two lines are perpendicular if the product of their slopes is equal to $-1$.

**Step 1:** **Find the slopes of the sides of the triangle.**

SLOPE OF SIDE $AB$ (BETWEEN POINTS $A(6,1)$ AND $B(2,7)$):

The formula for the slope of a line joining two points $(x_1, y_1)$ and $(x_2, y_2)$ is:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

For points $A(6,1)$ and $B(2,7)$:

$$m_{AB} = \frac{7-1}{2-6} = \frac{6}{-4} = -\frac{3}{2}$$

SLOPE OF SIDE $BC$ (BETWEEN POINTS $B(2,7)$ AND $C(-6,-7)$):

For points $B(2,7)$ and $C(-6,-7)$:

$$m_{BC} = \frac{-7-7}{-6-2} = \frac{-14}{-8} = \frac{7}{4}$$

SLOPE OF SIDE $AC$ (BETWEEN POINTS $A(6,1)$ AND $C(-6,-7)$):

For points $A(6,1)$ and $C(-6,-7)$:

$$m_{AC} = \frac{-7-1}{-6-6} = \frac{-8}{-12} = \frac{2}{3}$$

**Step 2:** **Check if any two sides are perpendicular.**

Two lines are perpendicular if the product of their slopes is $-1$.

We check the slopes of different pairs of sides:

CHECKING $AB$ AND $BC$:

$$m_{AB} \times m_{BC} = \left(-\frac{3}{2}\right) \times \frac{7}{4} = -\frac{21}{8}$$

Since this is not equal to $-1$, the lines $AB$ and $BC$ are not perpendicular.

CHECKING $AB$ AND $AC$:

$$m_{AB} \times m_{AC} = \left(-\frac{3}{2}\right) \times \frac{2}{3} = -1$$

Since this is equal to $-1$, the lines $AB$ and $AC$ are **perpendicular**.

**Conclusion:**

Since the lines $AB$ and $AC$ are perpendicular, the triangle with vertices $A(6,1)$, $B(2,7)$, and $C(-6,-7)$ is a right triangle, with the right angle at vertex $A$.

**5.** **Two pairs of points are given. Find whether the two lines determined by these points are:**

**(i)** **parallel** **(ii)** **perpendicular**

**(iii)** **none.**

**(a)** **(1, –2), (2, 4) and (4, 1), (–8, 2)**

**(b)** **(–3, 4 ), (6, 2) and (4, 5), (–2, –7)**

**Sol.** **(a) Points: $(1,-2),(2,4)$ and $(4,1),(-8,2)$**

SLOPE OF LINE 1 (BETWEEN $(1,-2)$ AND $(2,4)$):

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-(-2)}{2-1} = \frac{4+2}{1} = \frac{6}{1} = 6$$

SLOPE OF LINE 2 (BETWEEN $(4,1)$ AND $(-8,2)$):

$$m_2 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-1}{-8-4} = \frac{1}{-12} = -\frac{1}{12}$$

CHECK FOR PARALLELISM:

For the lines to be parallel, their slopes must be equal. Since $m_1 = 6$ and $m_2 = -\frac{1}{12}$, the lines are **not parallel**.

CHECK FOR PERPENDICULARITY:

For the lines to be perpendicular, the product of their slopes must be $-1$:

$$m_1 \times m_2 = 6 \times -\frac{1}{12} = -\frac{6}{12} = -\frac{1}{2}$$

Since the product is not $-1$, the lines are **not perpendicular**.

**Conclusion for (a): The lines are neither parallel nor perpendicular.**

**(b) Points: $(-3,4),(6,2)$ and $(4,5),(-2,-7)$**

SLOPE OF LINE 1 (BETWEEN $(-3,4)$ AND $(6,2)$):

$$m_1 = \frac{2-4}{6-(-3)} = \frac{-2}{6+3} = \frac{-2}{9}$$

SLOPE OF LINE 2 (BETWEEN (4,5) AND (−2,−7)):

$$m_2 = \frac{-7-5}{-2-4} = \frac{-12}{-6} = 2$$

CHECK FOR PARALLELISM:

For the lines to be parallel, their slopes must be equal. Since $m_1 = \frac{-2}{9}$ and $m_2 = 2$, the lines are **not parallel**.

CHECK FOR PERPENDICULARITY:

For the lines to be perpendicular, the product of their slopes must be $-1$:

$$m_1 \times m_2 = \frac{-2}{9} \times 2 = \frac{-4}{9}$$

Since the product is not $-1$, the lines are **not perpendicular.**

**Conclusion for (b): The lines are neither parallel nor perpendicular.**

**Final Answer:**

- For (a): The lines are **neither parallel nor perpendicular.**
- For (b): The lines are **neither parallel nor perpendicular.**

6. **Find an equation of:**
   (a) **the horizontal line through (7,–9)**
   (b) **the vertical line through (–5, 3)**
   (c) **through A(–6, 5) having slope 7**
   (d) **through (8, –3) having slope 0**
   (e) **through (–8, 5) having slope undefined**
   (f) **through (–5, –3) and (9, –1)**
   (g) ***y*-intercept: –7 and slope: –5**
   (h) ***x*-intercept: –3 and *y*-intercept: 4**
   (i) ***x*-intercept: –9 and slope: –4**

**Sol.** To find the equations of lines, we use the general forms:

- **Horizontal line**: $y = c$, where $c$ is the $y$-coordinate.
- **Vertical line**: $x = c$, where $c$ is the $x$-coordinate.
- **Line with slope $m$ through point $(x_1, y_1)$**: Use the **point-slope form**:

$$y - y_1 = m(x - x_1)$$

- **Line through two points $(x_1, y_1)$ and $(x_2, y_2)$**: First calculate the slope $m = \frac{y_2-y_1}{x_2-x_1}$, then use the point-slope form.

