

Unit 4

Factorization and Algebraic Manipulation

EXERCISE 4.1

1. Factorize by identifying common factors.

(i) $6x + 12$

(ii) $15y^2 + 20y$

(iii) $-12x^2 - 3x$

(iv) $4a^2b + 8ab^2$

(v) $xy - 3x^2 + 2x$

(vi) $3a^2b - 9ab^2 + 15ab$

Solution:

(i) $6(x + 2)$

(ii) $5y(3y + 4)$

(iii) $-3x(4x + 1)$

(iv) $4ab(a + 2b)$

(v) $x(y - 3x + 2)$

(vi) $3ab(a - 3b + 5)$

2. Factorize and represent pictorially:

(i) $5x + 15$

(ii) $x^2 + 4x + 3$

(iii) $x^2 + 6x + 8$

(iv) $x^2 + 4x + 4$

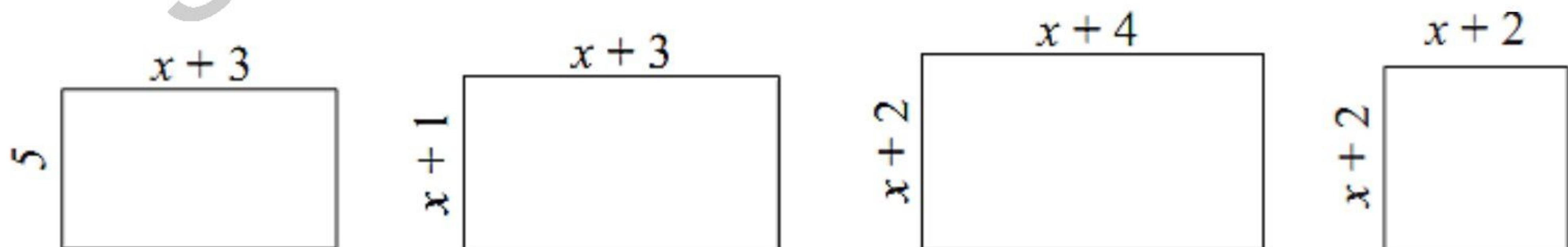
Solution

(i) $5(x+3)$

(ii) $x^2 + 4x + 3 = x^2 + 3x + x + 3 = x(x + 3) + 1(x + 3) = (x + 1)(x + 3)$

(iii) $x^2 + 6x + 8 = x^2 + 4x + 2x + 8 = x(x + 4) + 2(x + 4) = (x + 2)(x + 4)$

(iv) $x^2 + 4x + 4 = x^2 + 2x + 2x + 4 = x(x + 2) + 2(x + 2) = (x + 2)(x + 2) = (x + 2)^2$



3. Factorize:

- (i) $x^2 + x - 12$ (ii) $x^2 + 7x + 10$ (iii) $x^2 - 6x + 8$
(iv) $x^2 - x - 56$ (v) $x^2 - 10x - 24$ (vi) $y^2 + 4y - 12$
(vii) $y^2 + 13y + 36$ (viii) $x^2 - x - 2$

Solution

- (i) $x^2 + x - 12 = x^2 + 4x - 3x - 12 = x(x + 4) - 3(x + 4) = (x + 4)(x - 3)$
(ii) $x^2 + 7x + 10 = x^2 + 5x + 2x + 10 = x(x + 5) + 2(x + 5) = (x + 5)(x + 2)$
(iii) $x^2 - 6x + 8 = x^2 - 4x - 2x + 8 = x(x - 4) - 2(x - 4) = (x - 4)(x - 2)$
(iv) $x^2 - x - 56 = x^2 - 8x + 7x - 56 = x(x - 8) + 7(x - 8) = (x - 8)(x + 7)$
(v) $x^2 - 10x - 24 = x^2 - 12x + 2x - 24 = x(x - 12) + 2(x - 12) = (x - 12)(x + 2)$
(vi) $y^2 + 4y - 12 = y^2 + 6y - 2y - 12 = y(y + 6) - 2(y + 6) = (y + 6)(y - 2)$
(vii) $y^2 + 13y + 36 = y^2 + 9y + 4y + 36 = y(y + 9) + 4(y + 9) = (y + 9)(y + 4)$
(viii) $x^2 - x - 2 = x^2 - 2x + x - 2 = x(x - 2) + 1(x - 2) = (x - 2)(x + 1)$

4. Factorize:

- (i) $2x^2 + 7x + 3$ (ii) $2x^2 + 11x + 15$ (iii) $4x^2 + 13x + 3$
(iv) $3x^2 + 5x + 2$ (v) $3y^2 - 11y + 6$ (vi) $2y^2 - 5y + 2$
(vii) $4z^2 - 11z + 6$ (viii) $6 + 7x - 3x^2$

Solution

- (i) $2x^2 + 7x + 3 = 2x^2 + 6x + x + 3 = 2x(x + 3) + 1(x + 3) = (2x + 1)(x + 3)$
(ii) $2x^2 + 11x + 15 = 2x^2 + 6x + 5x + 15 = 2x(x + 3) + 5(x + 3) = (2x + 5)(x + 3)$
(iii) $4x^2 + 13x + 3 = 4x^2 + 12x + x + 3 = 4x(x + 3) + 1(x + 3) = (4x + 1)(x + 3)$
(iv) $3x^2 + 5x + 2 = 3x^2 + 3x + 2x + 2 = 3x(x + 1) + 2(x + 1) = (3x + 2)(x + 1)$
(v) $3y^2 - 11y + 6 = 3y^2 - 9y - 2y + 6 = 3y(y - 3) - 2(y - 3) = (3y - 2)(y - 3)$
(vi) $2y^2 - 5y + 2 = 2y^2 - 4y - y + 2 = 2y(y - 2) - 1(y - 2) = (2y - 1)(y - 2)$
(vii) $4z^2 - 11z + 6 = 4z^2 - 8z - 3z + 6 = 4z(z - 2) - 3(z - 2) = (4z - 3)(z - 2)$
(viii) $6 + 7x - 3x^2 = -3x^2 + 7x + 6 = -3x^2 + 9x - 2x + 6$
 $= 3x(-x + 3) + 2(-x + 3) = (3x + 2)(3 - x)$

