UNIT 9

Similar Figures

Students' learning outcomes

At the end of the unit, the students will be able to:

- Identify similarity of polygons. Area and volume of similar figures.
- Solve problems using the relationship between area's of similar figures and volume of similar solids.
- Geometrical properties of regular polygons, triangles and parallelograms.
- Solve real life problems that involve the properties of regular polygons, triangles and parallelograms (such as building architectural structures, fencing, tiling, painting and carpeting a room).

Example 1: If one pair of corresponding sides are parallel to each other, then the triangles so formed as shown in the figure are similar, i.e., In the figure, $\overline{AB}$ is parallel to $\overline{CD}$ and

$m\angle AOB = m\angle DOC$ (Vertically opposite angles)

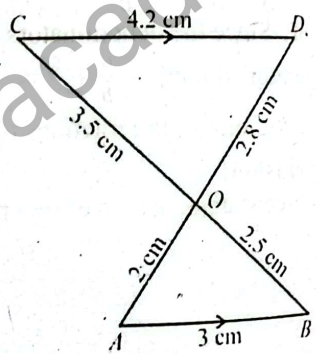
$m\angle A = m\angle D$ (Alternate angles of parallel lines)

$m\angle B = m\angle C$ (Alternate angles of parallel lines)

Since all three corresponding angles are equal,

So $\Delta OAB \sim \Delta ODC$

The ratio of corresponding sides are equal i.e.,



$$\frac{m\overline{OA}}{m\overline{OD}} = \frac{m\overline{AB}}{m\overline{DC}} = \frac{m\overline{OB}}{m\overline{OC}}$$

$$\frac{2}{2.8} = \frac{3}{4.2} = \frac{2.5}{3.5}$$

$$\frac{5}{7} = \frac{5}{7} = \frac{5}{7}$$

So, the triangles *OAB* and *ODC* are similar.

Example 2: In the triangles *XBC* and *XDE*, find the value of *x* and *y*.

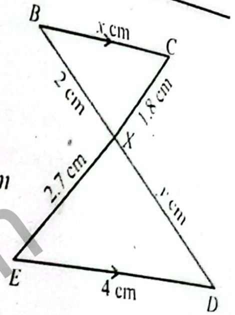
Solution: Since $\overline{BC}$ is parallel to $\overline{ED}$, so the triangles *XBC* and *XDE* are similar, so, the ratio of the corresponding sides are equal

$$\frac{m\overline{XB}}{m\overline{XD}} = \frac{m\overline{BC}}{m\overline{DE}} = \frac{m\overline{XC}}{m\overline{XE}}$$

$$\frac{2}{y} = \frac{x}{4} = \frac{1.8}{2.7}$$

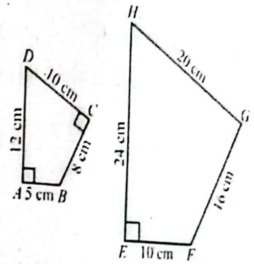
$$\frac{x}{4} = \frac{1.8}{2.7} \Rightarrow x = \frac{1.8}{2.7}\times 4 = 2.67\,cm$$

$$\frac{2}{y} = \frac{1.8}{2.7} \Rightarrow y = \frac{2.7}{1.8}\times 2 = 3\,cm$$



9.1.2 Similarity of Quadrilaterals

Example 3: The Quadrilateral *ABCD* has side lengths $m\overline{AB} = 5$cm, $m\overline{BC} = 8$, $m\overline{CD} = 10$ cm, $m\overline{AD} = 12$cm, and its angles are $m\angle A = 90°$, $m\angle B = 120°$ and $m\angle C = 90°$. Quadrilateral *EFGH* has side lengths $m\overline{EF} = 10$ cm, $m\overline{FG} = 16$ cm, $m\overline{GH} = 20$cm, $m\overline{EH} = 24$ cm and its angles are $m\angle E = 90°$, $m\angle F = 120°$ and $m\angle H = 60°$. Prove that the quadrilateral *ABCD* is similar to the quadrilateral *EFGH*. (Diagrams are not drawn to scale).



Solution: We see that in the quadrilateral *ABCD*:

$$m\angle D = 360° - (90° + 120° + 90°) = 60°.$$

In the quadrilateral *EFGH*, $m\angle G = 360° - (90° + 120° + 60°) = 90°$.

Now, check if the corresponding angles of the quadrilaterals are congruent:

$m\angle A = m\angle E = 90°$, $m\angle B = m\angle F = 120°$, $m\angle C = m\angle G = 90°$ and $m\angle D = m\angle H = 60°$.

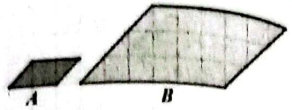
Next, check the ratios of the corresponding sides:

Ratio of $\overline{AB}$ to $\overline{EF}$: $\frac{m\overline{AB}}{m\overline{EF}} = \frac{5}{10} = \frac{1}{2}$, Ratio of $\overline{BC}$ to $\overline{FG}$: $\frac{m\overline{BC}}{m\overline{FG}} = \frac{8}{16} = \frac{1}{2}$

Ratio of $\overline{CD}$ to $\overline{GH}$: $\frac{m\overline{CD}}{m\overline{GH}} = \frac{10}{20} = \frac{1}{2}$ Ratio of $\overline{AD}$ to $\overline{EH}$: $\frac{m\overline{AD}}{m\overline{EH}} = \frac{12}{24} = \frac{1}{2}$.

Since the corresponding angles are congruent and the corresponding sides are proportional (with a ratio of $\frac{1}{2}$), so the quadrilateral *ABCD* is similar to the quadrilateral *EFGH*.

Example 4: Find whether the parallelograms are similar given that one of the angle between sides is 45° in both the parallelograms.



Solution: *Since* opposite angles in a parallelogram are equal and adjacent angles are supplementary, so the corresponding angles in both parallelograms (45°, 135°, 45°, and 135°) are equal. So, the parallelograms are similar.

Measure of the base of smaller parallelogram, b_1 = 2units

Measure of the base of larger parallelogram, b_2 = 6 units.

Measure of the height of smaller parallelogram, h_1 = 1 unit

Measure of the height of larger parallelogram, h_2 = 3 units.

Ratio of corresponding lengths are equal. i.e., $\frac{b_1}{b_2} = \frac{2}{6} = \frac{1}{3}$ and $\frac{h_1}{h_2} = \frac{1}{3}$

Therefore, $\frac{b_1}{b_2} = \frac{h_1}{h_2}$

Example 5: The perimeter of a regular octagon is 48 cm. Another octagon has sides that are 1.2 times the sides of the first octagon. What is the length of side of the second octagon?

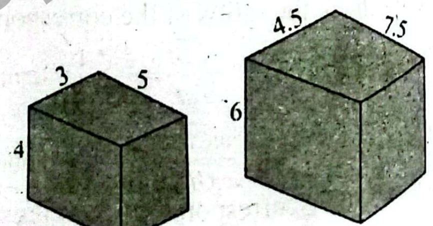
Solution: Perimeter of first regular octagon = 48 cm

Side length of first regular octagon = $\frac{48}{8}$ = 6 cm.

Side length of second regular octagon = 6 × 1.2 = 7.2 cm.

EXERCISE 9.1

1. Find whether the solids are similar. All lengths are in cm.



Sol: Small Cuboid Bigger Cuboid

4 × 3 × 5. 6 × 4.5 × 7.5

The lengths of the sides are in correspondence.

4 is in correspondence with 6

Ratio of 4 to 6 $= \frac{4}{6} = \frac{2}{3}$

3 is in correspondence with 4.5

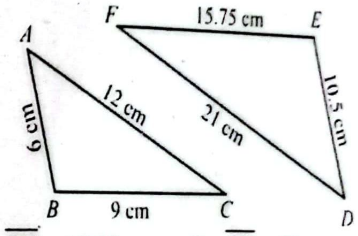
Ratio of 3 to 4.5 $= \frac{30}{45} = \frac{2}{3}$

5 is in correspondence with 7.5

Ratio of 7 to 7.5 $= \frac{5}{7.5} = \frac{2}{3}$

The ratio between the sides is equal, hence both the cuboids are similar.

2. In triangle *ABC*, the sides are given as $m\overline{AB}$ = 6cm, $m\overline{BC}$ = 9 cm and $m\overline{CA}$ = 12 cm. In triangle *DEF*, the sides are given as $m\overline{DE}$ = 10.5 cm, $m\overline{EF}$ = 15.75 cm, and $m\overline{FD}$ = 21 cm. Prove that the triangles are similar.



Sol: $\Delta ABC \approx \Delta DEF$

The correspondence is as follows

1. $AC \longleftrightarrow \overline{FD}$
2. $AB \longleftrightarrow \overline{ED}$ and
3. $\overline{BC} \longleftrightarrow EF$

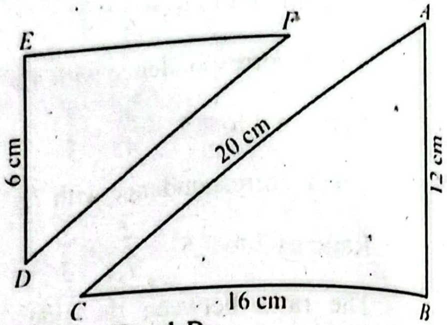
Ratio between 1st pair $= \frac{12}{21} = 0.571$

Ratio between 2nd pair $= \frac{6}{10.5} = 0.571$

Ratio between 3rd pair $= \frac{9}{15.75} = 0.571$

Since the ratio between the correspondences is the same. Hence the similarity between the two triangles is established.