(a) **Horizontal line through (7, −9):**

A horizontal line has constant $y$-value:

$$y = -9$$

or $y + 9 = 0$

(b) **Vertical line through (−5, 3):**

A vertical line has constant $x$-value:

$$x = -5$$

or $x + 5 = 0$

(c) **Through $A(-6, 5)$ having slope 7:**

Using point-slope form:

$$y - 5 = 7(x + 6)$$

Simplify:

$$y = 7x + 42 + 5 \Rightarrow y = 7x + 47$$

or $7x - y + 47 = 0$

(d) **Through (8, −3) having slope 0:**

A slope of 0 indicates a horizontal line:

$$y = -3$$

$$\text{or } y + 3 = 0$$

**(e) Through $(-8, 5)$ having undefined slope:**

An undefined slope indicates a vertical line:

$$x = -8$$

$$\text{or } x + 8 = 0$$

**(f) Through $(-5, -3)$ and $(9, -1)$:**

First, find the slope:

$$m = \frac{-1-(-3)}{9-(-5)} = \frac{-1+3}{9+5} = \frac{2}{14} = \frac{1}{7}$$

Now, use the point-slope form with point $(-5, -3)$:

$$y - (-3) = \frac{1}{7}(x - (-5))$$

Simplify:

$$y + 3 = \frac{1}{7}(x + 5)$$

$$= \frac{1}{7}x + \frac{5}{7} - 3 \Rightarrow y = \frac{1}{7}x - \frac{16}{7}$$

$$\text{or } x - 7y - 16 = 0$$

**(g) $y$-intercept: $-7$, slope: $-5$:**

Use the slope-intercept form $y = mx + b$:

$$y = -5x - 7$$

$$\text{or } 5x + y + 7 = 0$$

**(h) $x$-intercept: $-3$, $y$-intercept: $4$:**

Using the two intercepts, we can find the slope:

$$m = \frac{4-0}{0-(-3)} = \frac{4}{3}$$

Using slope-intercept form:

$$y = \frac{4}{3}x + 4$$

$$\text{or } 4x - 3y + 12 = 0$$

**(i) $x$-intercept: $-9$, slope: $-4$:**

At the $x$-intercept, $y = 0$, so the point is $(-9, 0)$. Use the point-slope form:

$$y - 0 = -4(x + 9)$$

Simplify: $\quad y = -4x - 36$

$$\text{or } 4x + y + 36 = 0$$

**7. Find an equation of the perpendicular bisector of the segment joining the points $A(3, 5)$ and $B(9, 8)$.**

**Sol.** To find the equation of the perpendicular bisector of the segment joining points $A(3,5)$ and $B(9,8)$, follow these steps:

**Step 1: Find the midpoint of the segment $AB$**

The **midpoint** $M$ of the segment joining two points $(x_1, y_1)$ and $(x_2, y_2)$ is given by the formula:

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Substitute $A(3,5)$ and $B(9,8)$ into the formula:

$$M = \left(\frac{3+9}{2}, \frac{5+8}{2}\right) = \left(\frac{12}{2}, \frac{13}{2}\right) = (6, 6.5)$$

**Step 2: Find the slope of the line $AB$**

The slope $m_{AB}$ of the line joining $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by:

$$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$$

Substitute the coordinates of $A(3,5)$ and $B(9,8)$:

$$m_{AB} = \frac{8-5}{9-3} = \frac{3}{6} = \frac{1}{2}$$

**Step 3: Find the slope of the perpendicular bisector**

The **slope** of the perpendicular bisector is the negative reciprocal of the slope of $AB$. Since the slope of $AB$ is $\frac{1}{2}$, the slope of the perpendicular bisector is:

$$m_{\text{perp}} = -\frac{1}{\frac{1}{2}} = -2$$

**Step 4: Use the point-slope form to find the equation of the perpendicular bisector**

The **point-slope form** of the equation of a line is:

$$y - y_1 = m(x - x_1)$$

Where $m$ is the slope, and $(x_1, y_1)$ is the point through which the line passes.

Substitute $m = -2$ and the midpoint $M(6, 6.5)$ into the point-slope form:

$$y - 6.5 = -2(x - 6)$$

Simplify the equation:

$$y - 6.5 = -2x + 12$$
$$y = -2x + 12 + 6.5$$
$$y = -2x + 18.5$$

**Final Answer:**

The equation of the perpendicular bisector is:

$$y = -2x + 18.5$$

or $4x + 2y - 37 = 0$

**8. Find an equation of the line through $(-4, -6)$ and perpendicular to a line having slope $\frac{-3}{2}$.**

**Sol.** To find the equation of the line through $(-4, -6)$ and perpendicular to a line with slope $m = \frac{-3}{2}$, follow these steps:

**Step 1: Slope of the perpendicular line**

The slope of a line perpendicular to a given line is the negative reciprocal of the original slope.

$$m_{\text{perpendicular}} = -\frac{1}{m} = -\frac{1}{\frac{-3}{2}} = \frac{2}{3}$$

**Step 2: Equation of the line**

Use the **point-slope form** of a line:

$$y - y_1 = m(x - x_1)$$

Here, $(x_1, y_1) = (-4, -6)$ and $m = \frac{2}{3}$.

Substitute these values:

$$y - (-6) = \frac{2}{3}(x - (-4))$$

$$y + 6 = \frac{2}{3}(x + 4)$$

**Step 3: Simplify the equation**

Distribute $\frac{2}{3}$:

$$y + 6 = \frac{2}{3}x + \frac{8}{3}$$

Subtract 6 from both sides: $y = \frac{2}{3}x + \frac{8}{3} - 6$

Express 6 as a fraction with a denominator of 3:

$$y = \frac{2}{3}x + \frac{8}{3} - \frac{18}{3}$$

Combine the constants:

$$y = \frac{2}{3}x - \frac{10}{3}$$

$$\text{or } 2x - 3y - 10 = 0$$

**9. Find an equation of the line through (11, -5) and parallel to a line with slope -24.**

**Sol.** To find the equation of the line through the point $(11, -5)$ and parallel to a line with slope $-24$, follow these steps:

**Step 1: Find the slope of the parallel line**

Since the lines are parallel, they have the same slope. Therefore, the slope of the new line is the same as the given slope, which is $-24$.

**Step 2: Use the point-slope form of the equation**

The **point-slope form** of the equation of a line is:

$$y - y_1 = m(x - x_1)$$

where $m$ is the slope and $(x_1, y_1)$ is a point on the line.

Substitute $m = -24$ and the point $(11, -5)$ into the point-slope form:

$$y - (-5) = -24(x - 11)$$

Simplify:

$$y + 5 = -24(x - 11)$$

Now, expand the right side:

$$y + 5 = -24x + 264$$

Subtract 5 from both sides to solve for $y$:

$$y = -24x + 264 - 5$$

$$y = -24x + 259$$

**Final Answer:**

The equation of the line through $(11, -5)$ and parallel to a line with slope $-24$ is:

$$y = -24x + 259$$

$$\text{or } 24x + y - 259 = 0$$

**10. Convert each of the following equations into slope intercept form, two intercept form and normal form:**

**(a)** $2x - 4y + 11 = 0$ **(b)** $4x + 7y - 2 = 0$

**(c)** $15y - 8x + 3 = 0$

**Sol.** To convert the given equations into **slope-intercept form**, **two-intercept form**, and **normal form**, follow these steps for each equation:

1. Forms of the Equation:

- **Slope-Intercept Form:**

$$y = mx + c$$

Rearrange to solve for $y$.

