

Chapter # 5

Linear Equations and Inequalities

Exercise # 5.2

Question # 1: Maximize $f(x, y) = 2x + 5y$; subject to the constraints:

$$2y - x \leq 8 \quad ; \quad x - y \leq 4 \quad ; \quad x \geq 0; y \geq 0$$

(a) Associated Equations

$$2y - x = 8 \quad \text{--- (A)} \quad \parallel \quad x - y = 4 \quad \text{--- (B)}$$

(b) x – Intercept

put $y = 0$ in equations (A) and (B)

$$2(0) - x = 8$$

$$-x = 8$$

$$x = -8$$

$$P_1 (-8, 0)$$

$$x - 0 = 4$$

$$x = 4$$

$$P_3 (4, 0)$$

(c) y – Intercept

put $x = 0$ in equations (A) and (B)

$$2y - 0 = 8$$

$$2y = 8$$

$$y = \frac{8}{2}$$

$$y = 4$$

$$P_2 (0, 4)$$

$$0 - y = 4$$

$$-y = 4$$

$$y = -4$$

$$P_4 (0, -4)$$

(d) Test Point (0,0)

put $x = 0, y = 0$ in given inequalities

$$2(0) - 0 \leq 8$$

$$0 \leq 8 \text{ (True)}$$

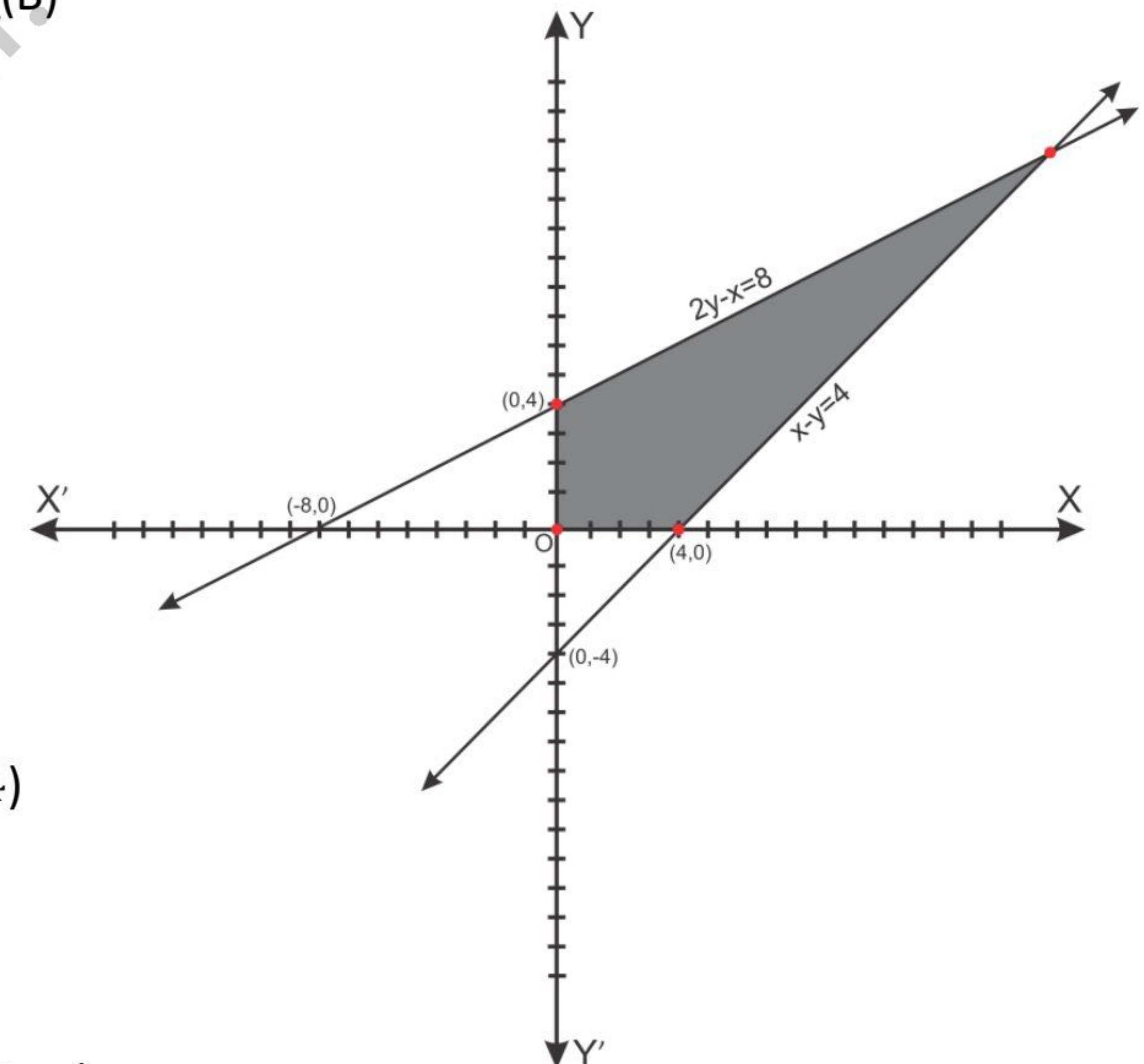
$$0 - 0 \leq 4$$

$$0 \leq 4 \text{ (True)}$$

Solution Regions lie towards the Origin

$$\therefore x \geq 0, y \geq 0$$

$\therefore$ Solution region will be feasible (I-Quadrant)



(e) Point of Intersection

$$2y - x = 8 \text{ _____ (A)}$$

$$x - y = 4 \text{ _____ (B)}$$

$$(A) + (B)$$

$$-x + 2y = 8$$

$$x - y = 4$$

$$y = 12$$

Put in equation (B)

$$x - 12 = 4$$

$$x = 4 + 12$$

$$x = 16$$

(f) Corner Points

$$(0,0), (0,4), (4,0), (16,12)$$

$$\because f(x,y) = 2x + 5y$$