3. In the given figure, $\Delta ABC \sim \Delta DEF$, $m\overline{AB} = 12$ cm, $m\overline{AC} = 20$ cm and $m\overline{BC} = 16$ cm. In ΔEF, $m\overline{DE} = 6$ cm. Find $m\overline{DF}$ and $m\overline{EF}$.



Sol: $\Delta DEF \approx \Delta ABC$. Find the sides E and D.

$$\frac{m\overline{AB}}{m\overline{DE}} = \frac{m\overline{BC}}{m\overline{EF}} = \frac{m\overline{AC}}{m\overline{DF}}$$

$$\frac{12}{6} = \frac{16}{\overline{EF}}$$

$$\overline{EF} = \frac{16 \times 6}{12} = 8$$

$$m\overline{EF} = 8$$

$$\frac{m\overline{AC}}{m\overline{DF}} = \frac{m\overline{AB}}{m\overline{DE}}$$

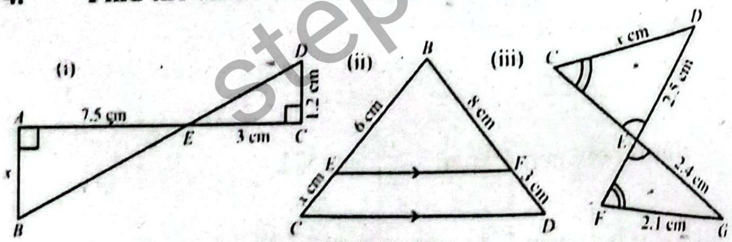
$$\frac{20}{m\overline{DE}} = \frac{12}{6}$$

$$m\overline{DF} = \frac{20 \times 6}{12} = 10$$

$$m\overline{DF} = 10$$

Thus, $m\overline{EF} = 8$ cm and $m\overline{DF} = 10$ cm

4. **Find the value of x in each of the following:**



Sol: (i) $\frac{x}{1.2} = \frac{7.5}{3}$

$$\therefore \quad x = \frac{7.5 \times 1.2}{3} = 3.0 \text{ cm}$$

(ii) EF // CD

$$\therefore \quad \frac{CE}{6} = \frac{DE}{8}$$

$$\frac{x}{6} = \frac{3}{8}$$

$$x = \frac{3}{8} \times 6 = \frac{9}{4} = 2.25$$

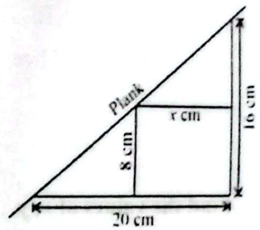
$$\therefore \quad x = 2.25 \text{ cm}$$

(iii) $\frac{x}{2.1} = \frac{25}{24}$

$$\therefore \quad x = \frac{2.1 \times 25}{24}$$

$$= 2.19 \text{ cm}$$

5. **A plank is placed straight upstairs that is 20 cm wide and 16 cm deep. A rectangular box of height 8 cm and width x cm is placed on a stair under the plank. Find the value of x.**



Sol: Δs AED and FTB are similar.

$$\therefore \quad \frac{20 - x}{8} = \frac{x}{8}$$

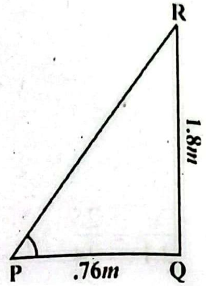
$$8x = 160 - 8x$$

$$8x + 8x = 160$$

$$16x = 160$$

$$x = 10 \text{ cm}$$

6. **A man who is 1.8 m tall casts a shadow of a 0.76m in length. If at the same time a telephone pole costs a 3m shadow, find the height of the pole.**



Sol: $\Delta PQR \cong \Delta ABC$

$m\angle A = m\angle P$

$m\angle B = m\angle Q = 90^o$

$BC = 3m$

$RQ = 1.8\ m$

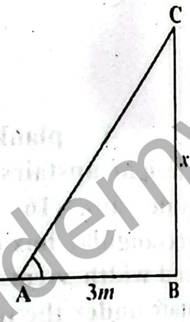
$PQ = .76\ m$

$AB = x\ m$

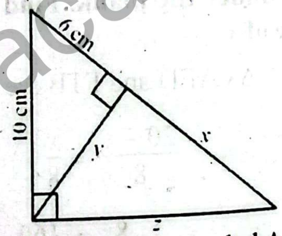
$$\frac{x}{3} = \frac{1.8 \times 3}{0.76}$$

$$\therefore \quad x = \frac{1.8 \times 3}{.76}$$

$$= 7.11\ m$$



7. **Find the values of x, y and z in the given figure.**



Sol: ABC is a right angled ΔABD is another right angled Δ.

In ΔABD

$\overline{AD} = 6cm$

$AB = 10\ cm$

$(BD)^2 = (AB)^2 - (AD)^2 = 100 - 36 = 64$

$\therefore \quad BD = y = 8cm.$

$\boxed{y = 8cm}$

$\Delta ABD \sim \Delta ABC$

$\angle ADB = BAC =$ Each 90^o

$\angle B \sim \angle B$ Common third angle

$\angle BAD = \angle C$

$\therefore \quad AB \sim BC; AD \sim AC; BD \sim AD$

$$\frac{AD}{AC} = \frac{BD}{AB}$$

$$\frac{8}{z} = \frac{6}{10}$$

$$\text{or } z = \frac{8 \times 10}{6} = \frac{80}{6}$$

$$= \frac{40}{3} = 13\frac{1}{3}$$

$$\boxed{z = 13\frac{1}{3}cm}$$

$$z^2 = x^2 + y^2$$

$$\frac{1600}{9} = x^2 + 64$$

$$x^2 = \frac{1600}{9} - 64$$

$$= \frac{1600 - 576}{9}$$

$$x^2 = \frac{1032}{9}$$

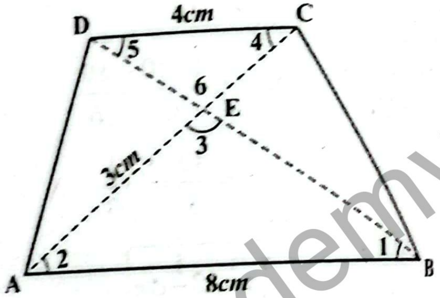
$$x = \frac{32}{3}$$

$$x = 10\frac{2}{3}\,cm$$

$$x = 10\frac{2}{3},\ y = 8,\ z = 13\frac{1}{3}$$

8. Draw an isosceles trapezoid $ABCD$ where $\overline{AB} \parallel \overline{CD}$ and $m\overline{AB} > m\overline{CD}$. Draw diagonals $\overline{AC}$ and $\overline{BD}$, intersecting at E. Prove that ΔABE is similar to ΔCDE. If $m\overline{AB} = 8$ cm, $m\overline{CD} = 4$ cm, and $m\overline{AE} = 3$ cm, find the length of $\overline{CE}$.

Sol:



1. Draw a line segment $\overline{AB}$ equal to 8cm.
2. Draw an other line segment CD equal to 4cm $\parallel$ to AB at any distance.
3. Join BC and DA.

ABCD is the required trapezium.

4. Join AC and BD diagonals crossing each other at E. Making angles $\angle 1$, $\angle 2$, $\angle 3$, $\angle 4$, $\angle 5$ and $\angle 6$ shown in the diagram.

Consider the Δs ABE and CDE.

$\angle 3 = \angle 6$ Vertically opposite angles.

$\angle 1 = \angle 5$ Alternate angles

$\angle 2 = \angle 4$ Alternate angles

$\Delta ABE \approx \Delta CDE$ ...(1)

$\frac{AB}{AE} = \frac{CD}{CE}$ Sides facing equal angles of similar triangle

$\frac{8}{AE} = \frac{4}{CE}$

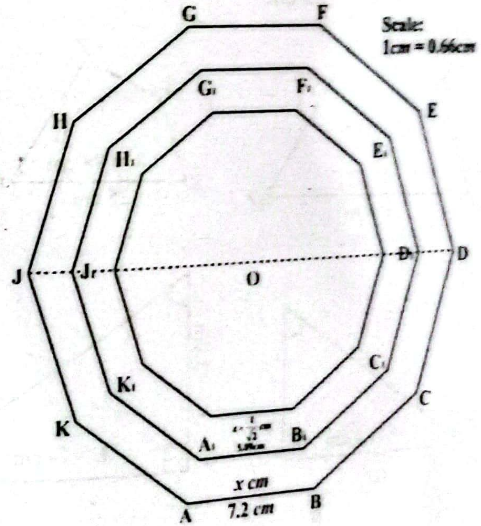
$\therefore\ CE = \frac{4 \times AE}{8} = \frac{\cancel{4} \times 3}{\cancel{8}_2} = \frac{3}{2} = 1.5$ ... (2)

Thus, the final answer is: $CE = 1.5$cm

9. A regular decagon has its side lengths decreased by a factor of $\frac{1}{\sqrt{2}}$. If the perimeter of the original decagon is 72*cm*. What is the side length of scaled decagon?