- **Two-Intercept Form:**

$$\frac{x}{a} + \frac{y}{b} = 1$$

Divide by the constant to express the intercepts.

- **Normal Form:**

$$x\cos\theta + y\sin\theta = p$$

Normalize the equation so that the coefficient of $x^2 + y^2$ is 1.

**(a) Equation: $2x - 4y + 11 = 0$**

(i) Slope-Intercept Form:

$$2x - 4y + 11 = 0 \Rightarrow -4y = -2x - 11 \Rightarrow y = \frac{1}{2}x + \frac{11}{4}$$

(ii) **Two-Intercept Form:**

Divide the equation by 11 to express it in intercept form:

$$\frac{2x}{11}-\frac{4y}{11}=1 \Rightarrow \frac{x}{\frac{11}{2}}-\frac{y}{\frac{11}{4}}=1$$

Rewriting:

$$\frac{x}{\frac{-11}{2}}+\frac{y}{\frac{11}{4}}=1$$

**(iii) Normal Form:**

Rewrite the equation in standard form $Ax+By+C=0$, where $A, B,$ and $C$ are divided by $\sqrt{A^2+B^2}$:

$$\sqrt{2^2+(-4)^2}=\sqrt{4+16}=\sqrt{20}=2\sqrt{5}$$

Normalize:

$$\frac{2x-4y+11}{2\sqrt{5}}=0 \Rightarrow \frac{x}{\sqrt{5}}-\frac{2y}{\sqrt{5}}+\frac{11}{2\sqrt{5}}=0$$

$$x\cos\alpha=\frac{-2}{2\sqrt{5}}<0 \text{ and } \sin\alpha=\frac{4}{2\sqrt{5}}>0$$

$$x\cos(116.57^\circ)+y\sin(116.57^\circ)=\frac{11}{2\sqrt{5}}$$

**(b) Equation: $4x+7y-2=0$**

(i) Slope-Intercept Form:

$$4x+7y-2=0 \Rightarrow 7y=-4x+2 \Rightarrow y=-\frac{4}{7}x+\frac{2}{7}$$

(ii) Two-Intercept Form:

Divide by 2 to express intercepts:

$$\frac{4x}{2}+\frac{7y}{2}=1 \Rightarrow \frac{x}{\frac{1}{2}}+\frac{y}{\frac{2}{7}}=1$$

(iii) Normal Form:

$$\sqrt{4^2+7^2}=\sqrt{16+49}=\sqrt{65}$$

Normalize:

$$\frac{4x+7y-2}{\sqrt{65}}=0 \Rightarrow \frac{4x}{\sqrt{65}}+\frac{7y}{\sqrt{65}}-\frac{2}{\sqrt{65}}=0$$

$$\text{or } x\cos(60.26^\circ)+y\sin(60.26^\circ)=\frac{2}{\sqrt{65}}$$

**(c) Equation: $15y-8x+3=0$**

**(i) Slope-Intercept Form:**

$$15y-8x+3=0 \Rightarrow 15y=8x-3 \Rightarrow y=\frac{8}{15}x-\frac{1}{5}$$

**(ii) Two-Intercept Form:** $\left(\frac{x}{a}+\frac{y}{b}=1\right)$

$$-8x+15y=-3$$

Divide both sides by $-3$

$$\frac{-8x}{-3}+\frac{15y}{-3}=\frac{-3}{-3}$$

$$\frac{8x}{3}-5y=1$$

$$\frac{8x}{3}+(-5y)=1$$

$$\frac{x}{\frac{3}{8}}+\frac{y}{\frac{-1}{5}}=1$$

**(iii) Normalize**

$$15y-8x+3=0$$

$$-8x+15y=-3$$

$$\because \sqrt{(-8)^2+(15)^2}=\pm\sqrt{64+225}\pm\sqrt{289}=\pm 17$$

To make R.H.S positive

Dividing both sides by "−17"

$$\frac{-8x}{-17}+\frac{15y}{-17}=\frac{-3}{-17}$$

$$\frac{8x}{17}+\frac{15y}{-17}=\frac{3}{17}$$

Comparing it with $x\cos\alpha+y\sin\alpha=p$

$$\Rightarrow \cos\alpha=\frac{8}{17}>0 \text{ and } \sin\alpha=\frac{-15}{17}<0,$$

$\Rightarrow$ Angle $\alpha$ lies in $4^{th}$ quadrant and $\alpha=298.070$

$$\Rightarrow x\cos 298.07'' y\sin 298.07''=\frac{3}{17}$$

**11. In each of the following check whether the two lines are**

| | | | |
|---|---|---|---|
| **(i)** | **parallel** | **(ii)** | **perpendicular** |
| **(iii)** | **neither parallel nor perpendicular** | | |
| **(a)** | $2x+y-3=0$ | ; | $4x+2y+5=0$ |
| **(b)** | $3y=2x+5$ | ; | $3x+2y-8=0$ |
| **(c)** | $4y+2x-1=0$ | ; | $x-2y-7=0$ |

***Solution:*** To determine whether the given lines are **parallel**, **perpendicular**, or **neither**, we compare their slopes.

***Key Steps:***

- Rewrite each line in **slope-intercept form** $y=mx+c$ to identify the slope $m$.
- Check:
  - **Parallel lines**: Slopes are equal $m_1=m_2$.
  - **Perpendicular lines**: Slopes are negative reciprocals $m_1\cdot m_2=-1$.
  - **Neither**: If neither condition is satisfied.

**(a) Lines:**

- $2x+y-3=0$

  Rewrite in slope-intercept form:

  $$y=-2x+3 \quad \text{(slope: } m_1=-2\text{).}$$

- $4x+2y+5=0$

  Rewrite in slope-intercept form:

  $$2y=-4x-5 \Rightarrow y=-2x-\frac{5}{2} \quad (\text{slope}: m_2=-2).$$

**Comparison:**

Since $m_1=m_2=-2$, the lines are **parallel**.