EXERCISE 4.2

1. Factorize each of the following expressions:

(i) $4x^4 + 81y^4$ (ii) $a^4 + 64b^4$ (iii) $x^4 + 4x^2 + 16$

(iv) $x^4 - 14x^2 + 1$ (v) $x^4 - 30x^2y^2 + 9y^4$ (vi) $x^4 - 7x^2y^2 + y^4$

Solution

1.(i) $4x^4 + 81y^4$

$$= (2x^2)^2 + (9y^2)^2 = (2x^2)^2 + (9y^2)^2 + 2(2x^2)(9y^2) - 2(2x^2)(9y^2)$$

$$= (2x^2 + 9y^2)^2 - 36x^2y^2 = (2x^2 + 9y^2)^2 - (6xy)^2$$

$$= (2x^2 + 9y^2 - 6xy)(2x^2 + 9y^2 + 6xy)$$

1.(ii) $a^4 + 64b^4$

$$= (a^2)^2 + (8b^2)^2 = (a^2)^2 + (8b^2)^2 + 2(a^2)(8b^2) - 2(a^2)(8b^2)$$

$$= (a^2 + 8b^2)^2 - 16a^2b^2 = (a^2 + 8b^2)^2 - (4ab)^2$$

$$= (a^2 + 8b^2 + 4ab)(a^2 + 8b^2 - 4ab)$$

1.(iii) $x^4 + 4x^2 + 16$

$$= x^4 + 8x^2 - 4x^2 + 16 = x^4 + 8x^2 + 16 - 4x^2$$

$$= (x^2 + 4)^2 - (2x)^2$$

$$= (x^2 + 4 - 2x)(x^2 + 4 + 2x)$$

1.(iv) $x^4 - 14x^2 + 1$

$$= x^4 - 16x^2 + 2x^2 + 1 = x^4 + 2x^2 + 1 - 16x^2$$

$$= (x^2 + 1)^2 - (4x)^2$$

$$= (x^2 + 1 - 4x)(x^2 + 1 + 4x)$$

$$1.(v) x^4 - 30x^2y^2 + 9y^4$$

$$= x^4 - 36x^2y^2 + 6x^2y^2 + 9y^4 = x^4 + 6x^2y^2 + 9y^4 - 36x^2y^2$$

$$= (x^2 + 3y^2)^2 - (6xy)^2$$

$$= (x^2 - 6xy + 3y^2)(x^2 + 6xy + 3y^2)$$

$$1.(vi) x^4 - 7x^2y^2 + y^4$$

$$= x^4 - 9x^2y^2 + 2x^2y^2 + y^4 = x^4 + 2x^2y^2 + y^4 - 9x^2y^2$$

$$= (x^2 + y^2)^2 - (3xy)^2$$

$$= (x^2 - 3xy + y^2)(x^2 + 3xy + y^2)$$

2. Factorize each of the following expressions:

$$(i) (x+1)(x+2)(x+3)(x+4) + 1 \quad (ii) (x+2)(x-7)(x-4)(x-1) + 17$$

$$(iii) (2x^2 + 7x + 3)(2x^2 + 7x + 5) + 1 \quad (iv) (3x^2 + 5x + 3)(3x^2 + 5x + 5) - 3$$

