

Chapter # 1

Real Numbers

Exercise # 1.2

Question # 1: Rationalize the denominator of the following:

$$\begin{aligned}
 \text{(i). } & \frac{13}{4+\sqrt{3}} \\
 &= \frac{13}{4+\sqrt{3}} \times \frac{4-\sqrt{3}}{4-\sqrt{3}} \quad \because a^2 - b^2 = (a+b)(a-b) \\
 &= \frac{13(4-\sqrt{3})}{(4)^2 - (\sqrt{3})^2} \\
 &= \frac{13(4-\sqrt{3})}{16-3} \\
 &= \frac{13(4-\sqrt{3})}{13} \\
 &= 4 - \sqrt{3} \quad \text{(Answer)}
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii). } & \frac{\sqrt{2}-1}{\sqrt{5}} \\
 &= \frac{\sqrt{2}-1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} \\
 &= \frac{(\sqrt{2} \times \sqrt{5} - \sqrt{5})}{(\sqrt{5})^2} \\
 &= \frac{(\sqrt{10} - \sqrt{5})}{5} \quad \text{(Answer)}
 \end{aligned}$$

$$\begin{aligned}
 \text{(v). } & \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} \\
 &= \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} \\
 &= \frac{(\sqrt{3}-\sqrt{2})^2}{(\sqrt{3})^2 - (\sqrt{2})^2} \quad \because (a-b)^2 = a^2 + b^2 - 2ab \\
 & \quad \quad \quad a^2 - b^2 = (a+b)(a-b) \\
 &= \frac{(\sqrt{3})^2 + (\sqrt{2})^2 - 2(\sqrt{3})(\sqrt{2})}{3-2} \\
 &= \frac{3+2-2\sqrt{3} \times \sqrt{2}}{1} \\
 &= 5 - 2\sqrt{6} \quad \text{(Answer)}
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii). } & \frac{\sqrt{2}+\sqrt{5}}{\sqrt{3}} \\
 &= \frac{\sqrt{2}+\sqrt{5}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \\
 &= \frac{(\sqrt{2} \times \sqrt{3} + \sqrt{5} \times \sqrt{3})}{(\sqrt{3})^2} \\
 &= \frac{(\sqrt{6} + \sqrt{15})}{3} \quad \text{(Answer)}
 \end{aligned}$$

$$\begin{aligned}
 \text{(iv). } & \frac{6-4\sqrt{2}}{6+4\sqrt{2}} \\
 &= \frac{6-4\sqrt{2}}{6+4\sqrt{2}} \times \frac{6-4\sqrt{2}}{6-4\sqrt{2}} \\
 &= \frac{(6-4\sqrt{2})^2}{(6)^2 - (4\sqrt{2})^2} \quad \because (a-b)^2 = a^2 + b^2 - 2ab \\
 & \quad \quad \quad a^2 - b^2 = (a+b)(a-b) \\
 &= \frac{(6)^2 + (4\sqrt{2})^2 - 2(6)(4\sqrt{2})}{36 - (16 \times 2)} \\
 &= \frac{36 + (16 \times 2) - 48\sqrt{2}}{36 - 32} \\
 &= \frac{36 + 32 - 48\sqrt{2}}{4} \\
 &= \frac{68 - 48\sqrt{2}}{4} \\
 &= \frac{4(17 - 12\sqrt{2})}{4} \\
 &= 17 - 12\sqrt{2} \quad \text{(Answer)}
 \end{aligned}$$

$$\begin{aligned}
 \text{(vi). } & \frac{4\sqrt{3}}{\sqrt{7}+\sqrt{5}} \\
 &= \frac{4\sqrt{3}}{\sqrt{7}+\sqrt{5}} \times \frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}-\sqrt{5}} \\
 &= \frac{4\sqrt{3}(\sqrt{7}-\sqrt{5})}{(\sqrt{7})^2 - (\sqrt{5})^2} \quad \because a^2 - b^2 = (a+b)(a-b) \\
 &= \frac{4\sqrt{3}(\sqrt{7}-\sqrt{5})}{7-5} \\
 &= \frac{4\sqrt{3}(\sqrt{7}-\sqrt{5})}{2} \\
 &= 2\sqrt{3}(\sqrt{7} - \sqrt{5}) \quad \text{(Answer)}
 \end{aligned}$$

Question # 2: Simplify the following:

$$\begin{aligned}
 \text{(i). } & \left(\frac{81}{16}\right)^{-\frac{3}{4}} \\
 &= \left(\frac{16}{81}\right)^{\frac{3}{4}} \\
 &= \frac{2^{\cancel{4} \times \frac{3}{4}}}{3^{\cancel{4} \times \frac{3}{4}}} \\
 &= \frac{2^3}{3^3} \\
 &= \frac{8}{27} \quad \text{(Answer)}
 \end{aligned}$$