**(b) Lines:**

- $3y=2x+5$

  Rewrite in slope-intercept form:

  $$y=\frac{2}{3}x+\frac{5}{3} \quad \left(\text{slope: } m_1=\frac{2}{3}\right).$$

- $3x+2y-8=0$

  Rewrite in slope-intercept form:

  $$2y=-3x+8 \Rightarrow y=-\frac{3}{2}x+4 \quad \left(\text{slope}: m_2=-\frac{3}{2}\right)$$

**Comparison:**
Since $m_1 \cdot m_2 = \frac{2}{3} \cdot -\frac{3}{2} = -1$, the lines are **perpendicular.**

**(c) Lines:**

- $4y + 2x - 1 = 0$
  Rewrite in slope-intercept form:

$$4y = -2x + 1 \Rightarrow y = -\frac{1}{2}x + \frac{1}{4} \quad \left(\text{slope}: m_1 = -\frac{1}{2}\right)$$

- $x - 2y - 7 = 0$
  Rewrite in slope-intercept form:

$$y = \frac{1}{2}x - \frac{7}{2} \quad \left(slope: m_2 = \frac{1}{2}\right)$$

**Comp a:**
Sinc $\neq m_2$ and $m_1 \times m_2 \neq -1$

**n parallel nor perpendicular.**

**Find an equation of the line through (−4, 7) and parallel to the line $2x - 7y + 4 = 0$.**

**Sol.** To find the equation of the line through $(-4,7)$ and parallel to the line $2x - 7y + 4 = 0$, follow these steps:

Step 1: Find the slope of the given line

Rewrite the given equation $2x - 7y + 4 = 0$ in slope-intercept form $y = mx + c$, where $m$ is the slope.

$$2x - 7y + 4 = 0 \Rightarrow -7y = -2x - 4 \Rightarrow y = \frac{2}{7}x + \frac{4}{7}$$

Thus, the slope of the given line is $m = \frac{2}{7}$.

Step 2: Use the same slope for the parallel line

Since parallel lines have the same slope, the line through $(-4,7)$ will also have a slope of $m = \frac{2}{7}$.

Step 3: Use the point-slope form to find the equation of the line

The point-slope form of the equation is:

$$y - y_1 = m(x - x_1)$$

where $(x_1, y_1) = (-4,7)$ and $m = \frac{2}{7}$.

Substitute these values:

$$y - 7 = \frac{2}{7}(x + 4)$$

Step 4: Simplify the equation

Distribute the slope $\frac{2}{7}$:

$$y - 7 = \frac{2}{7}x + \frac{8}{7}$$

Add 7 to both sides:

$$y = \frac{2}{7}x + \frac{8}{7} + 7$$

Express 7 as $\frac{49}{7}$:

$$y = \frac{2}{7}x + \frac{8}{7} + \frac{49}{7}$$

Combine the constants: $y = \frac{2}{7}x + \frac{57}{7}$

Final Answer:

The equation of the line through $(-4,7)$ and parallel to the line $2x - 7y + 4 = 0$ is:

$$y = \frac{2}{7}x + \frac{57}{7}$$

or $\quad 2x - 7y + 57 = 0$

13. **Find an equation of the line through (5, −8) and perpendicular to the join of $A(-15, 8)$, $B(10, 7)$.**

***Solution:*** A(−15, −18), B(10, 7)

$$\text{Slope of } \overline{AB} = \frac{x_2 - x_1}{y_2 - y_1} = \frac{7-(-8)}{10(-15)}$$

$$= \frac{7-8}{10+15} = \frac{15}{25}$$

The Slope of line perpendicular to $\overline{AB} = m_2$ we know that for perpendicular lines,

$$m_1 \times m_2 = -1$$

$$\frac{3}{5} \times m_2 = -1$$

$$\boxed{m_2 = \frac{-5}{3}}$$

Since line passes through point (5, −8), so we can take $(x_1, y_1) = (5, -8)$

The point – slope of equation is:

$$y - y_1 = m_2(x - x)$$

$$y - (-8) = \frac{-5}{3}(x - 5)$$

$$3(y + 8) = -5(x - 5)$$

$$3y + 24 = -5x + 25$$

$$\Rightarrow 3y + 24 + 5x - 25 = 0$$

$$\Rightarrow 5x + 3y - 1 = 0$$

**Example 14:** On a map, Town $A$ is at coordinates (2,3) and Town $B$ is at (−4,−1). What is the distance between the two towns?

**Solution:** Use the distance formula:

$$d = \sqrt{(x_2 - x_1) + (y_2 - y_1)^2}$$

Substitute the values:

$$d = \sqrt{(-4-2)^2 + (-1-3)^2} = \sqrt{(-6)^2 + (-4)^2} = \sqrt{36+16} = \sqrt{52} \approx 7.21$$

Thus, the distance between Town $A$ and Town $B$ is approximately 7.21 units.

**Example 15:** Suppose two cities, City $A$ and City $B$, are represented by the coordinates (3, 4) and (7, 1) on a map. Find the straight-line distance between the two cities.

**Solution:** We apply the distance formula:

$$d = \left|\overline{AB}\right| = \sqrt{(7-3)^2 + (1-4)^2}$$

$$d = \left|\overline{AB}\right| = \sqrt{(4)^2 + (-3)^2}$$

$$d = \left|\overline{AB}\right| = \sqrt{16+9} = \sqrt{25} = 5$$

Thus, the straight line distance between City $A$ and City $B$ is 5 units.

**Example 16:** An Engineer is building a bridge between two points on a riverbank. Suppose the coordinates of the two points where the bridge will start and end are (2, 5) and (8, 9). Find the coordinates of the midpoint, which will represent the center of the bridge.

**Solution:** We apply the midpoint formula:

$$M = \left(\frac{2+8}{2}, \frac{5+9}{2}\right)$$

$$= \left(\frac{10}{2}, \frac{14}{2}\right) = (5, 7)$$

Thus, the center of the bridge is at the point (5, 7)

**Example 17:** A landscaper is designing a triangular garden with corners at points $A(2, 3)$, $B(5, 7)$, and C(6, 2). Calculate the lengths of the sides of the triangle.

**Solution:** Use the **distance formula** to find the length of each side: $\quad d = \sqrt{(x_2 + x_1)^2 + (y_2 - y_1)^2}$

$$|\overline{AB}| = \sqrt{(5-2)^2+(7-3)^2}$$
$$|\overline{AB}| = \sqrt{(3)^2+(4)^2}$$
$$|\overline{AB}| = \sqrt{9+16} = \sqrt{25} = 5 \text{ units}$$
$$|\overline{BC}| = \sqrt{(6-5)^2+(2-7)^2}$$
$$|\overline{BC}| = \sqrt{(1)^2+(-5)^2}$$
$$|\overline{BC}| = \sqrt{1+25} = \sqrt{26} = 5.10 \text{ units}$$
$$|\overline{AC}| = \sqrt{(6-2)^2+(2-3)^2}$$
$$|\overline{AC}| = \sqrt{(4)^2+(-1)^2}$$
$$|\overline{AC}| = \sqrt{16+1} = \sqrt{17} = 4.12 \text{ units}$$

Thus, the lengths of the sides of the triangular garden are:
$m\overline{AB} = 5$ units, $m\overline{BC} \approx 55.10$ units $m\overline{AC} \approx 4.12$ units

**Example 18:** A pilot needs to travel from city $A(50, 60)$ to city $B(120, 150)$. Determine the heading angle the plane should take relative to the east direction.