Sol: Parameter of the Decagon = 72 cm

No. of sides of Decagon = 10



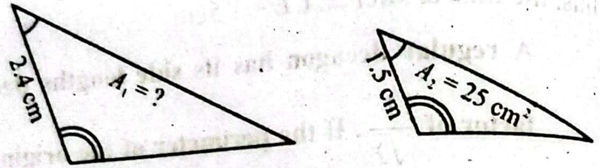
$$x = \frac{72}{12} \times \frac{1}{\sqrt{2}}$$

$$= \frac{6}{\sqrt{2}}$$

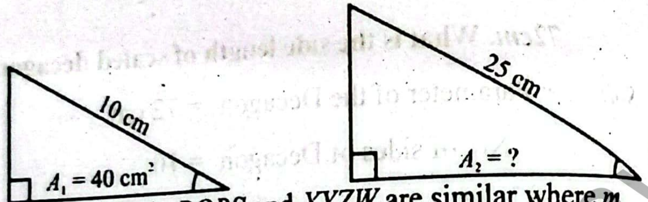
$$= \frac{6 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{6\sqrt{2}}{2} = 3\sqrt{2}\text{cm}$$

Example 6: Find the unknown value in the following:

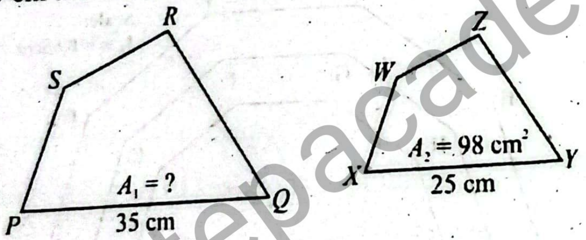
(i)



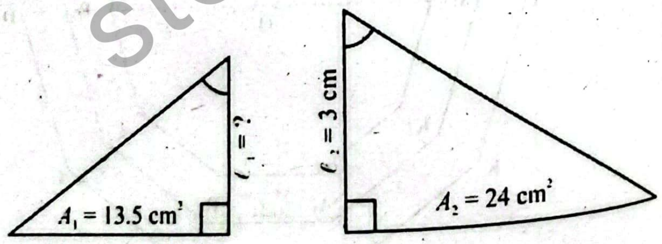
(ii)



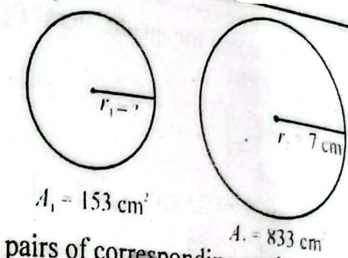
(iii) The quadrilaterals $PQRS$ and $XYZW$ are similar where $m\overline{PQ} = 35$ cm and $m\overline{XY} = 25$ cm.



(iv)



(v)



Solution: (i) Since two pairs of corresponding angles are equal i.e., triangles are similar. We use the formula for ratio of areas of similar figures.

$$\frac{A_1}{A_2} = \left(\frac{\ell_1}{\ell_2}\right)^2$$

Here $\ell_1 = 2.4$ cm, $\ell_2 = 1.5$ cm, $A_2 = 25$ cm², $A_1 = ?$

$$\frac{A_1}{25} = \left(\frac{2.4}{1.5}\right)^2$$

$$\frac{A_1}{25} = \left(\frac{8}{5}\right)^2$$

$$A_1 = \frac{64}{25} \times 25 = 64\text{cm}^2$$

(ii) Apply formula:

$$\frac{A_1}{A_2} = \left(\frac{\ell_1}{\ell_2}\right)^2$$

Here $\ell_1 = 10$ cm, $\ell_2 = 25$ cm, $A_1 = 40$ cm², $A_2 = ?$

$$\frac{40}{A_2} = \left(\frac{10}{25}\right)^2$$

$$\frac{40}{A_2} = \left(\frac{2}{5}\right)^2$$

$$\frac{40}{A_2} = \frac{4}{25}$$

$$A_2 = 40 \times \frac{25}{4} = 250\text{cm}^2$$

(iii) It is given that the quadrilateral $PQRS$ is similar to quadrilateral $XYZW$.

$$\frac{A_1}{A_2}=\left(\frac{\ell_1}{\ell_2}\right)^2$$

Here $\ell_1 = 35$ cm, $\ell_2 = 25$ cm, $A_1 = ?$, $A_2 = 98\text{ cm}^2$

$$\frac{A_1}{98}=\left(\frac{35}{25}\right)^2$$

$$\frac{A_1}{98}=\left(\frac{7}{5}\right)^2$$

$$A_1=\frac{49}{25}\times 98=192.08\text{cm}^2$$

(iv) Since two pairs of corresponding angles in both triangles are equal, so triangles are similar.

$$\therefore \quad \frac{A_1}{A_2}=\left(\frac{\ell_1}{\ell_2}\right)^2$$

Here $\ell_1 = ?$, $\ell_2 = 3$ cm, $A_1 = 13.5\text{ cm}^2$, $A_2 = 24\text{cm}^2$

$$\frac{13.5}{24}=\left(\frac{\ell_1}{3}\right)^2$$

$$\frac{135}{240}=\left(\frac{\ell_1}{3}\right)^2$$

$$\frac{9}{16}=\left(\frac{\ell_1}{3}\right)^2$$

$$\sqrt{\left(\frac{\ell_1}{3}\right)^2}=\sqrt{\frac{9}{16}} \quad \text{(Taking square root)}$$

$$\frac{\ell_1}{3}=\frac{3}{4}$$

$$\ell_1=\frac{9}{4}$$

$$=2.25cm$$

(v) For similar spheres

$$\frac{A_1}{A_2}=\left(\frac{\ell_1}{\ell_2}\right)^2$$

Here $r_1 = ?$, $r_2 = 7$ cm , $A_1 = 153\text{ cm}^2$, $A_2 = 833\text{ cm}^2$

$$\frac{153}{833}=\left(\frac{r_1}{7}\right)^2$$

$$\frac{9}{49}=\left(\frac{r_1}{7}\right)^2$$

$$\sqrt{\left(\frac{r_1}{7}\right)^2}=\sqrt{\frac{9}{49}} \quad \text{(Taking square root)}$$

$$\frac{r_1}{7}=\frac{3}{7} \quad \Rightarrow r_1 = 3\text{cm}$$

Example 7: Two polygons are similar with a ratio of corresponding sides being $\frac{3}{5}$. If the area of the smaller polygon is 54 cm^2, find the area of the larger polygon.

Solution: The ratio of the areas of two similar polygons is the square of the ratio of corresponding sides.

So, $\dfrac{\text{Area of larger polygon}}{\text{Area of smaller polygon}}=\left(\frac{5}{3}\right)^2=\frac{25}{9}$

Therefore, Area of larger polygon $=\frac{25}{9}\times 54=150\text{ cm}^2$

Example 8: Given that $\overline{BC} \parallel \overline{DE}$, prove that the triangles ABC and ADE are similar.

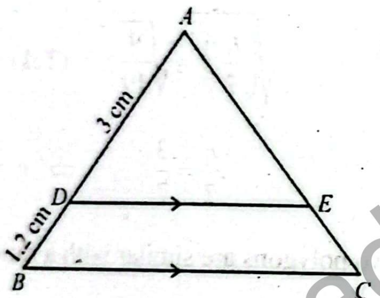
(i) If $m\overline{AB} = 3$ cm and $m\overline{BD} = 1.2$ cm, find the ratio of area of ΔABC to the area of ΔADE.

(ii) If area of ΔADE is 125 cm^2, find the area of ΔABC and area of trapezium $BCED$

Solution: Since $m\angle A = m\angle A$ (common), $m\angle B = m\angle D$ and $m\angle C = m\angle E$ (Corresponding angles of parallel lines $\overline{BC}$ and $\overline{DE}$).

Hence ΔABC is similar to ΔADE.

(i) Ratio of sides $= \dfrac{m\overline{AB}}{m\overline{AD}} = \dfrac{3+1.2}{3} = \dfrac{4.2}{3} = \dfrac{7}{5}$



$\therefore \quad \dfrac{\text{Area of } \Delta ABC}{\text{Area of } \Delta ADE} = \left(\dfrac{\ell_1}{\ell_2}\right)^2 = \left(\dfrac{7}{5}\right)^2 = \dfrac{49}{25}$

(ii) Area of $\Delta ADE = 125$ cm^2

$\dfrac{\text{Area of } \Delta ABC}{125} = \dfrac{49}{25}$

$\Rightarrow \quad$ Area of $\Delta ABC = \dfrac{49}{25} \times 125 = 245$ cm^2

Area of trapezium $BCED$ = Area of ΔABC – Area of ΔADE

$= 245 - 125 = 120$ cm^2

EXERCISE 9.2

1. Find the ratio of the areas of similar figures if the ratio of their corresponding lengths are:

(i) 1 : 3 (ii) 3 : 4 (iii) 2 : 7

(iv) 8 : 9 (v) 6 : 5

Sol: If the ratio of the length is $\dfrac{l_1}{l_2}$. Then ratio of their areas is