$$(v) (x+1)(x+2)(x+3)(x+6) - 3x^2 \quad (vi) (x+1)(x-1)(x+2)(x-2) - 16x^2$$

Solution

$$2.(i) (x+1)(x+2)(x+3)(x+4) + 1$$

$$= (x+1)(x+4)(x+2)(x+3) + 1$$

$$= (x^2 + 5x + 4)(x^2 + 5x + 6) + 1$$

$$= (y+4)(y+6) + 1 = y^2 + 10y + 24 + 1 = y^2 + 10y + 25$$

$$= (y+5)^2 = (x^2 + 5x + 5)^2$$

$$2.(ii) (x+2)(x-7)(x-4)(x-1) + 17$$

$$= (x+2)(x-7)(x-4)(x-1) + 17$$

$$= (x^2 - 5x - 14)(x^2 - 5x + 4) + 17$$

$$= (y-14)(y+4) + 17 = y^2 - 10y - 56 + 17 = y^2 - 10y - 39$$

$$= y^2 - 13y + 3y - 39 = y(y-13) + 3(y-13)$$

$$= (y-13)(y+3) = (x^2 - 5x - 13)(x^2 - 5x + 3)$$

$$\mathbf{2.(iii) (2x^2 + 7x + 3)(2x^2 + 7x + 5) + 1}$$

$$= (2x^2 + 7x + 3)(2x^2 + 7x + 5) + 1$$

$$= (y + 3)(y + 5) + 1 = y^2 + 8y + 15 + 1 = y^2 + 8y + 16$$

$$= (y + 4)^2 = (2x^2 + 7x + 4)^2$$

$$\mathbf{2.(iv) (3x^2 + 5x + 3)(3x^2 + 5x + 5) - 3}$$

$$= (3x^2 + 5x + 3)(3x^2 + 5x + 5) - 3$$

$$= (y + 3)(y + 5) - 3 = y^2 + 8y + 15 - 3 = y^2 + 8y + 12$$

$$= y^2 + 6y + 2y + 12 = y(y + 6) + 2(y + 6)$$

$$= (y + 6)(y + 2) = (3x^2 + 5x + 6)(3x^2 + 5x + 2)$$

$$\mathbf{2.(v) (x + 1)(x + 2)(x + 3)(x + 6) - 3x^2}$$

$$= (x + 1)(x + 6)(x + 2)(x + 3) - 3x^2$$

$$= (x^2 + 7x + 6)(x^2 + 5x + 6) - 3x^2$$

$$= (x^2 + 6 + 7x)(x^2 + 6 + 5x) - 3x^2$$

$$= (y + 7x)(y + 5x) - 3x^2 = y^2 + 12xy + 35x^2 - 3x^2$$

$$= y^2 + 12xy + 32x^2 = y^2 + 8xy + 4xy + 32x^2 = y(y + 8x) + 4x(y + 8x)$$

$$= (y + 8x)(y + 4x) = (x^2 + 8x + 6)(x^2 + 4x + 6)$$

$$\mathbf{2.(vi) (x + 1)(x - 1)(x + 2)(x - 2) - 16x^2}$$

$$= (x + 1)(x + 2)(x - 1)(x - 2) - 16x^2$$

$$= (x^2 + 3x + 2)(x^2 - 3x + 2) - 16x^2$$

$$= (x^2 + 2 + 3x)(x^2 + 2 - 3x) - 16x^2$$

$$= (y + 3x)(y - 3x) - 16x^2 = y^2 - 9x^2 - 16x^2 = y^2 - 25x^2$$

$$= (y - 5x)(y + 5x) = (x^2 - 5x + 2)(x^2 + 5x + 2)$$

3. Factorize:

(i) $8x^3 + 12x^2 + 6x + 1$

(ii) $27a^3 + 108a^2b + 144ab^2 + 64b^3$

(iii) $x^3 + 18x^2y + 108xy^2 + 216y^3$

(iv) $8x^3 - 125y^3 + 150xy^2 - 60x^2y$

Solution

3.(i) $8x^3 + 12x^2 + 6x + 1$

$$= (2x)^3 + 3(2x)^2(1) + 3(2x)(1)^2 + (1)^3$$

$$= (2x + 1)^3$$

Remember!

$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

3.(ii) $27a^3 + 108a^2b + 144ab^2 + 64b^3$

$$= (3a)^3 + 3(3a)^2(4b) + 3(3a)(4b)^2 + (4b)^3$$

$$= (3a + 4b)^3$$

3.(iii) $x^3 + 18x^2y + 108xy^2 + 216y^3$

$$= (x)^3 + 3(x)^2(6y) + 3(x)(6y)^2 + (6y)^3$$

$$= (x + 6y)^3$$

3.(iv) $8x^3 - 125y^3 + 150xy^2 - 60x^2y$

$$= (2x)^3 + (-5y)^3 + 3(2x)(-5y)^2 + 3(2x)^2(-5y)$$

$$= (2x - 5y)^3$$

4. Factorize:

(i) $125a^3 - 1$

(ii) $64x^3 + 125$

(iii) $x^6 - 27$

(iv) $1000a^3 + 1$

(v) $343x^3 + 216$

(vi) $27 - 512y^3$

Solution

4.(i) $125a^3 - 1$

$$= (5a)^3 - (1)^3$$

$$= (5a - 1)[(5a)^2 + (5a)(1) + (1)^2]$$

$$= (5a - 1)(25a^2 + 5a + 1)$$

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

4.(ii) $64x^3 + 125$

$$= (4x)^3 + (5)^3$$

$$= (4x + 5)[(4x)^2 - (4x)(5) + (5)^2]$$

$$= (4x + 5)(16x^2 - 20x + 25)$$

4.(iii) $x^6 - 27$

$$= (x^2)^3 - (3)^3$$

$$= (x^2 - 3)[(x^2)^2 + (x^2)(3) + (3)^2]$$

$$= (x^2 - 3)(x^4 + 3x^2 + 9)$$

4.(iv) $1000a^3 + 1$

$$= (10a)^3 + (1)^3$$

$$= (10a + 1)[(10a)^2 - (10a)(1) + (1)^2]$$

$$= (10a + 1)(100a^2 - 10a + 1)$$

4.(v) $343x^3 + 216$

$$= (7x)^3 + (6)^3$$

$$= (7x + 6)[(7x)^2 - (7x)(6) + (6)^2]$$

$$= (7x + 6)(49x^2 - 42x + 36)$$

4.(vi) $27 - 512y^3$

$$= (3)^3 - (8y)^3$$

$$= (3 - 8y)[(3)^2 + (3)(8y) + (8y)^2]$$

$$= (3 - 8y)(9 + 24y + 64y^2)$$

EXERCISE 4.3

1. Find HCF by factorization method.

(i) $21x^2y, 35xy^2$

(ii) $4x^2 - 9y^2, 2x^2 - 3xy$

(iii) $x^3 - 1, x^2 + x + 1$

(iv) $a^3 + 2a^2 - 3a, 2a^3 + 5a^2 - 3a$

(v) $t^2 - 3t - 4, t^2 + 5t + 4, t^2 - 1$

(vi) $x^2 + 15x + 56, x^2 + 5x - 24, x^2 + 8x$

Solution

1.(i) $21x^2y, 35xy^2$

$$21x^2y = 3 \times 7 \times x \times x \times y$$

$$35xy^2 = 5 \times 7 \times x \times y \times y$$

$$\text{HCF} = 7 \times x \times y = 7xy$$

1.(ii) $4x^2 - 9y^2, 2x^2 - 3xy$

$$4x^2 - 9y^2 = (2x)^2 - (3y)^2 = (2x - 3y)(2x + 3y)$$

$$2x^2 - 3xy = x(2x - 3y)$$

$$\text{HCF} = 2x - 3y$$

1.(iii) $x^3 - 1, x^2 + x + 1$

$$x^3 - 1 = (x - 1)(x^2 + x + 1)$$

$$x^2 + x + 1 = x^2 + x + 1$$

$$\text{HCF} = x^2 + x + 1$$

1.(iv) $a^3 + 2a^2 - 3a, 2a^3 + 5a^2 - 3a$

$$a^3 + 2a^2 - 3a = a(a^2 + 2a - 3) = a(a^2 + 3a - a - 3) = a(a + 3)(a - 1)$$

$$2a^3 + 5a^2 - 3a = a(2a^2 + 5a - 3) = a(2a^2 + 6a - a - 3) = a(a + 3)(2a - 1)$$