**Solution:** The heading angle can be calculated using the slope:

$$m = \frac{150-60}{120-50} = \frac{90}{70} = \frac{9}{7}$$

Let $\theta$ be the required angle, then

$$\tan\theta = m = \frac{9}{7}$$
$$\theta = \tan^{-1}\left(\frac{9}{7}\right)$$
$$\theta = \tan^{-1}(1.2857)$$
$$\theta \approx 52.13^\circ$$

Thus, the plane should take a heading angle of $52.13^\circ$ north of east.

**Example 19:** Abdul Hadi is traveling from point $A$(Latitude $10^\circ$ $N$, Longitude $50^\circ$ $E$) to point $B$(Latitude $20^\circ$ $N$, Longitude $60^\circ$ $E$). Find the midpoint of his journey in terms of latitude and longitude.

**Solution:** Given that

Point $A$(Latitude $10^\circ$ $N$, Longitude $50^\circ$ $E$)
Point $B$(Latitude $20^\circ$ $N$, Longitude $60^\circ$ $E$)

$$\text{Midpoint latitude} = \frac{10^\circ + 20^\circ}{2} = 15^\circ N$$
$$\text{Midpoint longitude} = \frac{50^\circ + 60^\circ}{2} = 55^\circ N$$

Thus, the midpoint of Abdul Hadi's journey would be a Latitude $15^\circ$ $N$, Longitude $55^\circ$ $E$.

**Example 20:** A landscaper is designing a straight pathway from $P(2, 3)$ to $Q(8, 9)$. What is the length of the pathway?

**Solution:** The length of the straight pathway can be found using the distance formula:

$$\text{Distance} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$= \sqrt{(8-2)^2+(9-3)^2}$$
$$= \sqrt{(6)^2+(6)^2}$$
$$= \sqrt{36+36} = 6\sqrt{2}$$

So, the length of the pathway is approximately $6\sqrt{2}$ units.

## EXERCISE 7.3

**1. If the houses of two friends is represented by coordinates (2, 6) and (9, 12) on a grid. Find the straight line distance between their houses if the grid units represent kilometres?**

**Sol.** To find the straight-line distance between two points on a grid, use the **distance formula**:

$$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Here, the coordinates of the two points are:

$$(x_1, y_1) = (2,6),\ (x_2, y_2) = (9,12)$$

**Step 1: Substitute the coordinates**

$$d = \sqrt{(9-2)^2 + (12-6)^2}$$

**Step 2: Simplify the differences**

$$d = \sqrt{(7)^2 + (6)^2}$$

**Step 3: Square the values**

$$d = \sqrt{49 + 36}$$

**Step 4: Add and simplify**

$$d = \sqrt{85}$$

**Step 5: Approximate the square root**

$$d \approx 9.22$$

**Final Answer:**

The straight-line distance between their houses is approximately:

$$\boxed{9.22\ kilometres}$$

2. **Consider a straight trail (represented by coordinate plane) that starts at point (5, 7) and ends at point (15, 3). What are the coordinates of the midpoint?**

**Sol.** To find the midpoint of a line segment with endpoints $(x_1, y_1)$ and $(x_2, y_2)$, we use the **midpoint formula:**

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Given the endpoints are (5,7) and (15,3):

**Step 1: Substitute the coordinates into the midpoint formula**

$$\left(\frac{5+15}{2}, \frac{7+3}{2}\right)$$

**Step 2: Simplify**

$$\left(\frac{20}{2}, \frac{10}{2}\right) = (10,5)$$

**Final Answer:**

The midpoint of the trail is:

$$\boxed{(10,5)}$$

3. **An architect is designing a park with two buildings located at (10, 8) and (4, 3) on the grid. Calculate the straight-line distance between the buildings. Assume the coordinates are in meters.**

**Sol.** To calculate the straight-line distance between two points $(x_1, y_1)$ and $(x_2, y_2)$, we use the **distance formula**:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

**Step 1: Identify the coordinates**

The coordinates of the buildings are:

$$(x_1, y_1) = (10,8) \quad \text{and} \quad (x_2, y_2) = (4,3)$$

**Step 2: Substitute the coordinates into the formula**

$$d = \sqrt{(4-10)^2 + (3-8)^2}$$

$$d = \sqrt{(-6)^2 + (-5)^2}$$

**Step 3: Square the differences**

$$d = \sqrt{36 + 25}$$

**Step 4: Add and simplify**

$$d = \sqrt{61}$$

**Step 5: Approximate the square root**

$$d \approx 7.81$$

**Final Answer:**

The straight-line distance between the buildings is approximately:

$$\boxed{7.81 \text{ meters}}$$

4. **A delivery driver needs to calculate the distance between two delivery locations. One location is at (7, 2) and the other at (12, 10) on the city grid map, where each unit represents kilometers. What is the distance between the two locations?**

**Sol.** To calculate the distance between two points $(x_1, y_1)$ and $(x_2, y_2)$ on a grid, we use the **distance formula:**

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

**Step 1: Identify the coordinates**

The coordinates of the two locations are:

$$(x_1, y_1) = (7,2) \quad \text{and} \quad (x_2, y_2) = (12,10)$$

**Step 2: Substitute the coordinates into the formula**

$$d = \sqrt{(12-7)^2 + (10-2)^2}$$

$$d = \sqrt{(5)^2 + (8)^2}$$

**Step 3: Square the differences**

$$d = \sqrt{25 + 64}$$

**Step 4: Add and simplify**

$d = 89$

**Step 5: Approximate the square root**

$d \approx 9.43$

**Final Answer:**

The distance between the two locations is approximately:

$$\boxed{9.43 \text{ kilometers}}$$

5. **The start and end points of a race track are given by coordinates (3, 9) and (9, 13). What is the midpoint of the track.**

**Sol.** To find the midpoint of the race track, we use the **midpoint formula**, which gives the point halfway between two given coordinates $(x_1, y_1)$ and $(x_2, y_2)$:

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

**Step 1: Identify the coordinates**

The start and end points of the race track are:

$$(x_1, y_1) = (3,9) \quad \text{and} \quad (x_2, y_2) = (9,13)$$

**Step 2: Substitute the coordinates into the midpoint formula**

$$\left(\frac{3+9}{2}, \frac{9+13}{2}\right)$$

**Step 3: Simplify the calculations**

$$\left(\frac{12}{2}, \frac{22}{2}\right) = (6,11)$$

**Final Answer:**

The midpoint of the race track is:

$$\boxed{(6,11)}$$

6. **The coordinates of two points on a road are A (3, 4) and B (7, 10). Find the midpoint of the road.**

**Sol.** To find the **midpoint** of the road between the two points $A(3,4)$ and $B(7,10)$, we use the **midpoint formula**:

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

**Step 1: Identify the coordinates**

The coordinates of the points are:

$$A(3,4) \quad \text{and} \quad B(7,10)$$

**Step 2: Substitute the coordinates into the formula**

$$\left(\frac{3+7}{2}, \frac{4+10}{2}\right)$$

**Step 3: Simplify the calculations**

$$\left(\frac{10}{2}, \frac{14}{2}\right) = (5,7)$$

**Final Answer:**

The midpoint of the road is:

$$\boxed{(5,7)}$$

7. **A ship is navigating from Port A located at (12°N, 65°W) to Port B at (20°N, 45°W). If the ship travels along the shortest path on the surface of the Earth, calculate the straight line distance between the points.**

**Sol.** Location of port $A(12^\circ\text{N}, 65^\circ\text{W}) = (x_1, y_1)$

Location of port $B(20^\circ N, 45^\circ W) = (x_2, y_2)$

By using distance formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$|\overline{AB}| = \sqrt{(20-12)^2 + (45-65)^2}$$

$$|\overline{AB}| = \sqrt{(8)^2 + (20)^2} = \sqrt{64+400}$$

$$= \sqrt{464} = 21.54 \text{ units}$$

**8. Farah is fencing around a rectangular field with corners at (0, 0), (0, 5), (8, 5) and (8, 0). How much fencing material will she need to cover the entire perimeter of the field?**

***Solution:*** To determine how much fencing material Farah will need, we first need to calculate the perimeter of the rectangular field.

Step 1: Use the distance formula to find the lengths of the sides

The coordinates of the four corners of the rectangular field are:

$$(0,0), (0,5), (8,5), (8,0)$$

**Length of the first side:**

From (0,0) to (0,5), the horizontal distance is $0 - 0 = 0$ and the vertical distance is $5 - 0 = 5$. Thus, the length of this side is 5 units.

**Length of the second side:**

From (0,5) to (8,5), the horizontal distance is $8 - 0 = 8$ and the vertical distance is $5 - 5 = 0$. Thus, the length of this side is 8 units.

**Length of the third side:**

From (8,5) to (8,0), the horizontal distance is $8 - 8 = 0$ and the vertical distance is $5 - 0 = 5$. Thus, the length of this side is 5 units.

**Length of the fourth side:**

From (8,0) to (0,0), the horizontal distance is $8 - 0 = 8$ and the vertical distance is $0 - 0 = 0$. Thus, the length of this side is 8 units.

**Step 2: Calculate the perimeter**

The perimeter $P$ of the rectangle is the sum of the lengths of all four sides:

$$P = 2 \times (5 + 8) = 2 \times 13 = 26$$

**Final Answer:**

Farah will need **26 units** of fencing material to cover the entire perimeter of the field.

**9. An airplane is flying from City X at $(40^\circ \text{N}, 100^\circ \text{W})$ to City *Y* at $(50^\circ \text{N}, 80^\circ \text{W})$. Use coordinate geometry, calculate the shortest distance between these two cities.**

***Sol:*** By using distance formula

$$d = \sqrt{(x_2 - x_1) + (y_2 - y_1)^2}$$

Putting the values

$$|\overline{XY}| = \sqrt{(50-40)^2 + (80-100)^2}$$

$$= \sqrt{(10)^2 + (-20)^2}$$

$$= \sqrt{100+400}$$

$$= \sqrt{500}$$

$$= \sqrt{100 \times 5}$$

$$= 10\sqrt{5} \approx 22.4 \text{ units}$$

**10. A land surveyor is marking out a rectangular plot of land with corners at (3, 1), (3, 6), (8, 6), and (8, 1). Calculate the perimeter.**

**Sol.** Let coordinates of corner are A(3, 1), B(3, 6), C(8, 6), D(8, 1)

First we find length and width of rectangular plot by using the distance formula.

$$d = \sqrt{(x_2 - x_1) + (y_2 - y_1)^2}$$

Finding length $\overline{AB}$

$$|\overline{AB}| = \sqrt{(3-3)^2 + (6-1)^2}$$
$$= \sqrt{(0)^2 + (5)^2}$$
$$= \sqrt{0+25} = \sqrt{25} = 5 \text{ units}$$

Finding width $\overline{BC}$

$$|\overline{BC}| = \sqrt{(8-3)^2 + (6-6)^2}$$
$$= \sqrt{(5)^2 + (0)^2} = \sqrt{25+0} = \sqrt{25} = 0 \text{ units}$$

We observe that rectangular plot is a square

Finding perimeter

$$\text{Perimeter} = 4|\overline{AB}| \qquad (\because P = 4\ell)$$
$$= 4(5 \text{ units})$$
$$= 20 \text{ units}$$

**11. A landscaper needs to install a fence around a rectangular garden. The garden has its corners at the coordinates: $A(0, 0)$, $B(5, 0)$, $C(5, 3)$ and $D(0, 3)$. How much fencing is required?**

**Sol.** First we find the length and width of rectangle using distance formula.

$$d = \sqrt{(x_2 - x_1) + (y_2 - y_1)^2}$$

Length of side $\overline{AB}$ : A(0, 0), B(5, 0)

$$L = |\overline{AB}| = \sqrt{(5-0)^2 + (0-0)^2}$$
$$= \sqrt{(5)^2 + (0)^2}$$
$$= \sqrt{25+0} = \sqrt{25}$$
$$= 5 \text{ units}$$

Length of side $\overline{BC}$ ; $B(5, 0)$, $C(5, 3)$

$$W = |\overline{BC}| = \sqrt{(5-5)^2 + (3-0)^2}$$
$$= \sqrt{(0)^2 + (3)^2}$$
$$= \sqrt{0+9} = \sqrt{9} = 3$$