$\dfrac{A_1}{A_2} = \left(\dfrac{l_1}{l_2}\right)^2$

(i) Ratio in length is $1 : 3 = \dfrac{1}{3}$

Ratio in Areas $= \left(\dfrac{1}{3}\right)^2 = \dfrac{1}{3}$

Ratio is 1 : 9

(ii) Ratio 3 : 4 or $\dfrac{3}{4}$

Ratio in Areas $= \left(\dfrac{3}{4}\right)^2 = \dfrac{9}{16}$

Ratio in 9 : 16

(iii) Ratio in length $= 2 : 7 = \dfrac{2}{7}$

Ratio in Areas $= \left(\dfrac{2}{7}\right)^2 = \dfrac{4}{49}$

Ratio in 4 : 49

(iv) Ratio in length = 8 : 9 or $\dfrac{8}{9}$

Ratio in Areas $= \left(\dfrac{8}{9}\right)^2 = \dfrac{64}{81}$

Ratio = 64 : 81

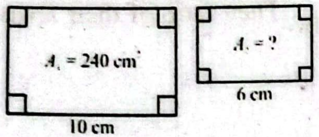
(v) Ratio in length = 6 : 5 or $\frac{6}{5}$

Ratio in Areas = $\left(\frac{6}{5}\right)^2 = \frac{36}{25}$

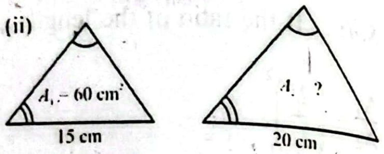
Ratio = 36 : 25

2. Find the unknowns in the following figures:

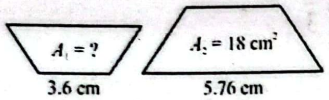
(i)



(ii)



(iii)



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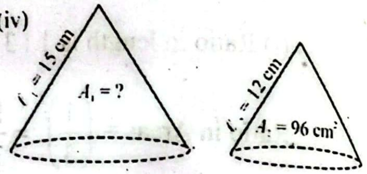

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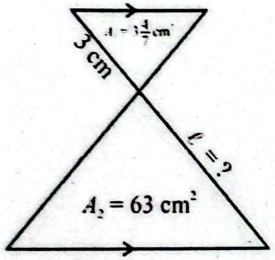
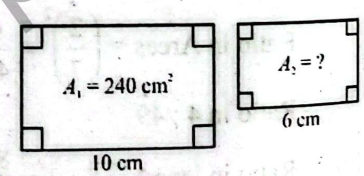


Sol: Ratio between lengths = $\frac{10}{6}$

(i) 1[st] Area A_1 = 240

2[nd] Area A_2 = ?

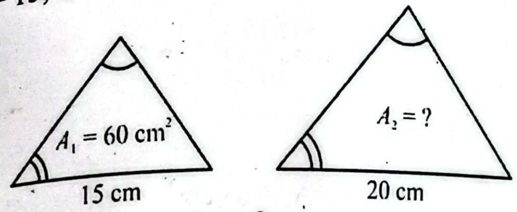


$\frac{A_1}{A_2} = \left(\frac{10}{6}\right)^2$

$\frac{240}{A_2} = \frac{100}{36}$

$A_2 = \frac{240 \times 36}{100}$ cm² = 86.4 cm²

(ii) $l_1 = 15, l_2 = 20$



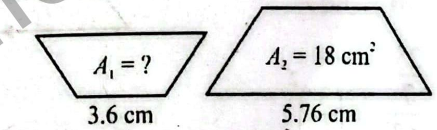
$A_1 = 60$, $A_2 = ?$

$\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2 = \left(\frac{15}{20}\right)^2 = \frac{225}{400}$

$\frac{60}{A_2} = \frac{225}{400}$, $A2 = \frac{60 \times 400}{225}$ cm²

$A_2 = 106.67\ cm^2$

(iii) $l_1 = 3.6 A_1 = ?$



$A_2 = 18$ cm² $l_2 = 5.76$

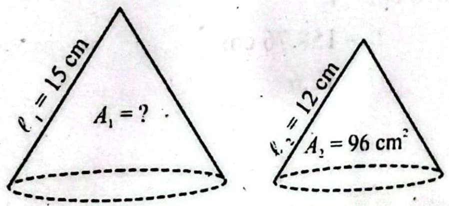
$\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2$

$A_1 = \left(\frac{l_1}{l_2}\right)^2 . A_2$

$= \left(\frac{3.6}{5.76}\right)^2 \times 18$

$A_1 = (.643)^2 \times 18 = 7.031\ cm^2$

(iv) $A_1 = ?$; $l_1 = 15, l_2 = 12$



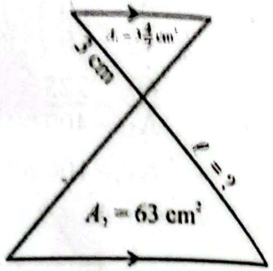
$A_2 = 96$

$$\frac{A_1}{A_2} = \left(\frac{15}{12}\right)^2 \quad \therefore A_1 = \left(\frac{15}{12}\right)^2 \times 96$$

$A_1 = 150\ cm^2$

(v) $A_1 = 3\frac{4}{7}\ cm^2$, $A_2 = 63\ cm^2$

Solution: $A_1 = 3\frac{4}{7}$, $l_1 = 3cm$



$A_1 = \frac{25}{7} cm^2, l_2 = ?$

$A_2 = 63cm$

Since figures are similar, therefore.

$$\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2$$

$$\frac{\frac{25}{7}}{63} = \left(\frac{3}{l_2}\right)^2$$

$$\frac{25}{7\times 63} = \frac{9}{l_2^2}$$

$$\frac{25}{441} = \frac{9}{l_2^2}$$

$$\Rightarrow \quad \frac{25}{441} = \frac{l_2^2}{9}$$

$$\frac{441}{25}\times 9 = l_2^2$$

$$\frac{3969}{25} = l_2^2$$

$158.76\ cm = l_2^2$

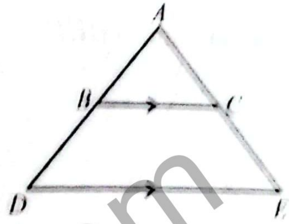
$\Rightarrow \quad l_2^2 = 158.76\ cm$

$\therefore \quad l = 12.6$

Given that area of $\Delta ABC = 36\ cm^2$ and $m\overline{AB} = 6\ cm$, $m\overline{BD} = 4\ cm$. Find

(a) the area of ΔADE

(b) the area of trapezium $BCED$



Sol: Area of $\Delta BC = 36cm^2 = A_1$

Measure of $AB = l_1 = 6cm$

Measure of $BD = l_2 = 4cm$

Find (a) Area of ADE Δ Area of Trap. DBCE.

Length of $AD = l_2$

$= AB + BD$

$= 6 + 4 = 10$

$\because \quad \overline{BC} \parallel DE$

$\therefore \quad \Delta ABC \approx \Delta ADE$

Hence $\frac{A_1}{A_2} = \left[\frac{l_1}{l_2}\right]^2$

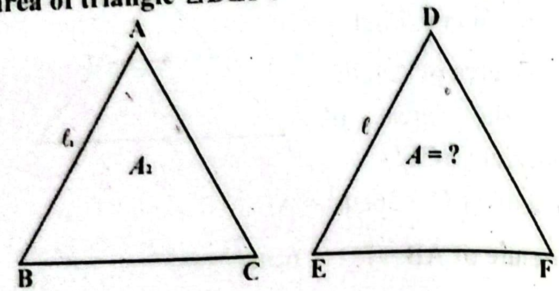
$$\frac{36}{A_2} = \left(\frac{6}{10}\right)^2 = \frac{36}{100}$$

$$\therefore \quad A_2 = \frac{36\times 100}{36} = 100$$

Now, $A_1 + BCED = 100\ cm^2$

$\therefore$ Area $BCED = 100 - 36 = 64\ cm^2$

4. **Given that ΔABC and ΔDEF are similar, with a scale factor of $k = 3$. If the area of ΔABC is $50cm^2$, find the area of triangle ΔDEF?**



Sol: $l_1 = 3l_2$

$$A_2 = 50 \text{ cm}^2$$

$$A_1 = ?$$

$$\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2 = (3)^2$$

$$\frac{A_1}{A_2} = \left(\frac{3\cancel{l_2}}{\cancel{l_2}}\right)^2 = (3)^2$$

$$A_1 = 9\,A_2 = 9 \times 50 = 450 \text{ cm}^2$$

5. **Quadrilaterals $ABCD$ and $EFGH$ are similar, with a scale factor of $k = \frac{1}{4}$. If the area of quadrilateral $ABCD$ is 64 cm^2, find the area of quadrilateral $EFGH$.**

Sol: Area ABCD $= 64 \text{ cm}^2 = A_1$

Area EFGH $= A_2 = ?$

Scale factor $k = \frac{1}{4}$

$$\therefore \quad \frac{A_1}{A_2} = k^2$$

$$\therefore \quad A_1 = \left(\frac{1}{4}\right)^2 = A_2$$

or $\quad A_2 = 16\,A_1 = 64 \times 16$

$$A_2 = 1024 \text{ cm}^2$$

6. **The areas of two similar triangles are $16cm^2$ and $25cm^2$. What is the ratio of a pair of corresponding sides?**

Sol: $A_1 = 16 \text{ cm}^2$; $A_2 = 25 \text{ cm}^2$

Let the corresponding lengths be l_1 and l_2

$$\left(\frac{l_1}{l_2}\right)^2 = \frac{16}{25} \quad \text{or} \quad \frac{l_1}{l_2} = \frac{4}{5}$$

Hence, scale factor $= \frac{4}{5}$.