$$\text{put } x = 0, y = 0$$

$$f(0,0) = 2(0) + 5(0) = 0 + 0 = 0$$

$$\text{put } x = 0, y = 4$$

$$f(0,4) = 2(0) + 5(4) = 0 + 20 = 20$$

$$\text{put } x = 4, y = 0$$

$$f(4,0) = 2(4) + 5(0) = 8 + 0 = 8$$

$$\text{put } x = 16, y = 12$$

$$f(16,12) = 2(16) + 5(12) = 32 + 60 = 92$$

Hence, $f(x,y)$ is maximized at $(16,12)$

Question # 2: Maximize $f(x,y) = x + 3y$; subject to the constraints:

$$2x + 5y \leq 30 \quad ; \quad 5x + 4y \leq 20 \quad ; \quad x \geq 0; y \geq 0$$

(a) Associated Equations

$$2x + 5y = 30 \text{ _____ (A)}$$

$$5x + 4y = 20 \text{ _____ (B)}$$

(b) x – Intercept

put $y = 0$ in equations (A) and (B)

$$2x + 5(0) = 30$$

$$2x = 30$$

$$x = \frac{30}{2}$$

$$x = 15$$

$$P_1 (15,0)$$

$$5x + 4(0) = 20$$

$$5x = 20$$

$$x = \frac{20}{5}$$

$$x = 4$$

$$P_3 (4,0)$$

(c) y – Intercept

put $x = 0$ in equations (A) and (B)

$$\begin{aligned}
 2(0) + 5y &= 30 \\
 5y &= 30 \\
 y &= \frac{30}{5} \\
 y &= 6 \\
 P_2 (0,6)
 \end{aligned}$$

$$\begin{aligned}
 5(0) + 4y &= 20 \\
 4y &= 20 \\
 y &= \frac{20}{4} \\
 y &= 5 \\
 P_4 (0,5)
 \end{aligned}$$

(d) **Test Point (0,0)**

put $x = 0, y = 0$ in given inequalities

$$\begin{aligned}
 2(0) + 5(0) &\leq 30 \\
 0 &\leq 30 \text{ (True)}
 \end{aligned}$$

$$\begin{aligned}
 5(0) + 4(0) &\leq 20 \\
 0 &\leq 20 \text{ (True)}
 \end{aligned}$$

