Chapter # 2

Logarithms

Exercise # 2.4

Question # 1: Without using calculator, evaluate the following:

(i). $\log_2 18 - \log_2 9$

$= \log_2(2 \times 9) - \log_2 9$

$= \log_2 2 + \cancel{\log_2 9} - \cancel{\log_2 9}$

$= \log_2 2$ $\quad \because \log_a a = 1$

$= 1$ (Answer)

(ii). $\log_2 64 + \log_2 2$

2	64
2	32
2	16
2	8
2	4
2	2
	1

$= \log_2 2^6 + \log_2 2$

$= 6\log_2 2 + \log_2 2$ $\quad \because \log_a a = 1$

$= 6(1) + 1$

$= 6 + 1 = 7$ (Answer)

(iii). $\frac{1}{3}\log_3 8 - \log_3 18$

2	18
3	9
3	3
	1

$= \frac{1}{3}\log_3 2^3 - \log_3(2 \times 3^2)$

$= \log_3(2^{\cancel{3}})^{\frac{1}{\cancel{3}}} - (\log_3 2 + \log_3 3^2)$

$= \cancel{\log_3 2} - \cancel{\log_3 2} - 2\log_3 3$

$= -2\log_3 3$ $\quad \because \log_a a = 1$

$= -2(1) = -2$ (Answer)

(iv). $2\log 2 + \log 25$

$= \log 2^2 + \log 25$

$= \log 4 + \log 25$

$= \log 4 \times 25$

$= \log 100$

$= \log 10^2$

$= 2\log 10$ $\quad \because \log_{10} 10 = 1$

$= 2(1) = 2$ (Answer)

(v). $\frac{1}{3}\log_4 64 + 2\log_5 25$

4	64
4	16
4	4
	1

$= \frac{1}{3}\log_4 4^3 + 2\log_5 5^2$

$= \log_4(4^{\cancel{3}})^{\frac{1}{\cancel{3}}} + 2 \times 2\log_5 5$

$= \log_4 4 + 4\log_5 5$ $\quad \because \log_a a = 1$

$= 1 + 4(1)$

$= 1 + 4 = 5$ (Answer)

(vi). $\log_3 12 + \log_3 0.25$

$= \log_3(3 \times 4) + \log_3\left(\frac{\cancel{25}^{1}}{\cancel{100}_{4}}\right)$

$= \log_3 3 + \log_3 4 + \log_3\left(\frac{1}{4}\right)$ $\quad \because \log_a a = 1$

$= 1 + \cancel{\log_3 4} + \log_3 1 - \cancel{\log_3 4}$

$= 1 + \log_3 1$ $\quad \because \log_a 1 = 0$

$= 1 + 0 = 1$ (Answer)

Question # 2: Write the following as a single logarithm:

(i). $\frac{1}{2}\log 25 + 2\log 3$

$= \log(25)^{\frac{1}{2}} + \log 3^2$

$= \log(\sqrt{25} \times 9)$

$= \log(5 \times 9)$

$= \log 45$ (Answer)

(ii). $\log 9 - \log\frac{1}{3}$

$= \log\frac{9}{1/3}$

$= \log\frac{9 \times 3}{1}$

$= \log 27$ (Answer)

(iii). $\log_5 b^2 . \log_a 5^3$

$= 2\log_5 b \times 3\log_a 5$ $\quad \because \log_b x = \frac{\log_a x}{\log_a b}$

$= 2 \times 3\left(\frac{\log b}{\cancel{\log 5}} \times \frac{\cancel{\log 5}}{\log a}\right)$

$= 6\left(\frac{\log b}{\log a}\right)$ $\quad \because \frac{\log_a x}{\log_a b} = \log_b x$

$= 6\log_a b$ (Answer)

(iv). $2\log_3 x + \log_3 y$

$= \log_3 x^2 + \log_3 y$

$= \frac{\log x^2}{\log 3} + \frac{\log y}{\log 3}$

$= \frac{\log x^2 + \log y}{\log 3}$

$= \frac{\log x^2 y}{\log 3}$ $\quad \because \frac{\log_a x}{\log_a b} = \log_b x$

$= \log_3 x^2 y$ (Answer)

(v). $4\log_5 x - \log_5 y + \log_5 z$

$= \log_5 x^4 + \log_5 z - \log_5 y$

$= \log_5 \frac{x^4 z}{y}$ (Answer)

(vi). $2\ln a + 3\ln b - 4\ln c$

$= \ln a^2 + \ln b^3 - \ln c^4$

$= \ln \frac{a^2 b^3}{c^4}$ (Answer)

Question # 3: Expand the following using laws of logarithms:

(i). $\log\left(\frac{11}{5}\right)$

$= \log 11 - \log 5$ (Answer)