7. **The areas of two similar triangles are $144cm^2$ and $81cm^2$. If the base of the large triangle is $30cm$, find the corresponding base of the smaller triangle.**

Sol: Area $\quad A_1 = 144$

$$A_2 = 81$$

Let the K factor $= x = \frac{l_1}{l_2}$

$$\frac{A_1}{A_2} = x^2$$

or $\quad x^2 = \frac{144}{81}$

$$x = \frac{12}{9} = \frac{l_1}{l_2}$$

Hence, scale factor is $\frac{12}{9}$.

$$l_1 = 30$$

$\therefore \quad \frac{l_1}{l_2} = 30 \quad$ or $\quad l_2 = \frac{9l_1}{12}$

or $\quad l_2 = \frac{{}^{3}\cancel{9} \times \cancel{30}^{15}}{\cancel{12}_{\cancel{4}\,2}} = \frac{45}{2}$

$$= 22.5 \text{ cm}$$

8. A regular heptagon is inscribed in a larger regular heptagon and each side of the larger heptagon is 1.7 times the side of the smaller heptagon. If the area of the smaller heptagon is known to be 100 cm^2, find the area of the larger heptagon.

Sol: Let the side of the smaller heptagon = l_1

= side of the larger heptagon = $1.7\, l_1$

$$l_2 = 1.7\, l_1$$

Area of smaller heptagon $A_1 = 100$

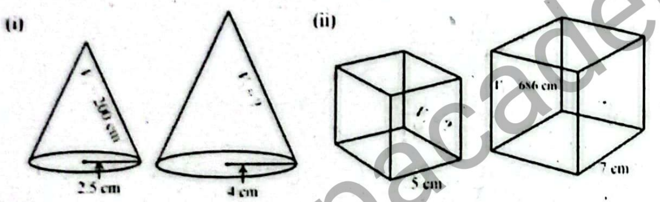
Then Area of bigger heptagon $= A_2$

$$\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2 = \left(\frac{l_1}{1.7\, l_1}\right)^2 = \left(\frac{1}{1.7}\right)^2$$

$$A_2 = \frac{A_1}{\left(\frac{1}{1.7}\right)^2} = 100 \times (1.7)^2$$

$$= 100 \times 2.89 = 289 \text{ cm}^2$$

Example 9: Find the unknown volume in the following similar solids:



Solution: (i) $\ell_1 = 2.5$ cm, $\ell_2 = 4$ cm

$$\frac{V_1}{V_2} = \left(\frac{\ell_1}{\ell_2}\right)^3$$

$$\frac{200}{V_2} = \left(\frac{2.5}{4}\right)^3$$

$$\frac{200}{V_2} = \left(\frac{5}{8}\right)^3$$

$$V_2 = 200 \times \frac{512}{125}$$

$$V_2 = 819.2 \text{ cm}^3$$

(ii) Using formula $\frac{V_1}{V_2} = \left(\frac{\ell_1}{\ell_2}\right)^3$

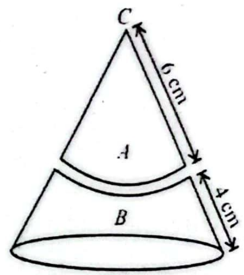
$$\frac{V_1}{686} = \left(\frac{5}{7}\right)^3 \qquad \begin{bmatrix} \ell_1 = 5 \text{ cm}, \ell_2 = 7 \text{ cm} \\ V_1 = ?, V_2 = 686 \text{ cm}^3 \end{bmatrix}$$

$$\frac{V_1}{686} = \frac{125}{343}$$

$$V_1 = \frac{125}{343} \times 686$$

$$= 250 \text{ cm}^3$$

Example 10: A solid cone C is cut into two pieces A and B with sloping edges 6 cm and 4cm. Find the ratio of:



(i) the diameters of the bases of the cones A and C.

(ii) the area of the bases of the cones A and C.

(iii) the volumes of the two cones A and C.

(iv) If volume of cone A is 72 cm^3, find the volume of solid B.

Solution: Let diameter of cone $A = d_1$

Diameter of cone $C = d_2$

(i) The ratios of the corresponding lengths are equal because of similarity of the cones.

$$\therefore \frac{d_1}{d_2} = \frac{\ell_1}{\ell_2} = \frac{6}{10}$$

$$= \frac{3}{5} \qquad \text{i.e., } \frac{\ell_1}{\ell_2} = \frac{3}{5}$$

(ii) $\dfrac{\text{Area of cone } A}{\text{Area of cone } C} = \left(\dfrac{\ell_1}{\ell_2}\right)^2$

$= \left(\dfrac{3}{5}\right)^2 = \dfrac{9}{25}$

(iii) $\dfrac{\text{Volume of cone } A}{\text{Volume of cone } C} = \left(\dfrac{\ell_1}{\ell_2}\right)^3$

$= \left(\dfrac{3}{5}\right)^3 = \dfrac{27}{125}$

(iv) V_1 = Volume of cone $A = 72 \text{ cm}^3$

V_2 = Volume of cone C = ?

$\therefore \quad \dfrac{V_1}{V_2} = \left(\dfrac{\ell_1}{\ell_2}\right)^3$

$\dfrac{72}{V_2} = \dfrac{27}{125}$

$V_2 = \dfrac{72 \times 125}{27} = 333\dfrac{1}{3} \text{ cm}^3$

Volume of solid B = Volume of cone C – Volume of cone A

$= 333\dfrac{1}{3} - 72 = 261\dfrac{1}{3} \text{ cm}^3$

Example 11: The mass of sack of rice is 50kg and height 4m. Find the mass of the similar sack of rice with height of 6m.

Solution: Mass of the smaller sack of rice $w_1 = 50$ kg

Height of smaller sack of rice $h_1 = 4m$

Mass of larger sack of rice w_2 = ?

Height of smaller sack of rice $h_2 = 6m$

Using formula $\dfrac{w_1}{w_2} = \left(\dfrac{h_1}{h_2}\right)^3$

$\dfrac{50}{w_2} = \left(\dfrac{4}{6}\right)^3$

$\dfrac{50}{w_2} = \dfrac{8}{27}$

$w_2 = \dfrac{27 \times 50}{8} = 168.75$ kg

Example 12: The ratio of the corresponding lengths of two similar cylindrical cans is 3 :2.

(i) The larger cylindrical can has surface area of 67.5 square metres. Find the surface area of the smaller cylindrical can.

(ii) The smaller cylindrical can has a volume of 132 cubic metres. Find the volume of larger tin can.

Solution: (i) Surface area of larger can = $A_1 = 67.5 \text{ m}^2$

Surface area of smaller can = A_2 = ?

Ratio of corresponding lengths is $\dfrac{\ell_1}{\ell_2} = \dfrac{3}{2}$

Using formula for areas of the similar figures:

$\dfrac{A_1}{A_2} = \left(\dfrac{\ell_1}{\ell_2}\right)^2$

$\dfrac{67.5}{A_2} = \left(\dfrac{3}{2}\right)^2 \quad \Rightarrow \quad A_2 = 67.5 \times \dfrac{4}{9} = 30 \; m^2$

(ii) Volume of smaller can = $V_2 = 132 \; m^3$

Volume of larger can = V_1 = ?

Using formula for volume of similar figures: $\dfrac{V_1}{V_2} = \left(\dfrac{\ell_1}{\ell_2}\right)^3$

$\dfrac{V_1}{132} = \left(\dfrac{3}{2}\right)^3 \quad \Rightarrow \quad V_1 = 132 \times \dfrac{27}{8} = 445.5 \; m^3$

EXERCISE 9.3

1. The radii of two spheres are in the ratio 3 : 4. What is the ratio of their volumes?

Sol: $\frac{V_1}{V_2} = \left(\frac{l_1}{l_2}\right)^3 = k^3, \frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2$

or $\frac{V_1}{V_2} = k^3$

$$\frac{r_1}{r_2} = \frac{3}{4}$$

$$\frac{V_1}{V_2} = \left(\frac{3}{4}\right)^3 = \frac{27}{64}$$

$$\therefore \quad V_1 : V_2 = \frac{27}{64}$$

2. Two regular tetrahedrons have volumes in the ratio 8:27. What is the ratio of their sides?

Sol: $V_1 : V_2 = 8 : 27$

$$\frac{V_1}{V_2} = \frac{8}{27}$$

If l_1 and l_2 are the sides of tetrahedrons then

$$\left(\frac{l_1}{l_2}\right)^3 = \frac{8}{27} \text{ or } \frac{l_1}{l_2} = \frac{2}{3}$$

3. Two right cones have volumes in the ratio 64 : 125. What is the ratio of:

(i) their heights (ii) their base areas?

Sol: $V_1 : V_2 = 64 : 125$

Volume of right circular cone of base radius r and height h is given by $V = \frac{1}{3}\pi r^2 h$.