2	16
2	8
2	4
2	2
	1

3	81
3	27
3	9
3	3
	1

$$\begin{aligned}
 \text{(ii). } & \left(\frac{3}{4}\right)^{-2} \div \left(\frac{4}{9}\right)^3 \times \frac{16}{27} \\
 &= \left(\frac{4}{3}\right)^2 \div \frac{4^3}{3^{2 \times 3}} \times \frac{4^2}{3^3} \\
 &= \frac{4^2}{3^2} \times \frac{3^6}{4^3} \times \frac{4^2}{3^3} \\
 &= 4^{2+2-3} \times 3^{6-2-3} \\
 &= 4 \times 3 \\
 &= 12 \quad \text{(Answer)}
 \end{aligned}$$

3	81
3	27
3	9
3	3
	1

$$\begin{aligned}
 \text{(iii). } & (0.027)^{-\frac{1}{3}} \\
 &= \left(\frac{27}{1000}\right)^{-\frac{1}{3}} \\
 &= \left(\frac{1000}{27}\right)^{\frac{1}{3}} \\
 &= \left(\frac{10^3}{3^3}\right)^{\frac{1}{3}} \\
 &= \frac{10^{3 \times \frac{1}{3}}}{3^{3 \times \frac{1}{3}}} \\
 &= \frac{10}{3} \quad \text{(Answer)}
 \end{aligned}$$

3	27
3	9
3	3
	1

$$\begin{aligned}
 \text{(iv). } & \sqrt[7]{\frac{x^{14} \times y^{21} \times z^{35}}{y^{14} z^7}} \\
 &= (x^{14} y^{21-14} z^{35-7})^{\frac{1}{7}} \\
 &= (x^{14} y^7 z^{28})^{\frac{1}{7}} \\
 &= x^{14 \times \frac{1}{7}} y^{7 \times \frac{1}{7}} z^{28 \times \frac{1}{7}} \\
 &= x^2 y z^4 \quad \text{(Answer)}
 \end{aligned}$$

$$\begin{aligned}
 \text{(v). } & \frac{5 \cdot (25)^{n+1} - 25 \cdot (5)^{2n}}{5 \cdot (5)^{2n+3} - (25)^{n+1}} \\
 &= \frac{5 \cdot 5^{2(n+1)} - 5^2 \cdot 5^{2n}}{5 \cdot 5^{2n+3} - 5^{2(n+1)}} \\
 &= \frac{5 \cdot 5^{2n+2} - 5^{2n+2}}{5 \cdot 5^{2n+2} - 5^{2n+2}} \\
 &= \frac{5^{2n+2}(5-1)}{5^{2n+2}(5-1)} \\
 &= \frac{4}{25-1} \\
 &= \frac{4}{24} \\
 &= \frac{1}{6} \quad \text{(Answer)}
 \end{aligned}$$

$$\begin{aligned}
 \text{(vi). } & \frac{(16)^{x+1} + 20(4^{2x})}{(2)^{x-3} \times 8^{x+2}} \\
 &= \frac{(2^4)^{x+1} + 20(2^{2 \times 2x})}{2^{x-3} \times 2^{3(x+2)}} \\
 &= \frac{2^{4x+4} + 20(2^{4x})}{2^{x-3} \times 2^{3x+6}} \\
 &= \frac{2^{4x} \cdot 2^4 + 20(2^{4x})}{2^{x-3+3x+6}} \\
 &= \frac{2^{4x}(2^4 + 20)}{2^{4x+3}} \\
 &= \frac{2^4(16+20)}{2^3} \\
 &= \frac{36}{8} \\
 &= \frac{9}{2} \quad \text{(Answer)}
 \end{aligned}$$

$$\begin{aligned}
 \text{(vii). } & (64)^{\frac{-2}{3}} \div (9)^{\frac{-3}{2}} \\
 &= (4^3)^{\frac{-2}{3}} \div (3^2)^{\frac{-3}{2}} \\
 &= 4^{-2} \div 3^{-3} \\
 &= \frac{4^{-2}}{3^{-3}} \\
 &= \frac{3^3}{4^2} \\
 &= \frac{27}{16} \quad \text{(Answer)}
 \end{aligned}$$