Solution Regions lie towards the Origin

(e) **Corner Points**

$(0,0), (4,0), (0,5)$

$$\because f(x, y) = x + 3y$$

put $x = 0, y = 0$

$$f(0,0) = 0 + 3(0) = 0 + 0 = 0$$

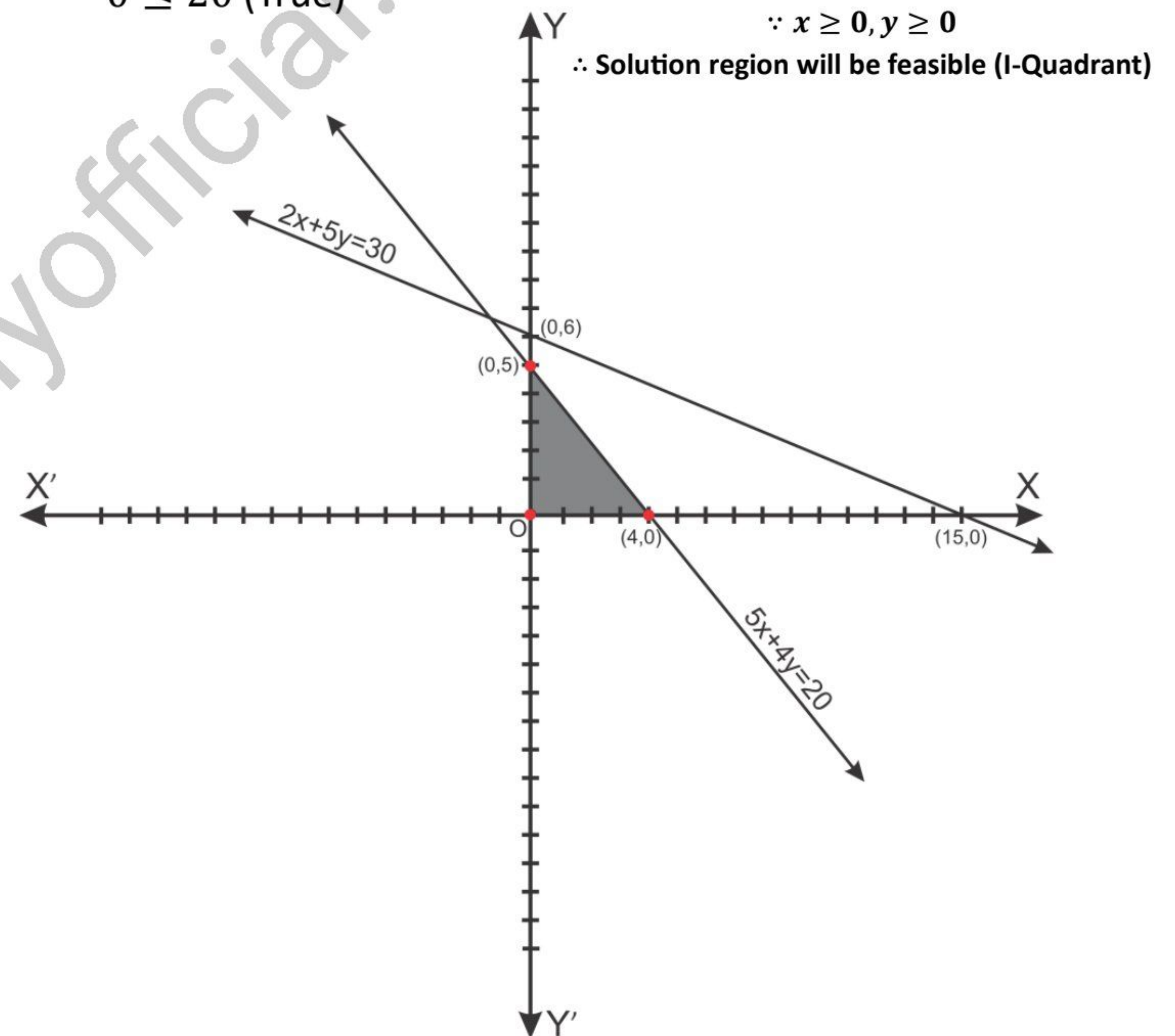
put $x = 4, y = 0$

$$f(4,0) = 4 + 3(0) = 4 + 0 = 4$$

put $x = 0, y = 5$

$$f(0,5) = 0 + 3(5) = 0 + 15 = 15$$

Hence, $f(x, y)$ is maximized at $(0,5)$



Question # 3: Maximize $z = 2x + 3y$; subject to the constraints:

$$2x + y \leq 4 \quad ; \quad 4x - y \leq 2 \quad ; \quad x \geq 0; y \geq 0$$

(a) Associated Equations

$$2x + y = 4 \text{ _____ (A)} \quad \parallel \quad 4x - y = 2 \text{ _____ (B)}$$