(ii). $\log_5 \sqrt{8a^6}$

$= \log_5 (2^3 a^6)^{\frac{1}{2}}$

$= \log_5 \left(2^{3\times\frac{1}{2}}\, a^{\overset{3}{\cancel{6}}\times\frac{1}{\cancel{2}}}\right)$

$= \log_5 2^{\frac{3}{2}}\ a^3$

$= \log_5 2^{\frac{3}{2}} + \log_5 a^3$

$= \frac{3}{2}\log_5 2 + 3\log_5 a$ (Answer)

(v). $\ln \sqrt[3]{16x^3}$

2	16
2	8
2	4
2	2
	1

$= \ln (2^4 x^3)^{\frac{1}{3}}$

$= \ln 2^{4\times\frac{1}{3}}\, x^{\cancel{3}\times\frac{1}{\cancel{3}}}$

$= \ln 2^{\frac{4}{3}}\, x$

$= \ln 2^{\frac{4}{3}} + \ln x$

$= \frac{4}{3}\ln 2 + \ln x$ (Answer)

(iii). $\ln\left(\frac{a^2 b}{c}\right)$

$= \ln a^2 + \ln b - \ln c$

$= 2\ln a + \ln b - \ln c$ (Answer)

(iv). $\log\left(\frac{xy}{z}\right)^{\frac{1}{9}}$

$= \frac{1}{9}\left[\log\left(\frac{xy}{z}\right)\right]$

$= \frac{1}{9}(\log x + \log y - \log z)$ (Answer)

(vi). $\log_2\left(\frac{1-a}{b}\right)^5$

$= 5\log_2\left(\frac{1-a}{b}\right)$

$= 5[\log_2(1-a) - \log_2 b]$ (Answer)

Question # 4: Evaluate the value of 'x' in the following equations:

(i). $\log 2 + \log x = 1$

$\log 2x = 1$

$10^1 = 2x$

$\frac{\overset{5}{\cancel{10}}}{\cancel{2}} = x$

$x = 5$ (Answer)

(iii). $(81)^x = (243)^{x+2}$

3	243
3	81
3	27
3	9
3	3
	1

$(3^4)^x = (3^5)^{x+2}$

$\cancel{3}^{4x} = \cancel{3}^{5x+10}$

$4x = 5x + 10$

$4x - 5x = 10$

$-x = 10$

$x = -10$ (Answer)

(v). $\log(5x - 10) = 2$

$10^2 = 5x - 10$

(ii). $\log_2 x + \log_2 8 = 5$

$\log_2 8x = 5$

$2^5 = 8x$

$32 = 8x$

$\frac{\overset{4}{\cancel{32}}}{\cancel{8}} = x$

$x = 4$ (Answer)

(iv). $\left(\frac{1}{27}\right)^{x-6} = 27$

$(27^{-1})^{x-6} = 27$

$27^{-x+6} = 27^1$

$-x + 6 = 1$

$-x = 1 - 6$

$-x = -5$

$x = 5$ (Answer)

(vi). $\log_2(x+1) - \log_2(x-4) = 2$

$$100 = 5x - 10$$
$$100 + 10 = 5x$$
$$110 = 5x$$
$$\frac{\cancel{110}^{22}}{\cancel{5}} = x$$
$$x = 22 \qquad \text{(Answer)}$$

$$\log_2\left(\frac{x+1}{x-4}\right) = 2$$
$$2^2 = \frac{x+1}{x-4}$$
$$4(x-4) = x+1$$
$$4x - 16 = x + 1$$
$$4x - x = 1 + 16$$
$$3x = 17$$
$$x = \frac{17}{3}$$
$$x = 5\frac{2}{3} \qquad \text{(Answer)}$$

Question # 5: Find the value of the following with the help of logarithm table:

(i). $\frac{\mathbf{3.68 \times 4.21}}{\mathbf{5.234}}$

Let,

$$x = \frac{3.68\times4.21}{5.234}$$

Taking 'log' on both sides

$$logx = log\left(\frac{3.68\times4.21}{5.234}\right)$$
$$logx = log3.68 + log4.21 - log5.234$$
$$logx = 0.5658 + 0.6243 - 0.7188$$
$$logx = 0.4713$$

Taking '$antilog$' on both sides

$$\cancel{antilog} \times \cancel{log}x = anti\log 0.4713$$
$$x = 2.9601 \qquad \text{(Answer)}$$

(ii). $\mathbf{4.67 \times 2.11 \times 2.397}$

Let,

$$x = 4.67 \times 2.11 \times 2.397$$

Taking 'log' on both sides

$$logx = \log(4.67 \times 2.11 \times 2.397)$$
$$logx = log4.67 + log2.11 + log2.397$$
$$logx = 0.6693 + 0.3243 + 0.3797$$
$$logx = 1.3733$$