We know that fencing required is equal to the perimeter of rectangular garden. So

$$\text{Perimeter} = 2(L + W)$$
$$P = 2[5 + 3]$$
$$P = 2(8)$$
$$P = 16 \text{ units}$$

## REVIEW EXERCISE 7

**1. Choose the correct option.**

(i) The equation of a straight line in the slope-intercept form is written as:

(a) $y = m(x + c)$ (b) y-yi = m(x - xi)

(c) $y = c + mx$ (d) $ax + by + c = 0$

(ii) The gradients of two parallel lines are:

(a) equal (b) zero

(c) negative reciprocals of each other

(d) always undefined

(iii) If the product of the gradients of two lines is –1, then the lines are:

(a) Parallel (b) perpendicular

(c) Collinear (d) coincident

(iv) Distance between two points $P(1, 2)$ and $Q(4, 6)$ is:

(a) 5 (b) 6 (c) $\sqrt{13}$ (d) 4

(v) The midpoint of a line segment with endpoints (–2, 4) and (6, –2) is:

(a) (4, 2) (b) (2, 1) (c) (1, 1) (d) (0, 0)

(vi) A line passing through points (1, 2) and (4, 5) is:

(a) $y = x + 1$  (b) $y = 2x + 3$

(c) $y = 3x - 2$  (d) $y = x + 2$

(vii) The equation of a line in point-slope form is:

(a) $y = m(x + c)$  (b) $y - y_1 = m(x - x_1)$

(c) $y = c + mx$  (d) $ax + by + c = 0$

(viii) $2x + 3y - 6 = 0$ in the slope-intercept form is:

(a) $y = \frac{-2}{3}x + 2$  (b) $y = \frac{2}{3}x - 2$

(c) $y = \frac{2}{3}x + 1$  (d) $y = \frac{-2}{3}x - 2$

(ix) The equation of a line in symmetric form is:

(a) $\frac{x}{a} + \frac{y}{b} = 1$  (b) $\frac{x - x_1}{1} + \frac{y - y_1}{m} = \frac{z - z_1}{1}$

(c) $ax + by + c = 0$ (d) $y - y_1 = m(x - x_1)$

(x) The equation of a line in normal form is:

(a) $y = mx + c$  (b) $\frac{x}{a} + \frac{y}{b} = 1$

(c) $\frac{x - x_1}{\cos\alpha} = \frac{y - y_1}{\sin\alpha}$  (d) $x\cos\alpha + y\sin\alpha = p$

***Answers:***

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| (i) | c | (ii) | a | (iii) | b | (iv) | a | (v) | b |
| (vi) | a | (vii) | b | (viii) | a | (ix) | c | (x) | d |

**Solve the following:**

**2. Find the distance between two points $A(2, 3)$ and $B(7, 8)$ on a coordinate plane.**

**Sol. Step 1: Use the Distance Formula**

The coordinates of the two points are:

- Point A: (2,3)
- Point B: (7,8)

The distance formula is given by:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Where $(x_1, y_1)$ and $(x_2, y_2)$ are the coordinates of the two points.

**Step 2: Substitute the Coordinates into the Formula**

Substitute the coordinates of points A and B into the formula:

$$d = \sqrt{(7 - 2)^2 + (8 - 3)^2}$$

$$d = \sqrt{5^2 + 5^2} = \sqrt{25 + 25} = \sqrt{50}$$

**Step 3: Simplify the Expression**

$$d = \sqrt{50} = \sqrt{25 \times 2} = 5\sqrt{2}$$

**3. Find the midpoint of the line segment joining the points (4, –2) and (–6, 3).**

**Sol. Step 1: Use the Midpoint Formula**

The midpoint $M(x_m, y_m)$ of a line segment joining two points $(x_1, y_1)$ and $(x_2, y_2)$ is given by the formula:

$$M(x_m, y_m) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

**Step 2: Substitute the Coordinates of the Points**

The coordinates of the two points are:

- Point 1: (4, –2)
- Point 2: (–6,3)

Substitute the values into the formula:

$$M(x_m, y_m) = \left(\frac{4 + (-6)}{2}, \frac{-2 + 3}{2}\right)$$

$$M(x_m, y_m) = \left(\frac{-2}{2}, \frac{1}{2}\right)$$

$$M(x_m, y_m) = (-1, 0.5) \text{ or } \left(-1, \frac{1}{2}\right)$$

**Final Answer:**

The midpoint of the line segment joining the points (4, –2) and (–6,3) is $\boxed{(-1, 0.5)}$ or $\left(-1, \frac{1}{2}\right)$

4. **Calculate the gradient (slope) of the line passing through the points (1, 2) and (4, 6).**

**Sol.** **Step 1: Use the Slope Formula**

The slope $m$ of a line passing through two points $(x_1, y_1)$ and $(x_2, y_2)$ is given by the formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

**Step 2: Substitute the Coordinates of the Points**

The coordinates of the two points are:

- Point 1: (1,2)
- Point 2: (4,6)

Substitute the values into the formula:

$$m = \frac{6-2}{4-1} = \frac{4}{3}$$

**Final Answer:**

The slope (gradient) of the line passing through the points (1,2) and (4,6) is $\boxed{\frac{4}{3}}$.

5. **Find the equation of the line in the form $y = mx + c$ that passes through the points (3, 7) and (5, 11).**

**Sol.** **Step 1: Find the Slope (Gradient) of the Line**

The coordinates of the two points are:

- Point 1: (3,7)
- Point 2: (5,11)

The slope $m$ of the line passing through these points is calculated using the formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Substituting the values:

$$m = \frac{11-7}{5-3} = \frac{4}{2} = 2$$

So, the slope $m = 2$.

**Step --: Use the Point-Slope Formula to Find the Equation**

The point-slope form of the equation of a line is:

$$y - y_1 = m(x - x_1)$$

We know the slope $m = 2$ and we can use one of the points, say (3,7), to substitute into the formula:

$$y - 7 = 2(x - 3)$$

**Step 3: Simplify the Equation**

Now, simplify to get the equation in slope-intercept form $y = mx + c$:

$$y - 7 = 2(x - 3)$$

$$y - 7 = 2x - 6$$

$$y = 2x + 1$$

**Final Answer:**

The equation of the line in the form $y = mx + c$ that passes through the points (3,7) and (5,11) is:

$$\boxed{y = 2x + 1}$$

6. **If two lines are parallel and one line has a gradient of $\frac{2}{3}$, what is the gradient of the other line?**

**Sol.** **Parallel Lines:**

When two lines are parallel, they have the same gradient (slope). Given that the gradient of one line is $\frac{2}{3}$, the gradient of the other parallel line will also be $\frac{2}{3}$.