(b) x – Intercept

put $y = 0$ in equations (A) and (B)

$$2x + 0 = 4$$

$$2x = 4$$

$$x = \frac{4}{2}$$

$$x = 2$$

$$P_1 (2,0)$$

$$4x - 0 = 2$$

$$4x = 2$$

$$x = \frac{2}{4}$$

$$x = \frac{1}{2}$$

$$P_3 \left(\frac{1}{2}, 0\right)$$

(c) y – Intercept

put $x = 0$ in equations (A) and (B)

$$2(0) + y = 4$$

$$0 + y = 4$$

$$y = 4$$

$$P_2 (0,4)$$

$$4(0) - y = 2$$

$$-y = 2$$

$$y = -2$$

$$P_4 (0, -2)$$

(d) Test Point (0,0)

put $x = 0, y = 0$ in given inequalities

$$2(0) + 0 \leq 4$$

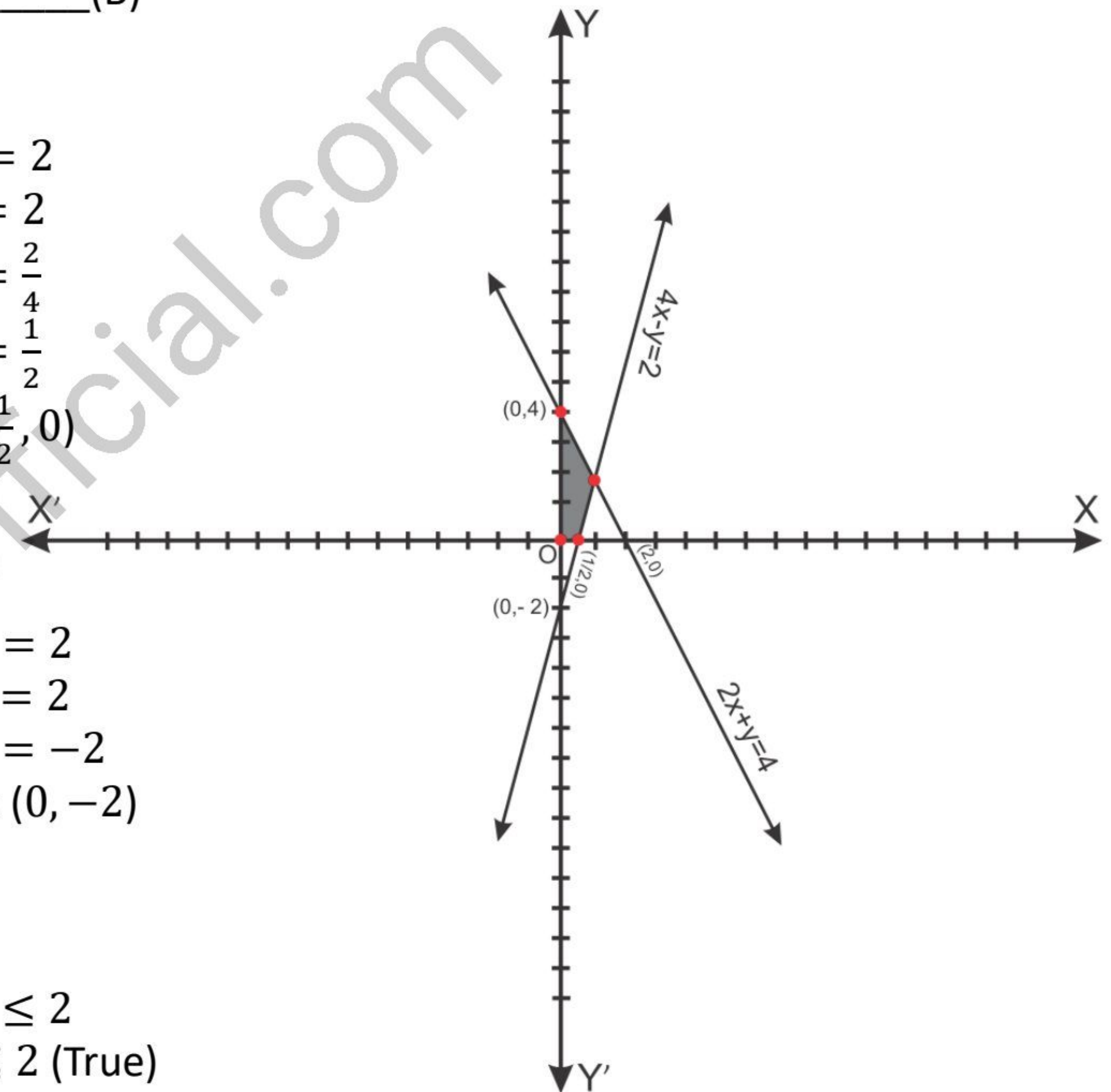
$$0 \leq 4 \text{ (True)}$$

$$4(0) - 0 \leq 2$$

$$0 \leq 2 \text{ (True)}$$

Solution Regions lie towards the Origin

$\therefore x \geq 0, y \geq 0$
 $\therefore$ Solution region will be feasible (I-Quadrant)



(e) Point of Intersection

$$2x + y = 4 \text{ _____ (A)}$$

$$4x - y = 2 \text{ _____ (B)}$$

$$(A) + (B)$$

$$2x + y = 4$$

$$4x - y = 2$$

$$\hline 6x = 6$$

$$x = \frac{6}{6}$$

$$x = 1$$

Put in equation (A)

$$2(1) + y = 4$$

$$2 + y = 4$$

$$y = 4 - 2$$

$$y = 2$$

(f) Corner Points

$$(0,0), (0,4), (1,0), (1,2)$$

$$\because z = 2x + 3y$$

$$\text{put } x = 0, y = 0$$

$$z = 2(0) + 3(0) = 0 + 0 = 0$$