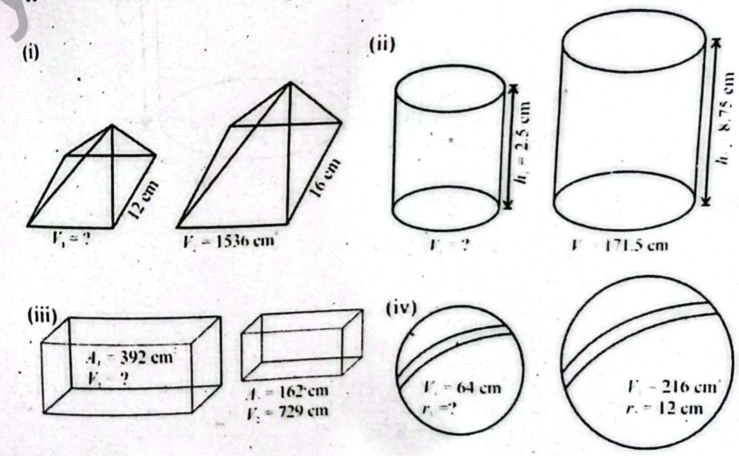
Ratio of height to $= h = \frac{3}{\pi r^2}$

$$\frac{V_1}{V_2} = \left(\frac{h_1}{h_2}\right)^3$$

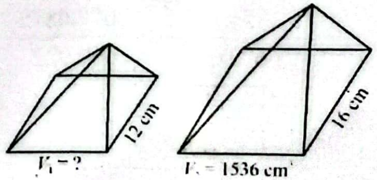
(i) or $\frac{h_1}{h_2} = \left(\frac{V_1}{V_2}\right)^{1/3} = \left(\frac{64}{125}\right)^{1/3} = \frac{4}{5}$

(ii) $\frac{A_1}{A_2} = \left(\frac{4}{5}\right)^2 = \frac{16}{25}$

4. Find the missing value in the following similar solids.



(i)



Sol: (i) $V_1 = ?$ $l_1 = 12$

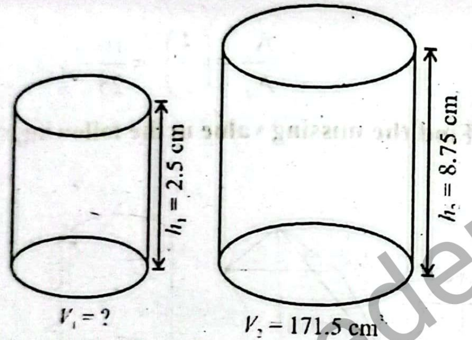
$V_2 = 1536$ $l_2 = 16$

$$\frac{V_1}{V_2} = \left(\frac{l_1}{l_2}\right)^3$$

$$V_1 = \left(\frac{l_1}{l_2}\right)^3 \times V_2 = \left(\frac{12}{16}\right)^3 \times 1536$$

$$V_2 = \frac{(12)^3}{(16)^3} \times 1536 = \frac{1728 \times 1536}{4096} = 648\ cm^3$$

(ii)



Sol: $l_1 = 2.5$ $l_2 = 8.75$

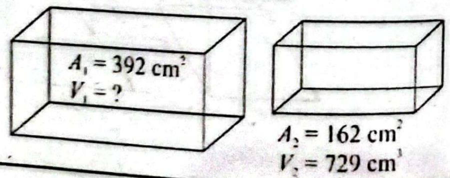
$V_1 = ?$ $V_2 = 171.5$

$$\frac{V_1}{V_2} = \left(\frac{l_1}{l_2}\right)^3$$

$$V_1 = \left(\frac{l_1}{l_2}\right)^3 \times V_2 = \left(\frac{2.5}{8.75}\right)^3 \times 171.5$$

$$= \frac{15.625 \times 171.5}{669.922} = \frac{2679.6875}{669.922} = 4\ cm^3$$

(iii)



Sol: $A_1 = 392$ $V_1 = ?$

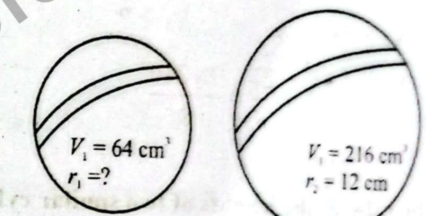
$A_2 = 162$ $V_2 = 729$

$$\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2 \quad \text{or} \quad \frac{l_1}{l_2} = \left(\frac{A_1}{A_2}\right)^{1/2}$$

$$\frac{V_1}{V_2} = \left(\frac{l_1}{l_2}\right)^3 = \left(\frac{A_1}{A_2}\right)^{3/2} = \left(\frac{392}{162}\right)^{3/2} = 2$$

$$V_1 = \left(\frac{392}{162}\right)^{3/2} \times 729 = 2744\ cm^3$$

(iv)



Sol: $V_1 = 64$ $r_1 = ?$

$V_2 = 216$ $r_2 = 12$

$$\left(\frac{r_1}{r_2}\right)^3 = \frac{64}{216}$$

$$\frac{r_1}{r_2} = \left(\frac{64}{216}\right)^{1/3} = \frac{4}{6} = \frac{2}{3}$$

$$r_1 = \frac{2}{3} \times 12 = 8\,\text{cm}$$

5. **The ratio of the corresponding lengths of two similar canonical cans is 3 : 2.**

(i) **The larger canonical can have surface area of 96 m^2. Find the surface area of the smaller canonical can.**

(ii) **The smaller canonical can have a volume of 240 m^3. Find the volume of larger canonical can.**

Sol: (i) $l_1 : l_2 = 3 : 2$ $\quad \frac{l_1}{l_2} = \frac{3}{2}$

$A_1 = 90$; $\quad A_2 = ?$

$$\left(\frac{3}{2}\right)^2 = \frac{A_1}{A_2} = A_2 = A_1 \times \left(\frac{2}{3}\right)^2$$

$$A_2 = {}^{10}\cancel{90} \times \frac{4}{\cancel{9}} = 40$$

$$96 \times \frac{4}{9} = 42.67\ m^2$$

(ii) $V_2 = 240$, $\quad V_1 = ?$

$$\frac{V_1}{V_2} = \left(\frac{3}{2}\right)^3 = \frac{27}{8} = \frac{7 \times 240}{8}$$

$$V_1 = 810\ m^3$$

6. **The ratio of the heights of two similar cylindrical water tanks is 5 : 3.**

(i) **If the surface area of the larger tank is 250 square metres, find the surface area of the smaller tank.**

(ii) **If the volume of the smaller tank is 270 cubic metres, find the volume of the larger tank.**

Sol: (i) $h : h = 5 : 3$ or $\frac{h_1}{h_2} = \frac{5}{3}$

$A_1 = 250$, $\quad A_2 = ?$

$$\frac{A_1}{A_2} = \left(\frac{h_2}{h_1}\right)^2 = \left(\frac{3}{5}\right)^2 = \frac{9}{25}$$

$$A_2 = \frac{9}{25} \times 250 = 90\ m^2$$

(ii) $V_2 = 270 \quad V_1 = ?$

$$\frac{V_1}{V_2} = \left(\frac{5}{3}\right)^3 = \frac{125}{27}$$

$$V_1 = \frac{125}{\cancel{27}} \times \cancel{270}^{10} = 1250\ m^3$$

Example 13: Find the measure of each interior angle of a regular pentagon.

Solution: Interior angle $= \frac{(n-2)\times 180^\circ}{n}$

$$= \frac{(5-2)\times 180^\circ}{5} = \frac{540^\circ}{5} = 108^\circ$$

Each exterior angle is: $\frac{360^\circ}{5} = 72^\circ$

Example 14: A tessellation is created using a combination of regular pentagons and decagons. Find the sum of the angles at a vertex where a pentagon and a decagon meet.

Solution:

Interior angle of regular decagon $= \frac{(n-2)\times 180^\circ}{n}$

$$= \frac{(10-2)\times 180^\circ}{10} = \frac{1440^\circ}{10} = 144^\circ$$

Interior angle of regular pentagon $= 108^\circ$

Sum of angles $= 144^\circ + 108^\circ = 252^\circ$. Since, angle sum 360°.

Tessellation cannot be done.

Example 15: A parallelogram-shaped room has a base of 10 metres and a height of 8m. Babar wants to carpet the room using rolls that cover 20 m^2 each. How many rolls of carpet do he need?

Solution:

The area of the parallelogram $= A =$ base $\times$ height $= 10 \times 8 = 80\text{m}^2$

Number of rolls needed: $\dfrac{80}{20} = 4$ rolls

Example 16: Find the area of the equilateral triangle ABC of side length s metres.

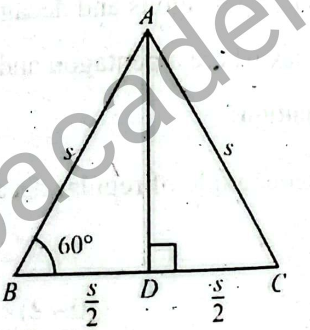
Solution: Draw perpendicular from A to side BC at point D. In the right angled triangle ABD:

Using trigonometric ratios: $\sin 60° = \dfrac{\text{Perpendicular}}{\text{Hypotenuse}}$

$$\frac{\sqrt{3}}{2} = \frac{m\overline{AD}}{s} \Rightarrow m\overline{AD} = \frac{\sqrt{3}}{2}s$$

$$\text{Area of triangle } ABC = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times s \times \frac{\sqrt{3}}{2}s$$

$$\text{Area of triangle } ABC = \frac{\sqrt{3}}{4}s^2$$



Example 17: Ali wants to create a floor design that uses regular hexagons (each with a side length of 1 metre) and equilateral triangles (each with a side length of 1 metre) to cover a rectangular area measuring 10 m by 5 m. Find how many hexagons and triangles Ali will need to complete the tessellation.