$$\text{HCF} = a(a + 3)$$

$$1.(v) t^2 - 3t - 4, t^2 + 5t + 4, t^2 - 1$$

$$t^2 - 3t - 4 = t^2 - 4t + t - 4 = t(t - 4) + 1(t - 4) = (t - 4)(t + 1)$$

$$t^2 + 5t + 4 = t^2 + 4t + t + 4 = t(t + 4) + 1(t + 4) = (t + 4)(t + 1)$$

$$t^2 - 1 = (t - 1)(t + 1)$$

$$\text{HCF} = t + 1$$

$$1.(vi) x^2 + 15x + 56, x^2 + 5x - 24, x^2 + 8x$$

$$x^2 + 15x + 56 = x^2 + 8x + 7x + 56 = x(x + 8) + 7(x + 8) = (x + 8)(x + 7)$$

$$x^2 + 5x - 24 = x^2 + 8x - 3x - 24 = x(x + 8) - 3(x + 8) = (x + 8)(x - 3)$$

$$x^2 + 8x = x(x + 8)$$

$$\text{HCF} = x + 8$$

2. Find HCF of the following expressions by using division method:

$$(i) \quad 27x^3 + 9x^2 - 3x - 10, 3x - 2 \quad (ii) \quad x^3 - 9x^2 + 23x - 15, x^2 - 4x + 3$$

$$(iii) \quad 2x^3 + 2x^2 + 2x + 2, 6x^3 + 12x^2 + 6x + 12$$

$$(iv) \quad 2x^3 - 4x^2 + 16x, x^3 - 4x, 3x^2 + 6x$$

Solution

$$2.(i) 27x^3 + 9x^2 - 3x - 10, 3x - 2$$

$$\begin{array}{r} 9x^2 + 9x + 5 \\ 3x - 2 \overline{) 27x^3 + 9x^2 - 3x - 10} \\ \underline{\pm 27x^3 \mp 18x^2} \\ 27x^2 - 3x - 10 \\ \underline{\pm 27x^2 \mp 18x} \\ 15x - 10 \\ \underline{\pm 15x \mp 10} \\ 0 \end{array}$$

$$\text{HCF} = 3x - 2$$

2.(ii) $x^3 - 9x^2 + 23x - 15, x^2 - 4x + 3$

$$\begin{array}{r}
 \overline{x - 5} \\
 x^2 - 4x + 3 \overline{) x^3 - 9x^2 + 23x - 15} \\
 \underline{\pm x^3 \mp 4x^2 \pm 3x} \\
 -5x^2 + 20x - 15 \\
 \underline{\mp 5x^2 \pm 20x \mp 15} \\
 0
 \end{array}$$

HCF = $x^2 - 4x + 3$

2.(iii) $2x^3 + 2x^2 + 2x + 2, 6x^3 + 12x^2 + 6x + 12$

$$\begin{array}{r}
 \overline{3} \\
 2x^3 + 2x^2 + 2x + 2 \overline{) 6x^3 + 12x^2 + 6x + 12} \\
 \underline{\pm 6x^3 \pm 6x^2 \pm 6x \pm 6} \\
 6x^2 + 6
 \end{array}$$

$$\begin{array}{r}
 \overline{\frac{x}{3}} \\
 6x^2 + 6 \overline{) 2x^3 + 2x^2 + 2x + 2} \\
 \underline{\pm 2x^3 \pm 2x} \\
 2x^2 + 2
 \end{array}$$

$$\begin{array}{r}
 \overline{3} \\
 2x^2 + 2 \overline{) 6x^2 + 6} \\
 \underline{\pm 6x^2 \pm 6} \\
 0
 \end{array}$$

and

HCF = $2x^2 + 2 = 2(x^2 + 1)$

2.(iv) $2x^3 - 4x^2 - 16x, x^3 - 4x, 3x^2 + 6x$

Let $3x^2 + 6x = 3(x^2 + 2x)$

3 is not common in both given polynomials, so ignoring it, we consider only $x^2 + 2x$

$$\begin{array}{r} x-2 \\ x^2+2x \overline{) x^3-4x} \\ \underline{\pm x^3 \quad \pm 2x^2} \\ -2x^2-4x \\ \underline{\mp 2x^2 \mp 4x} \\ 0 \end{array}$$

Now,

$$\begin{array}{r} 2x-8 \\ x^2+2x \overline{) 2x^3-4x^2-16x} \\ \underline{\pm 2x^3 \pm 4x^2} \\ -8x^2-16x \\ \underline{\mp 8x^2 \mp 16x} \\ 0 \end{array}$$

HCF = $x^2 + 2x = x(x + 2)$

3. Find LCM of the following expressions by using prime factorization method.

(i) $2a^2b, 4ab^2, 6ab$

(ii) $x^2 + x, x^3 + x^2$

(iii) $a^2 - 4a + 4, a^2 - 2a$

(iv) $x^4 - 16, x^3 - 4x$

(v) $16 - 4x^2, x^2 + x - 6, 4 - x^2$

Solution

3.(i) $2a^2b, 4ab^2, 6ab$

$$2a^2b = 2 \times a \times a \times b$$

$$4ab^2 = 2 \times 2 \times a \times b \times b$$

$$6ab = 2 \times 3 \times a \times b$$

$$\text{Common Factors} = 2 \times a \times b = 2ab$$

$$\text{non - Common Factors} = 2 \times 3 \times a \times b = 6ab$$

$$\text{LCM} = \text{CF} \times \text{NCF} = 2ab \times 6ab = 12a^2b^2$$

3.(ii) $x^2 + x, x^3 + x^2$

$$x^2 + x = x(x + 1)$$

$$x^3 + x^2 = x^2(x + 1) = x \times x \times (x + 1)$$

$$\text{Common Factors} = x(x + 1)$$

$$\text{non - Common Factors} = x$$

$$\text{LCM} = \text{CF} \times \text{NCF} = x(x + 1) \times x = x^2(x + 1)$$

3.(iii) $a^2 - 4a + 4, a^2 - 2a$

$$a^2 - 4a + 4 = (a - 2)^2 = (a - 2)(a - 2)$$

$$a^2 - 2a = a(a - 2)$$

$$\text{Common Factors} = (a - 2)$$

$$\text{non - Common Factors} = a(a - 2)$$

$$\text{LCM} = \text{CF} \times \text{NCF} = (a - 2) \times a(a - 2) = a(a - 2)^2$$