$$\text{put } x = 0, y = 4$$

$$z = 2(0) + 3(4) = 0 + 12 = 12$$

$$\text{put } x = 1, y = 0$$

$$z = 2(1) + 3(0) = 2 + 0 = 2$$

$$\text{put } x = 1, y = 2$$

$$z = 2(1) + 3(2) = 2 + 6 = 8$$

Hence, z is maximized at $(0,4)$

Question # 4: Maximize $z = 2x + y$; subject to the constraints:

$$x + y \geq 3 \quad ; \quad 7x + 5y \leq 35 \quad ; \quad x \geq 0; y \geq 0$$

(a) Associated Equations

$$x + y = 3 \text{ _____ (A)}$$

||

$$7x + 5y = 35 \text{ _____ (B)}$$

(b) x – Intercept

put $y = 0$ in equations (A) and (B)

$$x + 0 = 3$$

$$x = 3$$

$$P_1 (3,0)$$

$$7x + 5(0) = 35$$

$$7x = 35$$

$$x = \frac{35}{7}$$

$$x = 5$$

$$P_3 (5,0)$$

(c) y – Intercept

put $x = 0$ in equations (A) and (B)

$$\begin{aligned}0 + y &= 3 \\ y &= 3 \\ P_2(0, 3)\end{aligned}$$

$$\begin{aligned}7(0) + 5y &= 35 \\ 5y &= 35 \\ y &= \frac{35}{5} \\ y &= 7 \\ P_4(0, 7)\end{aligned}$$

(d) Test Point (0,0)

put $x = 0, y = 0$ in given inequalities

$$\begin{aligned}0 + 0 &\geq 3 \\ 0 &\geq 4 \text{ (False)}\end{aligned}$$

Solution region lies away from the origin

$$\begin{aligned}7(0) + 5(0) &\leq 35 \\ 0 &\leq 35 \text{ (True)}\end{aligned}$$