Solution: To find the area of an equilateral triangle with side length s, we can use the formula:

$$\text{Area of a triangle} = \frac{\sqrt{3}}{4} \cdot s^2$$

Multiple by 6 (since there are 6 triangles)

$$\text{Area of a hexagon} = \frac{6\sqrt{3}}{4} \cdot s^2 = \frac{3\sqrt{3}}{2} \cdot s^2$$

$$\text{Area of a hexagon} = \frac{3\sqrt{3}}{2} \times s^2 \approx \frac{3\sqrt{3}}{2} \times (1\text{ m})^2 \approx 2.598\text{ m}^2$$

$$\text{Area of an equilateral triangle} = \frac{\sqrt{3}}{4} \times s^2 \approx \frac{\sqrt{3}}{4} \times (1\text{ m})^2 \approx 0.433\text{ m}^2$$

Area of the rectangular floor $= 10\text{ m} \times 5\text{ m}$

$= 50\text{ m}^2$

Determine the arrangement: Assume a pattern where one hexagon is surrounded by 6 triangles. The area covered by one hexagon and the 6 surrounding triangles:

Total area covered by 1 hexagon and 6 triangles

$= 2.598\text{ m}^2 + 6 \times 0.433\text{ m}^2 \approx 2.598\text{ m}^2 + 2.598\text{ m}^2 = 5.196\text{ m}^2$

Calculate the total number of hexagons and triangles needed:

$$\text{Number of sets} = \frac{50\text{ m}^2}{5.196\text{ m}^2} \approx 9.62\text{ sets}$$

Rounding up, you can fit 10 sets of the pattern. Therefore, we need:

- Hexagons: 10
- Triangles: $10 \times 6 = 60$

Example 18: Falak plans to tile a square patio with an area of 100 square metres. He decides to use both square tiles and triangular tiles, each with an area of 0.25 square metres. If 60% of the tiles will be square and 40% will be triangular, how many tiles of each shape are needed?

Solution: $\text{Total number of tiles} = \dfrac{\text{Patio Area}}{\text{Tile Area}} = \dfrac{100}{0.25}$

$= 400$ tiles

Number of square tiles $= 400 \times 0.6 = 240$

Number of triangular tiles $= 400 \times 0.4 = 160$

EXERCISE 9.4

1. (i) What is the sum of the interior angles of a decagon (10-sided polygon)?
(ii) Calculate the measure of each interior angle of a regular hexagon,
(iii) What is each exterior angle of a regular pentagon?
(iv) If the sum of the interior angles of a polygon is 1260°, how many sides does the polygon have?

Sol: Interior angle of regular inside diagonals $\frac{n(x-3)}{2})$

Polygon $= \frac{180(x-2)}{n}$

Exterior angle $= \frac{360}{n}$

Where n is the number of rides

(i) $x = 10$ Interior angle $= \frac{(10-2)180}{10} = 144°$

Sum of all interior angles $= 1440°$

(ii) Hexagon n = 6 Each interior angle $= \frac{(6-2)180}{6}$

$= \frac{{}^{30}\cancel{180} \times 4}{\cancel{6}} = 120°$

(iii) Pentagon n = 5

Exterior angle $= \frac{360}{5} = 72°$

(iv) Total sum of interior angle = 1260°

$\therefore$ No. of sides $= (n-2)180°$

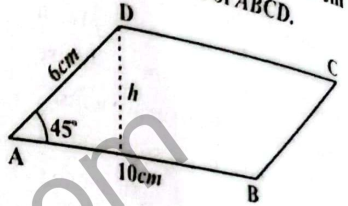
if n, then total sum $(n-2)180 = 1260$

$n - 2 = \frac{\cancel{126}\cancel{0}^{7}}{\cancel{18}\cancel{0}_{\cancel{3}}}$: $n = 7 + 2 = 9$

nine sided polygon.

2. **In a parallelogram $ABCD$, $m\overline{AB} = 10$ cm, $m\overline{AD} = 6$ cm and $m\angle BAD = 45°$. Calculate the area of $ABCD$.**

Sol: $\frac{h}{6} = \sin 45°$



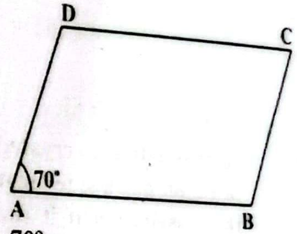
$h = \frac{6}{\sqrt{2}}$

Area $= 10 \times \frac{6}{\sqrt{2}}$

$= 30\sqrt{2}$

$= 42.42\ cm^2$

3. **In a parallelogram $ABCD$ if $m\angle DAB = 70°$, find the measures of all other angles in the parallelogram.**



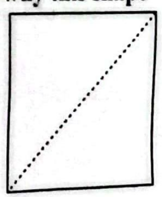
Sol: $\angle A = 70°$

$\angle C$ = opposite vertex = 70°

$\angle A + \angle B = 180$ $\therefore$ $\angle B = \angle D = 180 - 70$

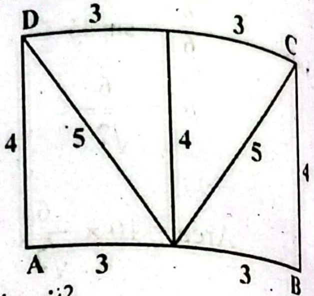
$\angle B = 110°$

4. **A shape is created by cutting a square in half diagonally and then attaching a right-angled triangle to the hypotenuse of each half. Explain why this shape can tessellate and calculate the interior angle of the new shape.**



Sol: Each half is again a right angle triangle. Hence, tessellate. a unit of 4 squares. Each joint of four covers 360°.

5. **A tessellation is created by repeatedly reflecting a basic shape. The basic shape is a right-angled triangle with sides of length 3, 4, and 5 units. Find: The minimum number of reflections needed to create a tessellation that covers a square with an area of 3600 square units.**



Sol: A unit of Tessellation is formed by joining 4 right angled Δs as shown in the diagram.

Total area of one unit $= 6m \times 4$ unit^2

$= 24$ unit^2

Total area $= 3600$ unit square

No. of units $= \dfrac{\overset{150}{\cancel{\overset{300}{\cancel{3600}}}}}{\cancel{24}_{\,2}} = 150$

No. of tessellation tiles $= 150 \times 4 = 600$

6. **A tessellation is created using regular hexagons. Each hexagon has a side length of 5cm. Find the total area of the tessellation if it consists of 25 hexagons and total perimeter of the outer edge of the tessellation, assuming it's a perfect hexagon.**

Sol: Area of a regular polygon of 6 sides (Hexagon) of side length $a = \dfrac{3\sqrt{3}a^2}{2}$

Side $= a = 5$cm

Area of 1 unit $= \dfrac{3\sqrt{3}\times 25}{2} = \dfrac{75\times\sqrt{3}}{2}$

Area of 25 units $= \dfrac{75\times\sqrt{3}\times 25}{2}$

$= \dfrac{187.5\times 1.732}{2}$

$= 1875 \times .866$ sq. cm $= 1623.8\ cm^2$

7. **A rectangular floor is 12m by 15m. How many square tiles, each $1m$ by $1m$, are needed to cover the floor?**

Sol: Area of Rectangular floor $= 12m \times 15m$

$= 180\ m^2$

Area of each role $= 1m \times 1m = 1m^2$

Number of tile $= \dfrac{180}{1} = 180$ tiles

8. **A rectangular wall is 10m tall and 120m wide. How many gallons of paint are needed to cover the wall, if one gallon covers $35m^2$?**

Sol: Area of the wall $= 10 \times 120 = 1200$m

Area covered by 1 gallon of petrol $= 35\ m^2$

Total no. of gallon of paints $= \dfrac{1200}{35} = 34.29$

Hence, 35 gallon of paint is the requirement.

9. **A rectangular wall has a length of 10 m and a width of 4 meters. If 1 litre of paint covers 7 m^2, how many liters of paint are needed to cover the wall?**

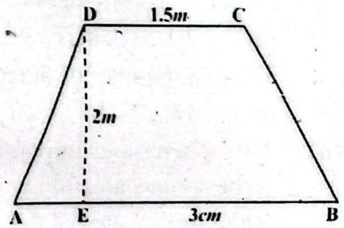
Sol: Area of wall $= 10 \times 4 = 40\ m^2$

Paint for $1m^2$ wall $= 7$ litres

Paint requirement $= \dfrac{40}{7} = 5.7$ litres

or 50 y = 6 lit.

10. **A window has a trapezoidal shape with parallel sides of $3m$ and $1.5m$ and a height of $2m$. Find the area of the window.**



Sol:

Area of the glass $= \left(\dfrac{3+1.5}{\cancel{2}}\right)\times \cancel{2}$

$= 4.5$ Sq.m.

REVIEW EXERCISE 9

1. Four options are given against each statement. Encircle the correct one.

(i) If two polygons are similar, then:
 (a) their corresponding angles are equal.
 (b) their areas are equal.
 (c) their volumes are equal.
 (d) their corresponding sides are equal.

(ii) The ratio of the areas of two similar polygons is:
 (a) equal to the ratio of their perimeters.
 (b) equal to the square of the ratio of their corresponding sides.
 (c) equal to the cube of the ratio of their corresponding sides.
 (d) equal to the sum of their corresponding sides.