$$3.(iv) x^4 - 16, x^3 - 4x$$

$$x^4 - 16 = (x^2 - 4)(x^2 + 4) = (x - 2)(x + 2)(x^2 + 4)$$

$$x^3 - 4x = x(x^2 - 4) = x(x - 2)(x + 2)$$

$$\text{Common Factors} = (x - 2)(x + 2)$$

$$\text{non - Common Factors} = x(x^2 + 4)$$

$$\text{LCM} = \text{CF} \times \text{NCF} = (x - 2)(x + 2) \times x(x^2 + 4) = x(x^4 - 16)$$

$$3.(v) 16 - 4x^2, x^2 + x - 6, 4 - x^2$$

$$16 - 4x^2 = 4(4 - x^2) = 4(2 - x)(2 + x)$$

$$x^2 + x - 6 = x^2 + 3x - 2x - 6 = (x + 3)(x - 2) = -(x + 3)(2 - x)$$

$$4 - x^2 = (2 - x)(2 + x)$$

$$\text{Common Factors} = (2 - x)(2 + x)$$

$$\text{non - Common Factors} = -4(x + 3)$$

$$\text{LCM} = \text{CF} \times \text{NCF} = (2 - x)(2 + x) \times -4(x + 3) = 4(x^2 - 4)(x + 3)$$

4. The HCF of two polynomials is $y - 7$ and their LCM is $y^3 - 10y^2 + 11y + 70$. If one of the polynomials is $y^2 - 5y - 14$, find the other.

Solution

$$\text{HCF} = y - 7$$

$$\text{LCM} = y^3 - 10y^2 + 11y + 70$$

$$p(y) = y^2 - 5y - 14$$

$$q(y) = ?$$

$$\text{Using formula: } p(y) \times q(y) = \text{HCF} \times \text{LCM}$$

$$(y^2 - 5y - 14) \times q(y) = (y - 7) \times (y^3 - 10y^2 + 11y + 70)$$

$$q(y) = \frac{(y-7) \times (y^3 - 10y^2 + 11y + 70)}{(y^2 - 5y - 14)}$$

$$q(y) = \frac{(y-7) \times (y^3 - 10y^2 + 11y + 70)}{(y-7)(y+2)}$$

$$q(y) = \frac{y^3 - 10y^2 + 11y + 70}{y+2}$$

$y + 2$	$\begin{array}{r} y^3 - 10y^2 + 11y + 70 \\ \pm y^3 \pm 2y^2 \\ \hline -12y^2 + 11y + 70 \\ \mp 12y^2 \mp 24y \\ \hline 35y + 70 \\ \pm 35y \pm 70 \\ \hline 0 \end{array}$	$y^2 - 12y + 35$
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$$q(y) = \frac{y^3 - 10y^2 + 11y + 70}{y + 2} = y^2 - 12y + 35$$

5. The LCM and HCF of two polynomial $p(x)$ and $q(x)$ are $36x^3(x + a)(x^3 - a^3)$ and $x^2(x - a)$ respectively. If $p(x) = 4x^2(x^2 - a^2)$, find $q(x)$.

Solution

$$\text{HCF} = x^2(x - a)$$

$$\text{LCM} = 36x^3(x + a)(x^3 - a^3)$$

$$p(x) = 4x^2(x^2 - a^2)$$

$$q(x) = ?$$

$$\text{Using formula: } p(x) \times q(x) = \text{HCF} \times \text{LCM}$$

$$4x^2(x^2 - a^2) \times q(x) = x^2(x - a) \times 36x^3(x + a)(x^3 - a^3)$$

$$q(x) = \frac{x^2(x - a) \times 36x^3(x + a)(x^3 - a^3)}{4x^2(x^2 - a^2)}$$

$$q(x) = 9x^3(x^3 - a^3)$$

6. The HCF and LCM of two polynomials is $(x + a)$ and $12x^2(x + a)(x^2 - a^2)$ respectively. Find the product of the two polynomials.