Solution region lies towards the origin

(e) Corner Points

$$(3, 0), (0, 3), (5, 0), (0, 7)$$

$$\because z = 2x + y$$

$$\text{put } x = 3, y = 0$$

$$z = 2(3) + 0 = 6 + 0 = 6$$

$$\text{put } x = 0, y = 3$$

$$z = 2(0) + 3 = 0 + 3 = 3$$

$$\text{put } x = 5, y = 0$$

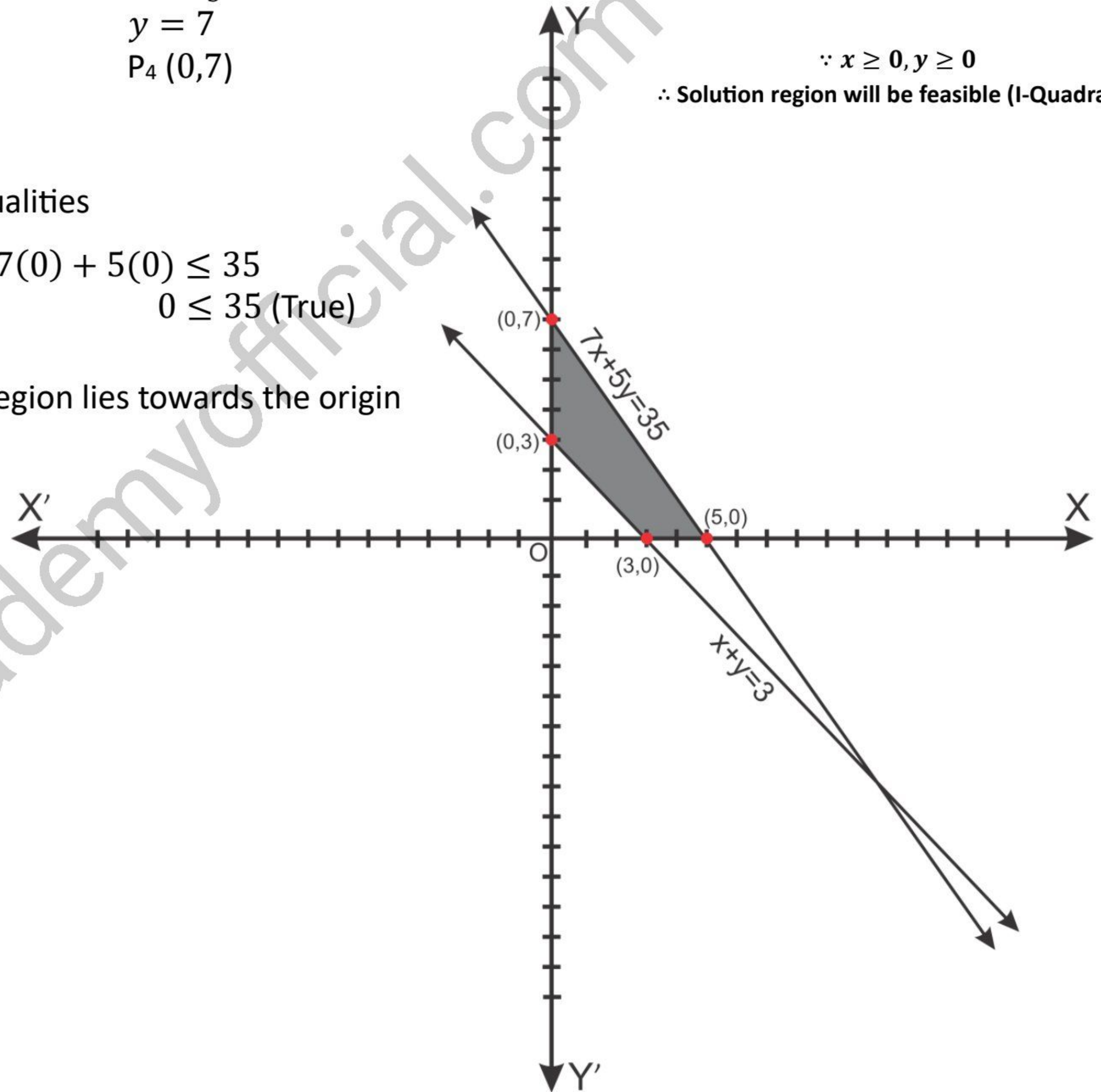
$$z = 2(5) + 0 = 10 + 0 = 10$$

$$\text{put } x = 0, y = 7$$

$$z = 2(0) + 7 = 0 + 7 = 7$$

Hence, z is minimized at $(0, 3)$

$\because x \geq 0, y \geq 0$
 $\therefore$ Solution region will be feasible (I-Quadrant)



Question # 5: Maximize the function defined as: $f(x, y) = 2x + 3y$; subject to the constraints:

$$2x + y \leq 10 \quad ; \quad x + 2y \leq 14 \quad ; \quad x \geq 0; y \geq 0$$

(a) Associated Equations

$$2x + y = 10 \text{ (A)} \quad \parallel \quad x + 2y = 14 \text{ (B)}$$

(b) x – Intercept

put $y = 0$ in equations (A) and (B)

$$\begin{aligned} 2x + 0 &= 10 \\ 2x &= 10 \\ x &= \frac{10}{2} \\ x &= 5 \\ P_1 (5, 0) \end{aligned}$$

$$\begin{aligned} x + 2(0) &= 14 \\ x + 0 &= 14 \\ x &= 14 \\ P_3 (14, 0) \end{aligned}$$