$$\begin{aligned}
 \text{(viii). } & \frac{3^n \times 9^{n+1}}{3^{n-1} \times 9^{n-1}} \\
 &= \frac{3^n \times 3^{2(n+1)}}{3^{n-1} \times 3^{2(n-1)}} \\
 &= \frac{3^{2n+2}}{3^{-1} \times 3^{2n-2}}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{3^{2n} \times 3^2 \times 3^1}{3^{2n} \times 3^{-2}} \\
 &= 3^{2+1+2} \\
 &= 3^5 = 243 \quad (\text{Answer})
 \end{aligned}$$

$$\begin{aligned}
 \text{(ix). } & \frac{5^{n+3} - 6 \times 5^{n+1}}{9 \times 5^n - 4 \times 5^n} \\
 &= \frac{5^n \times 5^3 - 6 \times 5^n \times 5^1}{5^n(9 - 4)} \\
 &= \frac{5^n(5^3 - 6 \times 5)}{5^n(9 - 4)} \\
 &= \frac{125 - 30}{5} \\
 &= \frac{95}{5} \\
 &= 19 \quad (\text{Answer})
 \end{aligned}$$

Question # 3: If $x = 3 + \sqrt{8}$ then find the value of:

$$\begin{aligned}
 \frac{x}{1} &= \frac{3 + \sqrt{8}}{1} \\
 \frac{1}{x} &= \frac{1}{3 + \sqrt{8}} \times \frac{3 - \sqrt{8}}{3 - \sqrt{8}} \\
 &= \frac{3 - \sqrt{8}}{3^2 - (\sqrt{8})^2} \\
 &= \frac{3 - \sqrt{8}}{9 - 8} \\
 &= \frac{3 - \sqrt{8}}{1} \\
 \frac{1}{x} &= 3 - \sqrt{8}
 \end{aligned}$$

$$\begin{aligned}
 \text{(i) } x + \frac{1}{x} &= 3 + \sqrt{8} + 3 - \sqrt{8} \\
 &= 6 \quad (\text{Answer})
 \end{aligned}$$

$$\begin{aligned}
 \text{(ii) } x - \frac{1}{x} &= 3 + \sqrt{8} - (3 - \sqrt{8}) \\
 &= \cancel{3} + \sqrt{8} - \cancel{3} + \sqrt{8} \\
 &= 2\sqrt{8} \quad (\text{Answer})
 \end{aligned}$$

$$\begin{aligned}
 \text{(iii) } x^2 + \frac{1}{x^2} & \\
 \because x + \frac{1}{x} &= 6
 \end{aligned}$$

Taking square on both sides

$$\begin{aligned}
 \left(x + \frac{1}{x}\right)^2 &= 6^2 \\
 (x)^2 + \left(\frac{1}{x}\right)^2 + 2(x)\left(\frac{1}{x}\right) &= 36 \\
 x^2 + \frac{1}{x^2} + 2 &= 36 \\
 x^2 + \frac{1}{x^2} &= 36 - 2 \\
 x^2 + \frac{1}{x^2} &= 34 \quad (\text{Answer})
 \end{aligned}$$

$$\begin{aligned}
 \text{(iv) } x^2 - \frac{1}{x^2} & \\
 x^2 - \frac{1}{x^2} &= \left(x + \frac{1}{x}\right)\left(x - \frac{1}{x}\right) \\
 &= (6)(2\sqrt{8}) \\
 &= 12\sqrt{8} \quad (\text{Answer})
 \end{aligned}$$

$$\begin{aligned}
 \text{(v) } x^4 + \frac{1}{x^4} & \\
 \because x^2 + \frac{1}{x^2} &= 34 \\
 \text{Taking square on both sides} & \\
 \left(x^2 + \frac{1}{x^2}\right)^2 &= (34)^2 \\
 (x^2)^2 + \left(\frac{1}{x^2}\right)^2 + 2(x^2)\left(\frac{1}{x^2}\right) &= 1156 \\
 x^4 + \frac{1}{x^4} + 2 &= 1156 \\
 x^4 + \frac{1}{x^4} &= 1156 - 2 \\
 x^4 + \frac{1}{x^4} &= 1154 \quad (\text{Answer})
 \end{aligned}$$

$$\begin{aligned}
 \text{(vi) } \left(x - \frac{1}{x}\right)^2 & \\
 &= (2\sqrt{8})^2 \\
 &= 4 \times 8 \\
 &= 32 \quad (\text{Answer})
 \end{aligned}$$