Solution

$$\text{HCF} = (x + a)$$

$$\text{LCM} = 12x^2(x + a)(x^2 - a^2)$$

$$p(x) \times q(x) = ?$$

$$\text{Using formula: } p(x) \times q(x) = \text{HCF} \times \text{LCM}$$

$$p(x) \times q(x) = (x + a) \times 12x^2(x + a)(x^2 - a^2)$$

$$p(x) \times q(x) = 12x^2(x + a)^2(x^2 - a^2) = 12x^2(x + a)^3(x - a)$$

EXERCISE 4.4

1. Find the square root of the following polynomials by factorization method:

(i) $x^2 - 8x + 16$

(ii) $9x^2 + 12x + 4$

(iii) $36a^2 + 84a + 49$

(iv) $64y^2 - 32y + 4$

(v) $200t^2 - 120t + 18$

(vi) $40x^2 + 120x + 90$

Solution

1.(i) $\sqrt{x^2 - 8x + 16} = ???$

$$x^2 - 8x + 16 = (x)^2 - 2(x)(4) + (4)^2 = (x - 4)^2$$

$$\sqrt{x^2 - 8x + 16} = \sqrt{(x - 4)^2}$$

$$\sqrt{x^2 - 8x + 16} = \pm(x - 4)$$

1.(ii) $\sqrt{9x^2 + 12x + 4} = ???$

$$9x^2 + 12x + 4 = (3x)^2 + 2(3x)(2) + (2)^2 = (3x + 2)^2$$

$$\sqrt{9x^2 + 12x + 4} = \sqrt{(3x + 2)^2}$$

$$\sqrt{9x^2 + 12x + 4} = \pm(3x + 2)$$

1.(iii) $\sqrt{36a^2 + 84a + 49} = ???$

$$36a^2 + 84a + 49 = (6a)^2 + 2(6a)(7) + (7)^2 = (6a + 7)^2$$

$$\sqrt{36a^2 + 84a + 49} = \sqrt{(6a + 7)^2}$$

$$\sqrt{36a^2 + 84a + 49} = \pm(6a + 7)$$

1.(iv) $\sqrt{64y^2 - 32y + 4} = ???$

$$64y^2 - 32y + 4 = (8y)^2 - 2(8y)(2) + (2)^2 = (8y - 2)^2$$

$$\sqrt{64y^2 - 32y + 4} = \sqrt{(8y - 2)^2}$$

$$\sqrt{64y^2 - 32y + 4} = \pm(8y - 2)$$

$$1.(v) \sqrt{200t^2 - 120t + 18} = ???$$

$$200t^2 - 120t + 18 = 2[100t^2 - 60t + 9]$$

$$200t^2 - 120t + 18 = 2[(10t)^2 - 2(10t)(3) + (3)^2] = 2(10t - 3)^2$$

$$\sqrt{64y^2 - 32y + 18} = \sqrt{2(10t - 3)^2}$$

$$\sqrt{64y^2 - 32y + 18} = \pm\sqrt{2}(10t - 3)$$

$$1.(vi) \sqrt{40x^2 + 120x + 90} = ???$$

$$40x^2 + 120x + 90 = 10(4x^2 + 12x + 9) = 10[(2x)^2 + 2(2x)(3) + (3)^2]$$

$$40x^2 + 120x + 90 = 10(2x + 3)^2$$

$$\sqrt{40x^2 + 120x + 90} = \sqrt{10(2x + 3)^2}$$

$$\sqrt{40x^2 + 120x + 90} = \pm\sqrt{10}(2x + 3)$$

2. Find the square root of the following polynomials by division method:

(i) $4x^4 - 28x^3 + 37x^2 + 42x + 9$

(ii) $121x^4 - 198x^3 - 183x^2 + 216x + 144$

(iii) $x^4 - 10x^3y + 27x^2y^2 - 10xy^3 + y^4$

(iv) $4x^4 - 12x^3 + 37x^2 - 42x + 49$

Solution

$$1.(i) \sqrt{4x^4 - 28x^3 + 37x^2 + 42x + 9} = ???$$

	$2x^2 - 7x - 3$
$2x^2$	$\begin{array}{r} 4x^4 - 28x^3 + 37x^2 + 42x + 9 \\ \underline{\pm 4x^4} \end{array}$
$4x^2 - 7x$	$\begin{array}{r} -28x^3 + 37x^2 \\ \underline{\mp 28x^3 \pm 49x^2} \end{array}$
$4x^2 - 14x - 3$	$\begin{array}{r} -12x^2 + 42x + 9 \\ \underline{\mp 12x^2 \pm 42x \pm 9} \\ 0 \end{array}$

$$\sqrt{4x^4 - 28x^3 + 37x^2 + 42x + 9} = \pm(2x^2 - 7x - 3)$$

1.(ii) $\sqrt{121x^4 - 198x^3 - 183x^2 + 216x + 144} = ???$

	$11x^2 - 9x - 12$
$11x^2$	$121x^4 - 198x^3 - 183x^2 + 216x + 144$ $\pm 121x^4$
$22x^2 - 9x$	$-198x^3 - 183x^2$ $\mp 198x^3 \pm 81x^2$
$22x^2 - 18x - 12$	$-264x^2 + 216x + 144$ $\mp 264x^2 \pm 216x \pm 144$
	0

$\sqrt{121x^4 - 198x^3 - 183x^2 + 216x + 144} = \pm(11x^2 - 9x - 12)$

1.(iii) $\sqrt{x^4 - 10x^3y + 27x^2y^2 - 10xy^3 + y^4} = ???$

	$x^2 - 5xy + y^2$
x^2	$x^4 - 10x^3y + 27x^2y^2 - 10xy^3 + y^4$ $\pm x^4$
$2x^2 - 5xy$	$-10x^3y + 27x^2y^2$ $\mp 10x^3y \pm 25x^2y^2$
$2x^2 - 10xy + y^2$	$2x^2y^2 - 10xy^3 + y^4$ $\pm 2x^2y^2 \mp 10xy^3 \pm y^4$
	0

$\sqrt{x^4 - 10x^3y + 27x^2y^2 - 10xy^3 + y^4} = \pm(x^2 - 5xy + y^2)$

1.(iv) $\sqrt{4x^4 - 12x^3 + 37x^2 - 42x + 49} = ???$

	$2x^2 - 3x + 7$
$2x^2$	$4x^4 - 12x^3 + 37x^2 - 42x + 49$ $\pm 4x^4$
$4x^2 - 3x$	$-12x^3 + 37x^2$ $\mp 12x^3 \pm 9x^2$
$4x^2 - 6x + 7$	$28x^2 - 42x + 49$ $\pm 28x^2 \mp 42x \pm 49$
	0

$\sqrt{4x^4 - 12x^3 + 37x^2 - 42x + 49} = \pm(2x^2 - 3x + 7)$

3. An investor's return $R(x)$ in rupees after investing x thousand rupees is given by quadratic expression:

$$R(x) = -x^2 + 6x - 8$$

Factorize the expression and find the investment levels that result in zero return.

Solution

$$R(x) = -x^2 + 6x - 8 = -x^2 + 4x + 2x - 8 = -x(x - 4) + 2(x - 4)$$

$$R(x) = (-x + 2)(x - 4)$$

For zero return $R(x) = 0$ we have $(-x + 2)(x - 4) = 0$

$$-x + 2 = 0 \quad \text{or} \quad x - 4 = 0$$

$$x = 2 \quad \text{or} \quad x = 4$$

Investment levels that result in zero return will be $x = 2$ and $x = 4$

4. A company's profit $P(x)$ in rupees from selling x units of a product is modeled by the cubic expression:

$$P(x) = x^3 - 15x^2 + 75x - 125$$

Find the break-even point(s), where the profit is zero.