**Answer for parallel lines:** The gradient of the other line is $\frac{2}{3}$.

7. **An airplane needs to fly from city A at coordinates (12, 5) to city B at coordinates (8, –4). Calculate the straight-line distance between these two cities.**

**Sol.** To calculate the straight-line distance between two points on a coordinate plane, we use the **distance formula**:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

**Step 1: Identify the coordinates of the two cities**

- City A: $(x_1, y_1) = (12,5)$
- City B: $(x_2, y_2) = (8,-4)$

**Step 2: Substitute the values into the distance formula**

$$d = \sqrt{(8-12)^2 + (-4-5)^2}$$

$$d = \sqrt{(-4)^2 + (-9)^2}$$

$$d = \sqrt{16+81}$$

$$d = \sqrt{97}$$

**Step 3: Calculate the result**

$$d \approx 9.85$$

**Final Answer:**

The straight-line distance between City A and City B is approximately $\boxed{9.85}$ miles.

8. **In a landscaping project, the path starts at (2, 3) and ends at (10, 7). Find the midpoint.**

**Sol.** To find the midpoint of a line segment with endpoints $(x_1, y_1)$ and $(x_2, y_2)$, we use the **midpoint formula**:

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

**Step 1: Identify the coordinates of the two endpoints**

- Starting point: $(x_1, y_1) = (2,3)$
- Ending point: $(x_2, y_2) = (10,7)$

**Step 2: Substitute the values into the midpoint formula**

$$\text{Midpoint} = \left(\frac{2+10}{2}, \frac{3+7}{2}\right)$$

$$\text{Midpoint} = \left(\frac{12}{2}, \frac{10}{2}\right)$$

$$\text{Midpoint} = (6,5)$$

**Final Answer:**

The midpoint of the garden path is $\boxed{(6,5)}$.

9. **A drone is flying from point (2, 3) to point (10, 15) on the grid. Calculate the gradient of the line along which the drone is flying and the total distance traveled.**

**Sol.** To find the **gradient (slope)** and the **distance** traveled by the drone, we'll use the following formulas:

**1. Gradient (Slope) Formula:**

The gradient $m$ of the line between two points $(x_1, y_1)$ and $(x_2, y_2)$ is given by:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

**2. Distance Formula:**

The distance $d$ between two points $(x_1, y_1)$ and $(x_2, y_2)$ is given by:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

**Step 1: Identify the coordinates of the two points**

- Starting point: $(x_1, y_1) = (2,3)$
- Ending point: $(x_2, y_2) = (10,15)$

**Step 2: Calculate the gradient (slope)**

$$m = \frac{15-3}{10-2} = \frac{12}{8} = \frac{3}{2}$$

So, the **gradient** of the line is $\frac{3}{2}$.

**Step 3: Calculate the distance traveled**

$$d = \sqrt{(10-2)^2 + (15-3)^2}$$

$$d = \sqrt{8^2 + 12^2}$$

$$d = \sqrt{64+144}$$

$$d = \sqrt{208} = \sqrt{16 \times 13}$$

$$= 4\sqrt{13} \text{ units}$$

**Final Answers:**

- The gradient of the line is $\boxed{\frac{3}{2}}$.
- The distance traveled by the drone is approximately $\boxed{4\sqrt{13}}$ units.

**10. For a line with a gradient of –3 and a y-intercept of 2, write the equation of the line in:**

**(a) Slope-intercept form**
**(b) Point-slope form (using the point (1, 2)**
**(c) Two-point form (using the points (1, 2) and (4,–7)**
**(d) Intercepts form**
**(e) Symmetric form**
**(f) Normal form**

**Sol.** We are given the following information about the line:

- Gradient (slope), $m = -3$
- y-intercept, $b = 2$

**(a) Slope-Intercept Form:**

The slope-intercept form of a line is:

$$y = mx + b$$

Substituting $m = -3$ and $b = 2$:

$$y = -3x + 2$$

**(b) Point-Slope Form:**

The point-slope form of the equation is:

$$y - y_1 = m(x - x_1)$$

Using the point $(x_1, y_1) = (1,2)$ and $m = -3$, we substitute into the point-slope form:

$$y - 2 = -3(x - 1)$$

**(c) Two-Point Form:**

The two-point form of a line is:

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

Using the points $(x_1, y_1) = (1,2)$ and $(x_2, y_2) = (4,-7)$, we substitute into the formula:

$$\frac{y - 2}{-7 - 2} = \frac{x - 1}{4 - 1}$$

**(d) Intercepts Form:**

The intercept form of the equation is:

$$\frac{x}{a} + \frac{y}{b} = 1$$

Where $a$ and $b$ are the x-intercept and y-intercept respectively.

The y-intercept is given as $b = 2$, and the x-intercept can be found by setting $y = 0$ in the slope-intercept form $y = -3x + 2$, so:

$$0 = -3x + 2 \quad \Rightarrow \quad x = \frac{2}{3}$$

Thus, $a = \frac{2}{3}$ and $b = 2$, so the equation becomes:

$$\frac{y}{2} + \frac{x}{\frac{2}{3}} = 1$$

**(e) Symmetric Form:**

The symmetric form of a line is:

$$y = -3x + 2$$

$$3x + y = 2$$

Divide both sides by $\sqrt{3^2 + 1^2} = \sqrt{10}$

$$\frac{3}{\sqrt{10}}x + \frac{1}{\sqrt{10}}y = \frac{2}{\sqrt{10}}$$

or $$\frac{y}{\sqrt{10}} + \frac{3x}{\sqrt{10}} = \frac{2}{\sqrt{10}}$$

## (f) Normal Form:

The normal form of a line is given by:

$$x\cos\theta + y\sin\theta = p$$

Where $\theta$ is the angle the line makes with the positive x-axis, and $p$ is the perpendicular distance from the origin to the line.

The slope of the line is $m = -3$, so the angle $\theta$ is:

$$\tan\theta = -3 \quad \Rightarrow \quad \theta = \tan^{-1}(-3)$$

For the normal form, we also need the perpendicular distance $p$. The perpendicular distance from the origin to the line $y = -3x + 2$ is calculated as:

$$p = \frac{|2|}{\sqrt{1^2 + (-3)^2}} = \frac{2}{\sqrt{10}}$$

Thus, the equation of the line in normal form is:

$$x\cos(\theta) + y\sin(\theta) = \frac{2}{\sqrt{10}}$$