(c) y – Intercept

put $x = 0$ in equations (A) and (B)

$$\begin{aligned} 2(0) + y &= 10 \\ 0 + y &= 10 \\ y &= 10 \\ P_2 (0, 10) \end{aligned}$$

$$\begin{aligned} 0 + 2y &= 14 \\ 2y &= 14 \\ y &= \frac{14}{2} \\ y &= 7 \\ P_4 (0, 7) \end{aligned}$$

(d) Test Point (0,0)

put $x = 0, y = 0$ in given inequalities

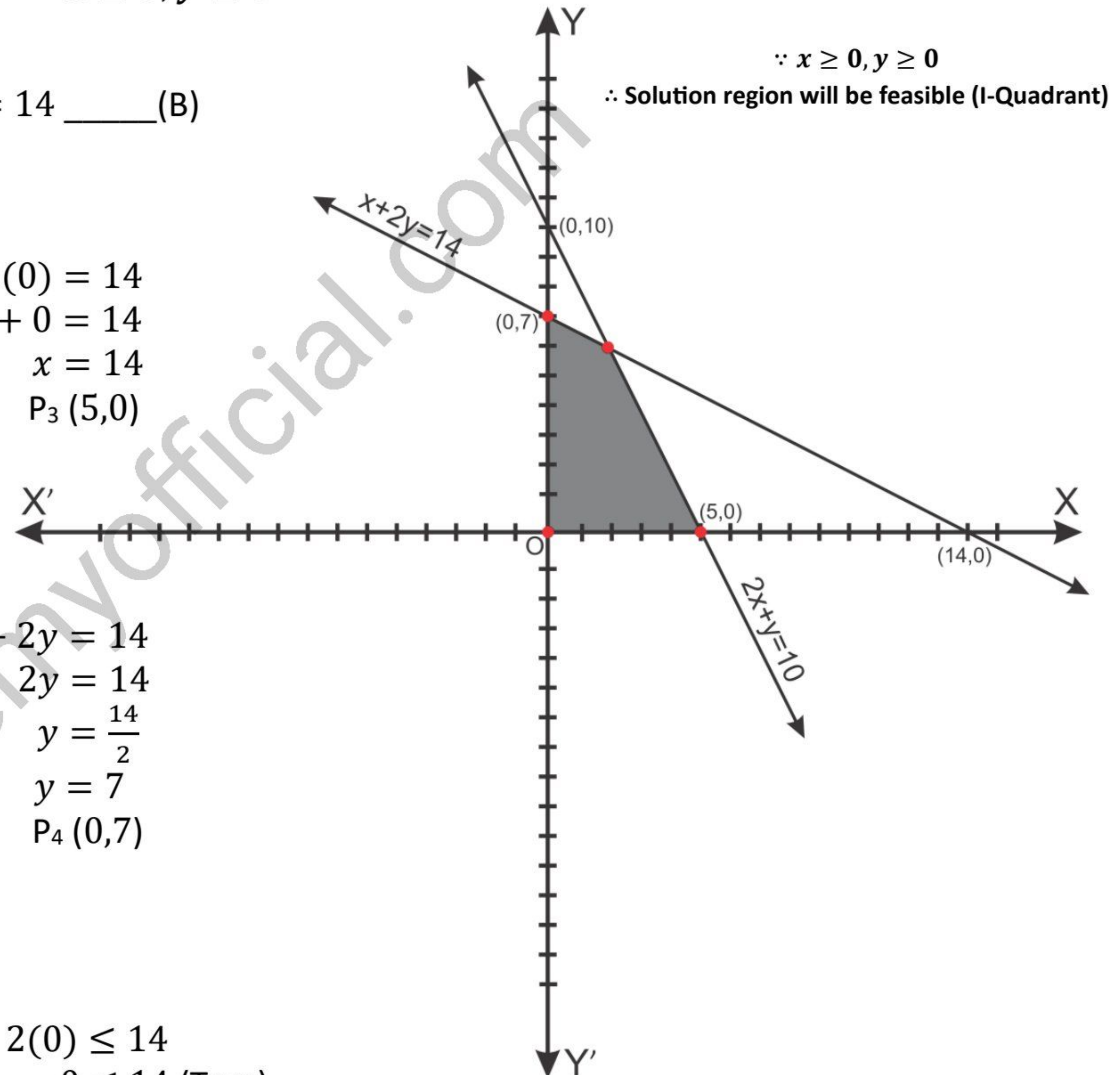
$$2(0) + 0 \leq 10$$

$$0 \leq 10 \text{ (True)}$$

$$0 + 2(0) \leq 14$$

$$0 \leq 14 \text{ (True)}$$

Solution Regions lie towards the origin



(e) Point of Intersection

$$2x + y = 10 ; x + 2y = 14 \text{ _____ (D)}$$

Multiply by '2'

$$4x + 2y = 20 \text{ _____ (C)}$$

$$(C) - (D)$$

$$4x + 2y = 20$$

$$\pm x \pm 2y = \pm 14$$

$$3x = 6$$

$$x = \frac{6}{3}$$

$$x = 2$$

Put in equation (D)

$$2 + 2y = 14$$

$$2y = 14 - 2$$

$$2y = 12$$

$$y = \frac{12}{2}$$

$$y = 6$$

(f) Corner Points

$$(0,0), (5,0), (0,7), (2,6)$$

$$\because f(x, y) = 2x + 3y$$

$$\text{put } x = 0, y = 0$$

$$f(0,0) = 2(0) + 3(0) = 0 + 0 = 0$$

$$\text{put } x = 5, y = 0$$

$$f(5,0) = 2(5) + 3(0) = 10 + 0 = 10$$

$$\text{put } x = 0, y = 7$$

$$f(0,7) = 2(0) + 3(7) = 0 + 21 = 21$$

$$\text{put } x = 2, y = 6$$

$$f(2,6) = 2(2) + 3(6) = 4 + 18 = 22$$

Hence, z is maximized at $(2,6)$

Question # 6: Find minimum and maximum values of $z = 3x + y$; subject to the constraints:

$$3x + 5y \geq 15 ; \quad x + 3y \leq 9 ; \quad x \geq 0; y \geq 0$$

(a) Associated Equations

$$3x + 5y = 15 \text{ _____ (A)}$$

$$x + 3y = 9 \text{ _____ (B)}$$

(b) x – Intercept

put $y = 0$ in equations (A) and (B)

$$3x + 5(0) = 15$$

$$3x = 15$$

$$x = \frac{15}{3}$$

$$x = 5$$

$$P_1 (5,0)$$

$$x + 3(0) = 9$$

$$x + 0 = 9$$

$$x = 9$$

$$P_3 (9,0)$$

(c) y – Intercept

put $x = 0$ in equations (A) and (B)

$$\begin{aligned}3(0) + 5y &= 15 \\5y &= 15 \\y &= \frac{15}{5} \\y &= 3 \\P_2 (0,3)\end{aligned}$$

$$\begin{aligned}0 + 3y &= 9 \\3y &= 9 \\y &= \frac{9}{3} \\y &= 3 \\P_4 (0,3)\end{aligned}$$

$$\therefore x \geq 0, y \geq 0$$

$\therefore$ Solution region will be feasible (I-Quadrant)

(d) Test Point (0,0)

put $x = 0, y = 0$ in given inequalities

$$\begin{aligned}3(0) + 5(0) &\geq 15 \\0 &\geq 15 \text{ (False)}\end{aligned}$$

Solution region lies away from the origin

$$\begin{aligned}0 + 3(0) &\leq 9 \\0 &\leq 9 \text{ (True)}\end{aligned}$$

Solution region lies towards the origin

(e) (Croner Points)

$$(0,3), (5,0), (9,0)$$

$$\therefore z = 3x + y$$

$$\text{put } x = 0, y = 3$$

$$z = 3(0) + 3 = 0 + 3 = 3$$

$$\text{put } x = 5, y = 0$$

$$z = 3(5) + 0 = 15 + 0 = 15$$

$$\text{put } x = 9, y = 0$$

$$z = 3(9) + 0 = 27 + 0 = 27$$

Hence, z is minimized at $(0,3)$ and maximized at $(9,0)$