(iii) If the volume of two similar solids is 125 cm^3 and 27 cm^3, the ratio of their corresponding heights is ________.
 (a) 3:5 (b) 5:3 (c) 25:9 (d) 9:25

(iv) The exterior angle of regular pentagon is:
 (a) 40° (b) 45° (c) 60° (d) 72°

(v) A parallelogram has an area of 64 cm^2 and a similar parallelogram has an area of 144 cm^2. If a side of the smaller parallelogram is 8 cm, the corresponding side of the larger parallelogram is:
 (a) 10 cm (b) 12 cm (c) 18 cm (d) 16 cm

(vi) The total number of diagonals in a polygon with 9 sides is:
 (a) 18 (b) 21 (c) 25 (d) 27

(vii) Two spheres are similar, and their radii are in the ratio 4:5. If the surface area of the larger sphere is 500K cm^2, what is the surface area of the smaller sphere?
 (a) 256π cm^2 (b) 320π cm^2 (c) 400π cm^2 (d) 405π cm^2

(viii) A regular polygon has an exterior angle of 30°. How many diagonals does the Polygon have?
 (a) 54 (b) 90 (c) 72 (d) 108

(ix) In a regular hexagon, the ratio of the length of a diagonal to the side length is:
 (a) $\sqrt{3}:1$ (b) 2 : 1 (c) 3 : 2 (d) 2 : 3

(x) A regular polygon has an interior angle of 165°. How many sides does it have?
 (a) 15 (b) 16 (c) 20 (d) 24

ANSWERS:

(i)	a	(ii)	b	(iii)	b	(iv)	d	(v)	b
(vi)	d	(vii)	b	(viii)	a	(ix)	b	(x)	d

2. If the sum of the interior angles of a polygon is 1080°, how many sides does the polygon has?

Sol: Sum of the interior angles of the polygon = $(n-2)180°$

Sum is given as 1080°

$$(n-2)180° = 1080°$$

$$n-2 = \frac{1080°}{180°} = 6$$

$$\therefore \quad n = 8$$

3. Two similar bottles are such that one is twice as high as the other. What is the ratio of their surface areas and their capacities?

Sol: $l_1 = l$; $l_2 = 2l$

$A_1 : A_2$ $\quad \frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2$

$$= \left(\frac{l}{2l}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

$$\frac{A_1}{A_2} = \frac{1}{4} \qquad A_1 : A_2 = 1 : 4$$

$$\frac{V_1}{V_2} = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

$V_1 : V_2 = 1 : 8$

4. **Each dimension of a model car is $\frac{1}{10}$ of the corresponding car dimension. Find the ratio of:**
   (a) **the areas of their wind screens**
   (b) **the capacities of their boots**
   (c) **the widths of the cars**
   (d) **the number of wheels they have.**

Sol: (a) Winds screen areas.

$$A_1 : A_2 = \left(\frac{1}{10}\right)^2 = 1:100$$

(*l*) Capacity of Boots

$$C_1 : C_2 = \left(\frac{1}{10}\right)^3 = 1:1000$$

(c) The width of the cars.

$$W_1 : W_2 = \left(\frac{1}{10}\right) = 1:10$$

(d) Number wheels same. 1 : 1

5. **Three similar jugs have heights 8 cm, 12 cm and 16 cm. If the smallest jug holds $\frac{1}{2}$ litre, find the capacities of the other two.**

Sol: $h_1 = 8$ cm, $h_2 = 12$ cm, $h_3 = 16$cm

$V_1 = \frac{1}{2}$ lit. $V_2 = ?$ $\qquad V_3 = ?$

$$\frac{V_2}{V_1} = \left(\frac{h_2}{h_1}\right)^3$$

$$\frac{V_2}{V_1} = \left(\frac{12}{8}\right)^3 \text{ or } V_2 = \left(\frac{12}{8}\right)^3 \times \frac{1}{2} \text{ litres}$$

$$= \frac{864}{512} \text{ litres}$$

$V_2 = 1.69$ litres

$$\frac{V_3}{V_1} = \left(\frac{h_3}{h_1}\right)^3 \text{ or } V_3 = \left(\frac{h_3}{h_1}\right)^3 \times V_1$$

$$= \left(\frac{16}{8}\right)^3 \times V_1$$

$$V_3 = 8 \times \frac{1}{2} \text{ litres} = 4 \text{ litres}$$

6. **Three similar drinking glasses have heights 7.5*cm*, 9*cm* and 10.5cm. If the tallest glass holds 343 millilitres, find the capacities of the other two.**

Sol: $h_1 = 7.5$cm $\quad h_2 = 9$cm $\quad h_3 = 10.5$cm

$V_3 = 343$ *ml*

Find V_1 and V_2.

$$\frac{V_3}{V_1} = \left(\frac{h_3}{h_1}\right)^3 \quad \therefore V_1 = V_3 \times \left(\frac{h_3}{h_1}\right)^3$$

$$= 343 \times \left(\frac{7.5}{10.5}\right)^3$$

$V_1 = 343 \times (.714)^3$

$= 343 \times .364 = 124.9$ *ml*

$$V_2 = V_3 \times \left[\frac{h_2}{h_3}\right]^3 = 125 \text{ ml}$$

$$= 343\left[\frac{9}{10.5}\right]^3$$

$= 343\ (.857)^3$

$= 343 \times 6294$

$V_2 = 215.89$ $\qquad$ i.e. 216 *ml*

7. **A toy manufacturer produces model cars which are similar in every way to the actual cars. If the ratio of the door area of the model to the door area of the car is 1 cm to 2500 cm, find:**

(a) **the ratio of their lengths**

(b) **the ratio of the capacities of their petrol tanks**

(c) **the width of the model, if the actual car is 150cm wide**

(d) **the area of the rear window of the actual car if the area of the rear window of the model is 3cm².**

Sol: Let the area of model = A_1

Area of actual car = A_2

$$A_1 : A_2 = 1 : 2500$$

Let of the model = l_1, let of actual.

$$\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2$$

(a) $$\frac{l_1}{l_2} = \left(\frac{A_1}{A_2}\right)^{1/2} = \left(\frac{1}{2500}\right)^{1/2} = \frac{1}{50}$$

$$l_1 : l_2 = 1 : 50$$

(b) Ratio of capacities of petrol tank

$$\frac{V_1}{V_2} = \left(\frac{l_1}{l_2}\right)^3 = \left(\frac{1}{50}\right)^3 = \frac{1}{125000}$$

$$V_1 : V_2 = 1 : 125000$$

(c) Width of Model = W_1

Width of Actual = W_2 = 150 cm

$$\frac{W_1}{W_2} = \frac{l_1}{l_2} \quad \text{or} \quad W_2 = \frac{l_1}{l_2}$$

$$= 150 \qquad W_1 = {}^{3}\cancel{150} \times \frac{1}{\cancel{50}}$$

$$= 3 \text{ cm}$$

(d) $$\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2 = \left(\frac{1}{50}\right)^2 = \frac{1}{2500}$$

$$\frac{3}{A_2} = \frac{1}{2500} \quad \text{or} \quad A_2 = 7500\text{cm}^2$$

8. **The ratio of the areas of two similar labels on two similar jars of coffee is 144 : 169. Find the ratio of**

(a) **the heights of the two jars**

(b) **their capacities.**

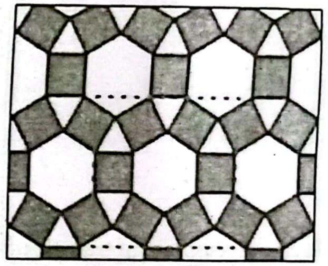
Sol: $A_1 : A_2 = 144 : 169$

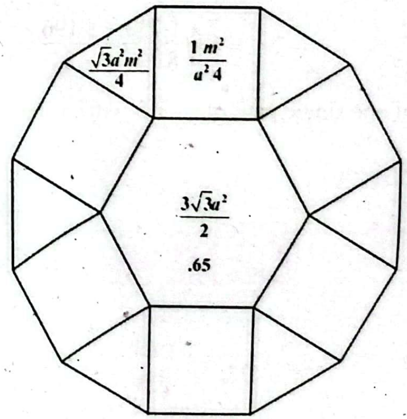
$$\left(\frac{h_1}{h_2}\right)^2 = \frac{A_1}{A_2} \quad \frac{144}{169}$$

(a) $$\frac{h_1}{h_2} = \frac{12}{13} \quad \text{or} \quad 12 : 13$$

(b) $$\frac{V_1}{V_2} = \left(\frac{12}{13}\right)^3 = \frac{1728}{2197} \quad \text{or} \quad 1728 : 2197$$

9. **A tessellation of tiles on a floor has been made using a repeating pattern of a regular hexagon, six squares and six equilateral triangles. Find the total area of a single pattern with side length** $\frac{1}{2}$ metre of each polygon.





Sol: Area of 1 square $= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} m^2$

Area of 6 squares $= 6 \times \frac{1}{4} = \frac{3}{2} m^2$

Area of one hexagon $= \frac{3\sqrt{3} \times \left(\frac{1}{2}\right)^2}{2} = \frac{3\sqrt{3}}{8}$

$\frac{3 \times 1.732}{8} = \frac{5.196}{8} = .65$

Area of 1 equivalent triangle $= \frac{3\sqrt{3}\ a^2}{4}$

$= \frac{3\sqrt{3} . \left(\frac{1}{2}\right)^2}{4}$

$= \frac{\sqrt{3}}{16} = \frac{1.732}{16} = .10825$

Area of 6 equilateral triangles$= \frac{\sqrt{3} \times 6}{16} = \frac{3\sqrt{3}}{8}$

$= \frac{3 \times 1.732}{8} = \frac{5.196}{8} = .65$

Area of one single pattern: $1.5 + .65 + .65 = 2.8m^2$