Solution

$$P(x) = x^3 - 15x^2 + 75x - 125$$

$$P(x) = (x)^3 - 3(x)^2(5) + 3(x)(5)^2 - (5)^3 = (x - 5)^3$$

Since profit is zero, using $P(x) = 0$ we have $(x - 5)^3 = 0$

After taking cube root on both sides we have $x = 5$

5. The potential energy $V(x)$ in an electric field varies as a cubic function of distance x , given by:

$$V(x) = 2x^3 - 6x^2 + 4x$$

Determine where the potential energy is zero.

Solution

$$V(x) = 2x^3 - 6x^2 + 4x$$

$$V(x) = 2x(x^2 - 3x + 2) = 2x(x - 2)(x - 1)$$

For zero potential energy, using $V(x) = 0$ we have $2x(x - 1)(x - 2) = 0$

Then $x = 0, x = 1, x = 2$

6. In structural engineering, the deflection $Y(x)$ of a beam is given by:

$$Y(x) = 2x^2 - 8x + 6$$

This equation gives the vertical deflection at any point x along the beam. Find the points of zero deflection.

Solution

$$Y(x) = 2x^2 - 8x + 6$$

$$Y(x) = 2(x^2 - 4x + 3)$$

$$Y(x) = 2(x^2 - 3x - x + 3)$$

$$Y(x) = 2(x - 1)(x - 3)$$

For zero potential deflection, using $Y(x) = 0$ we have $2(x - 1)(x - 3) = 0$

$$2 \neq 0 \text{ then } x - 1 = 0 \text{ or } x - 3 = 0$$

$$\text{Then } x = 1, x = 3$$

REVIEW EXERCISE 4

1. Four options are given against each statement. Encircle the correct option.
 - i. The factorization of $12x + 36$ is:
 (a) ☒ $12(x + 3)$ (b) $12(3x)$ (c) $12(3x + 1)$ (d) $x(12 + 36x)$
 - ii. The factors of $4x^2 - 12y + 9$ are:
 (a) $(2x + 3)^2$ (b) ☒ $(2x - 3)^2$
 (c) $(2x - 3)(2x + 3)$ (d) $(2 + 3x)(2 - 3x)^2$
 - iii. The HCF of a^3b^3 and ab^2 is:
 (a) a^3b^3 (b) ☒ ab^2 (c) a^4b^5 (d) a^2b
 - iv. The LCM of $16x^2$, $4x$ and $30xy$ is:
 (a) $480x^3y$ (b) $240xy$ (c) ☒ $240x^2y$ (d) $120x^4y$
 - v. Product of LCM and HCF = _____ of two polynomials.
 (a) sum (b) difference (c) ☒ product (d) quotient
 - vi. The square root of $x^2 - 6x + 9$ is:
 (a) ☒ $\pm(x - 3)$ (b) $\pm(x + 3)$ (c) $x - 3$ (d) $x + 3$
 - vii. The LCM of $(a - b)^2$ and $(a - b)^4$ is:
 (a) $(a - b)^2$ (b) $(a - b)^3$ (c) ☒ $(a - b)^4$ (d) $(a - b)^6$
 - viii. Factorization of $x^3 + 3x^2 + 3x + 1$ is:
 (a) ☒ $(x + 1)^3$ (b) $(x - 1)^3$
 (c) $(x + 1)(x^2 + x + 1)$ (d) $(x - 1)(x^2 - x + 1)$
 - ix. Cubic polynomial has degree:
 (a) 1 (b) 2 (c) ☒ 3 (d) 4
 - x. One of the factors of $x^3 - 27$ is:
 (a) ☒ $x - 3$ (b) $x + 3$ (c) $x^2 - 3x + 9$ (d) Both a and c