Question # 4: Find the rational number

P and Q such that: $\frac{8-3\sqrt{2}}{4+3\sqrt{2}} = p + q\sqrt{2}$

$$\begin{aligned}
 \frac{8-3\sqrt{2}}{4+3\sqrt{2}} \times \frac{4-3\sqrt{2}}{4-3\sqrt{2}} &= p + q\sqrt{2} \\
 \frac{8(4-3\sqrt{2}) - 3\sqrt{2}(4-3\sqrt{2})}{4^2 - (3\sqrt{2})^2} &= p + q\sqrt{2} \\
 \frac{32 - 24\sqrt{2} - 12\sqrt{2} + (3\sqrt{2})^2}{16 - (9 \times 2)} &= p + q\sqrt{2} \\
 \frac{32 - 36\sqrt{2} + (9 \times 2)}{16 - 18} &= p + q\sqrt{2} \\
 \frac{32 - 36\sqrt{2} + 18}{-2} &= p + q\sqrt{2}
 \end{aligned}$$

$$\frac{50-36\sqrt{2}}{-2} = p + q\sqrt{2}$$

$$\frac{50}{-2} - \frac{36\sqrt{2}}{-2} = p + q\sqrt{2}$$

$$-25 + 18\sqrt{2} = p + q\sqrt{2}$$

By comparing we get,

$$p = -25 \text{ and } q = 18$$

Question # 5: Simplify the following:

$$(i). \frac{(25)^{\frac{3}{2}} \times (243)^{\frac{3}{5}}}{(16)^{\frac{4}{4}} \times (8)^{\frac{3}{3}}}$$

$$= \frac{(5^2)^{\frac{3}{2}} \times (3^5)^{\frac{3}{5}}}{(2^4)^{\frac{4}{4}} \times (2^3)^{\frac{3}{3}}}$$

$$= \frac{5^3 \times 2^3}{2^5 \times 2^4}$$

$$= \frac{125 \times 27}{32 \times 16}$$

$$= \frac{3375}{512} \quad (\text{Answer})$$

3	243	2	16
3	81	2	8
3	27	2	4
3	9	2	2
3	3		1
	1		

$$(ii). \frac{54 \times \sqrt[3]{(27)^{2x}}}{9^{x+1} + 216(3^{2x-1})}$$

$$= \frac{54 \times (3^3)^{\frac{2x}{3}}}{3^{2(x+1)} + 216(3^{2x} \times 3^{-1})}$$

$$= \frac{54 \times 3^{2x}}{3^{2x+2} + \frac{216(3^{2x})}{3}}$$

$$= \frac{54 \times 3^{2x}}{3^{2x} \times 3^2 + 72(3^{2x})}$$

$$= \frac{54 \times 3^{2x}}{3^{2x}(9+72)}$$

$$= \frac{54}{81}$$

$$= \frac{2}{3} \quad (\text{Answer})$$

$$(iii). \sqrt{\frac{(216)^{\frac{2}{3}} \times (25)^{\frac{1}{2}}}{(0.04)^{\frac{-3}{2}}}}$$

$$= \sqrt{\frac{(2^3 \times 3^3)^{\frac{2}{3}} \times (5^2)^{\frac{1}{2}}}{\left(\frac{4}{100}\right)^{\frac{-3}{2}}}}$$

$$= \sqrt{\frac{(2^3)^{\frac{2}{3}} \times (3^3)^{\frac{2}{3}} \times 5^1}{\left(\frac{100}{4}\right)^{\frac{3}{2}}}}$$

2	216
2	108
2	54
3	27
3	9
3	3
	1

$$= \sqrt{\frac{2^2 \times 3^2 \times 5}{(25)^{\frac{3}{2}}}}$$

$$= \sqrt{\frac{2^2 \times 3^2 \times 5}{(5^2)^{\frac{3}{2}}}}$$

$$= \sqrt{\frac{2^2 \times 3^2 \times 5}{5^3}}$$

$$= \sqrt{\frac{2^2 \times 3^2}{5^{3-1}}}$$

$$= \sqrt{\frac{2^2 \times 3^2}{5^2}}$$

$$= \frac{2 \times 3}{5}$$

$$= \frac{6}{5} \quad (\text{Answer})$$

$$(iv). \left(a^{\frac{1}{3}} + b^{\frac{2}{3}}\right) \left(a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{2}{3}} + b^{\frac{4}{3}}\right)$$

$$\because a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$= \left(a^{\frac{1}{3}} + b^{\frac{2}{3}}\right) \left(\left(a^{\frac{1}{3}}\right)^2 - a^{\frac{1}{3}}b^{\frac{2}{3}} + \left(b^{\frac{2}{3}}\right)^2\right)$$

$$= \left(a^{\frac{1}{3}}\right)^3 + \left(b^{\frac{2}{3}}\right)^3$$

$$= a + b^2 \quad (\text{Answer})$$