Taking '$antilog$' on both sides

$$\cancel{antilog} \times \cancel{log}x = antilog1.3733$$
$$x = 23.6194 \qquad \text{(Answer)}$$

(iii). $\frac{\mathbf{(20.46)^2 \times (2.4122)}}{\mathbf{754.3}}$

Let,

$$x = \frac{(20.46)^2\times(2.4122)}{754.3}$$

Taking 'log' on both sides

$$logx = log\left[\frac{(20.46)^2\times(2.4122)}{754.3}\right]$$
$$logx = 2log20.46 + log2.4122 - log754.3$$
$$logx = 2(1.3109) + 0.3824 - 2.8775$$
$$logx = 2.6218 + 0.3824 - 2.8775$$
$$logx = 0.1267$$

Taking '$antilog$' on both sides

$$\cancel{antilog} \times \cancel{log}x = antilog0.1267$$
$$x = 1.3388 \qquad \text{(Answer)}$$

(iv). $\frac{\mathbf{\sqrt[3]{9.364} \times 21.64}}{\mathbf{3.21}}$

Let,

$$x = \frac{(9.364)^{1/3}\times21.64}{3.21}$$

Taking 'log' on both sides

$$logx = log\left[\frac{(9.364)^{1/3} \times 21.64}{3.21}\right]$$
$$logx = \frac{1}{3}log9.364 + log21.64 - log3.21$$
$$logx = \frac{1}{3}(0.9715) + 1.335 - 0.5065$$
$$logx = 0.3238 + 1.335 - 0.5065$$
$$logx = 1.1523$$

Taking '$antilog$' on both sides

$$\cancel{antilog} \times \cancel{log}x = antilog1.1523$$
$$x = 14.2003 \qquad \text{(Answer)}$$

Question # 6: The formula to measure the magnitude of earthquakes is given by $M = \log_{10}\left(\frac{A}{A_o}\right)$. If amplitude ($A$) is 10,000 and reference amplitude (A_o) is 10. What is the magnitude of the earthquake?

$$M = \log_{10}\left(\frac{A}{A_o}\right)$$

$$\text{put } A = 10000,\ A_o = 10$$

$$M = \log_{10}\left(\frac{10000}{10}\right)$$

$$M = \log_{10}(1000)$$

$$M = \log_{10} 10^3$$

$$M = 3\log_{10} 10 \qquad \because \log_a a = 1$$

$$M = 3(1)$$

$$M = 3 \qquad \text{(Answer)}$$

Question # 7: Abdullah invested Rs. 1,00,000 in a saving scheme and gains interest at the rate of 5% per annum so that the total value of this investment after t years is Rs y. This is modelled by an equation $y = 1,00,000(1.05)^t,\ t \geq 0$. Find after how many years the investment will be doubled?

$$\text{Investment Amount} = 100000 \text{ Rs}$$

$$\text{Rate} = 5\%$$

$$\text{Double Investment Year} = y = 200000 \text{ Rs}$$

$$\text{Time} = t = ?$$

$$\because y = 100000(1.05)^t$$

$$200000 = 100000(1.05)^t$$

$$\frac{200000}{100000} = (1.05)^t$$

$$2 = (1.05)^t$$

Taking 'log' on both sides

$$\log 2 = \log(1.05)^t$$

$$0.3010 = t(log 1.05)$$

$$0.3010 = t(0.0212)$$

$$t = \frac{0.3010}{0.0212}$$

$$t = 14 \text{ years} \qquad \text{(Answer)}$$

Question # 8: Huria is hiking up a mountain where the temperature (T) decreases by 3% (or a factor of 0.97) for every 100 meters gained in altitude. The initial temperature (T_i) at sea level is 20$^{\text{o}}$ C. Using the formula $T = T_i \times 0.97^{\frac{h}{100}}$, calculate the temperature at an altitude (h) of 500 meters.

$$\text{Initial Temperature} = T_i = 20^oC$$

$$\text{Altitude} = h = 500m$$

$$\text{Final Temperature} = T = ?$$

$$\because T = T_i \times (0.97)^{h/_{100}}$$

$$T = 20 \times (0.97)^{500/_{100}} \qquad \because h = 500$$

$$T = 20 \times (0.97)^5$$

Taking 'log' on both sides

$$\log T = log[20 \times (0.97)^5]$$

$$\log T = log20 + 5log0.97$$

$$\log T = 1.3010 + 5(-0.0132)$$

$$logT = 1.3010 - 0.066$$

$$logT = 1.235$$

Taking '$antilog$' on both sides

$$\cancel{antilog} \times \cancel{log}T = antilog1.235$$

$$T = 17.18^oC \qquad \text{(Answer)}$$