2. Factorize the following expressions:

(i) $4x^3 + 18x^2 - 12x$

(ii) $x^3 + 64y^3$

(iii) $x^3y^3 - 8$

(iv) $-x^2 - 23x - 60$

(v) $2x^2 + 7x + 3$

(vi) $x^4 + 64$

(vii) $x^4 + 2x^2 + 9$

(viii) $(x + 3)(x + 4)(x + 5)(x + 6) - 360$

(ix) $(x^2 + 6x + 3)(x^2 + 6x - 9) + 36$

Solution

2.(i) $4x^3 + 18x^2 - 12x$

$$= 2x(2x^2 + 9x - 6)$$

2.(ii) $x^3 + 64y^3$

$$= (x)^3 + (4y)^3$$

$$= (x + 4y)[(x)^2 - (x)(4y) + (4y)^2]$$

$$= (x + 4y)(x^2 - 4xy + 16y^2)$$

2.(iii) $x^3y^3 - 8$

$$= (xy)^3 - (2)^3$$

$$= (xy - 2)[(xy)^2 + (xy)(2) + (2)^2]$$

$$= (xy - 2)(x^2y^2 + 2xy + 4)$$

2.(iv) $-x^2 - 23x - 60$

$$= -(x^2 + 23x + 60) = -[x^2 + 20x + 3x + 60] = -[x(x + 20) + 3(x + 20)]$$

$$= -(x + 3)(x + 20)$$

2.(v) $2x^2 + 7x + 3$

$$= 2x^2 + 6x + x + 3 = 2x(x + 3) + 1(x + 3)$$

$$= (2x + 1)(x + 3)$$

2.(vi) $x^4 + 64$

$$= (x^2)^2 + (8)^2 + 2(x^2)(8) - 2(x^2)(8) = (x^2 + 8)^2 - 16x^2$$

$$= (x^2 + 8)^2 - (4x)^2 = (x^2 + 8 - 4x)(x^2 + 8 + 4x)$$

$$= (x^2 - 4x + 8)(x^2 + 4x + 8)$$

2.(vii) $x^4 + 2x^2 + 9$

$$= (x^2)^2 + 2x^2 + (3)^2$$

$$= (x^2)^2 + 2(x^2)(3) + (3)^2 + 2x^2 - 2(x^2)(3)$$

$$= (x^2 + 3)^2 - 4x^2$$

$$= (x^2 + 3)^2 - (2x)^2 = (x^2 + 3 - 2x)(x^2 + 3 + 2x)$$

$$= (x^2 - 2x + 3)(x^2 + 2x + 3)$$

2.(viii) $(x + 3)(x + 4)(x + 5)(x + 6) - 360$

$$= (x + 3)(x + 6)(x + 4)(x + 5) - 360$$

$$= (x^2 + 9x + 18)(x^2 + 9x + 20) - 360$$

$$= (y + 18)(y + 20) - 360 = y^2 + 38y + 360 - 360 = y^2 + 38y$$

$$= y(y + 38) = (x^2 + 9x)(x^2 + 9x + 38)$$

$$= x(x + 9)(x^2 + 9x + 38)$$

2.(ix) $(x^2 + 6x + 3)(x^2 + 6x - 9) + 36$

$$= (x^2 + 6x + 3)(x^2 + 6x - 9) + 36$$

$$= (y + 3)(y - 9) + 36 = y^2 - 6y - 27 + 36 = y^2 - 6y + 9$$

$$= (y)^2 - 2(y)(3) + (3)^2 = (y - 3)^2$$

$$= (x^2 + 6x - 3)^2$$

3. Find LCM and HCF by prime factorization method:

(i) $4x^3 + 12x^2, 8x^2 + 16x$

(ii) $x^3 + 3x^2 - 4x, x^2 - x - 6$

(iii) $x^2 + 8x + 16, x^2 - 16$

(iv) $x^3 - 9x, x^2 - 4x + 3$

Solution

3.(i) $4x^3 + 12x^2, 8x^2 + 16x$

$$4x^3 + 12x^2 = 4x^2(x + 3) = 4 \times x \times x \times (x + 3)$$

$$8x^2 + 16x = 8x(x + 2) = 2 \times 4 \times x \times (x + 2)$$

$$\text{Common Factors} = 4x$$

$$\text{Un - Common Factors} = x \times (x + 3) \times 2 \times (x + 2) = 2x(x + 3)(x + 2)$$

$$\text{HCF} = 4x$$

$$\text{LCM} = \text{CF} \times \text{UCF} = 4x \times 2x(x + 3)(x + 2) = 8x^2(x + 2)(x + 3)$$

3.(ii) $x^3 + 3x^2 - 4x, x^2 - x - 6$

$$x^3 + 3x^2 - 4x = x(x^2 + 3x - 4) = x(x^2 + 4x - x - 4) = x(x - 1)(x + 4)$$

$$x^2 - x - 6 = x^2 - 3x + 2x - 6 = (x - 3)(x + 2)$$

$$\text{Common Factors} = 1$$

$$\text{Un - Common Factors} = x(x - 1)(x + 4)(x - 3)(x + 2)$$

$$\text{HCF} = 1$$

$$\text{LCM} = \text{CF} \times \text{UCF} = x(x - 1)(x + 2)(x - 3)(x + 4) \quad \text{wrong answer in book}$$

3.(iii) $x^2 + 8x + 16, x^2 - 16$

$$x^2 + 8x + 16 = (x)^2 + 2(x)(4) + (4)^2 = (x + 4)^2 = (x + 4)(x + 4)$$

$$x^2 - 16 = (x)^2 - (4)^2 = (x - 4)(x + 4)$$

$$\text{Common Factors} = (x + 4)$$

$$\text{Un - Common Factors} = (x - 4)(x + 4)$$

$$\text{HCF} = (x + 4)$$

$$\text{LCM} = \text{CF} \times \text{UCF} = (x + 4) \times (x - 4)(x + 4) = (x - 4)(x + 4)^2$$

3.(iv) $x^3 - 9x, x^2 - 4x + 3$

$$x^3 - 9x = x(x^2 - 9) = x(x - 3)(x + 3)$$

$$x^2 - 4x + 3 = x^2 - 3x - x + 3 = (x - 3)(x - 1)$$

Common Factors = $(x - 3)$

Un - Common Factors = $x(x + 3)(x - 1)$

HCF = $(x - 3)$

LCM = CF $\times$ UCF = $(x - 3) \times x(x + 3)(x - 1)$

LCM = $x(x - 1)(x^2 - 9)$ **wrong answer in book**

4. Find square root by factorization and division method of the expression $16x^4 + 8x^2 + 1$.

Solution

Factorization:

$$16x^4 + 8x^2 + 1 = (4x^2)^2 + 2(4x^2)(1) + (1)^2$$

$$16x^4 + 8x^2 + 1 = (4x^2 + 1)^2$$

$$\sqrt{16x^4 + 8x^2 + 1} = \sqrt{(4x^2 + 1)^2}$$

$$\sqrt{16x^4 + 8x^2 + 1} = \pm(4x^2 + 1)$$

Division Method:

$4x^2 + 1$	$16x^4 + 8x^2 + 1$
$4x^2$	$16x^4$
$8x^2 + 1$	$8x^2 + 1$
	$\pm 8x^2 \pm 1$
	0

$$\sqrt{16x^4 + 8x^2 + 1} = \pm(4x^2 + 1)$$

5. Huria is analyzing the total cost of her loan, modeled by the expression $C(x) = x^2 - 8x + 15$, where x represents the number of years. What is the optimal repayment period for Huria's loan?

Solution

$$C(x) = x^2 - 8x + 15 = x^2 - 5x - 3x + 15 = (x - 5)(x - 3)$$

For optimal repayment, using $C(x) = 0$ we have $(x - 5)(x - 3) = 0$

We have $x = 3, x = 5$. That is 3 years or 5 years.