Linear Equations and Inequalities

Review Exercise # 5

Question # 1: Four options are given against each statement. Encircle the correct one.

#	Answer	#	Answer
i	C	vi	B
ii	C	vii	B
iii	C	viii	C
iv	D	ix	B
v	B	x	B

Question # 2: Solve and represent their solution on real line.

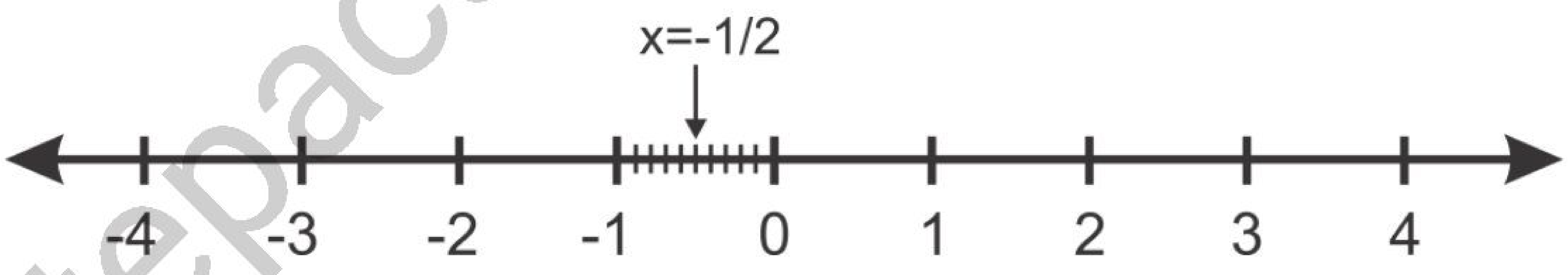
(i) $\frac{x+5}{3} = 1 - x$ ________ (A)

$$x + 5 = 3(1 - x)$$
$$x + 5 = 3 - 3x$$
$$x + 3x = 3 - 5$$
$$4x = -2$$
$$x = \frac{-\cancel{2}^{1}}{\cancel{4}_{2}}$$
$$x = \frac{-1}{2}$$

Check:

put $x = \frac{-1}{2}$ in equation (A)

$$\frac{\frac{-1}{2}+5}{3} = 1 - \left(-\frac{1}{2}\right)$$
$$\frac{9/2}{3} = 1 + \frac{1}{2}$$
$$\frac{\cancel{9}^{3}}{2} \times \frac{1}{\cancel{3}_{1}} = \frac{3}{2}$$
$$\frac{3}{2} = \frac{3}{2}$$
$$\text{S.S} = \left\{-\frac{1}{2}\right\}$$



(ii) $\frac{2x+1}{3} + \frac{1}{2} = 1 - \frac{x-1}{3}$ __________ (A)

Multiply by '6' on both sides

$$\cancel{6}^{2} \times \frac{2x+1}{\cancel{3}} + \cancel{6}^{3} \times \frac{1}{\cancel{2}} = 6 \times 1 - \cancel{6}^{2} \times \frac{x-1}{\cancel{3}}$$
$$2(2x + 1) + 3 = 6 - 2(x - 1)$$
$$4x + 2 + 3 = 6 - 2x + 2$$
$$4x + 5 = 8 - 2x$$
$$4x + 2x = 8 - 5$$
$$6x = 3$$

Check:

put $x = \frac{1}{2}$ in equation (A)

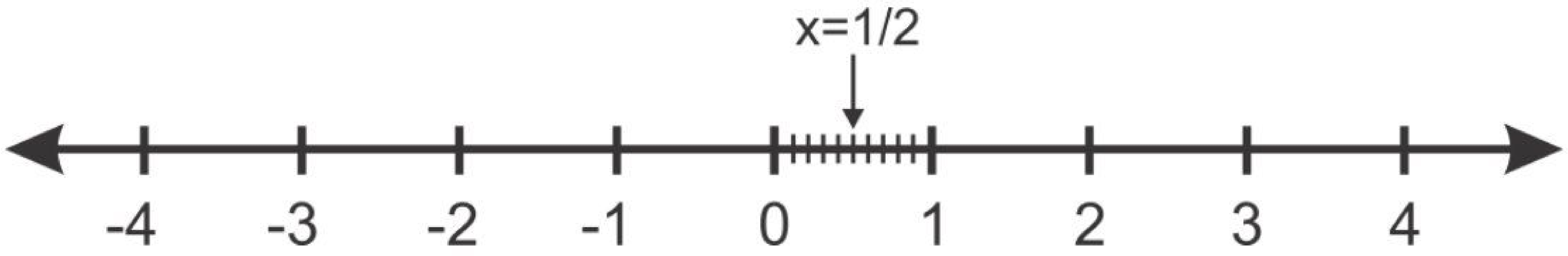
$$\frac{2\left(\frac{1}{2}\right)+1}{3} = 1 - \frac{\frac{1}{2}-1}{3}$$
$$\frac{1+1}{3} + \frac{1}{2} = 1 - \frac{-\frac{1}{2}}{3}$$
$$\frac{2}{3} + \frac{1}{2} = 1 + \frac{1}{2} \times \frac{1}{3}$$
$$\frac{4+3}{6} = 1 + \frac{1}{6}$$

$x = \frac{\cancel{3}^{1}}{\cancel{6}_{2}}$

$x = \frac{1}{2}$

$\frac{7}{6} = \frac{7}{6}$

$S.S = \left\{\frac{1}{2}\right\}$



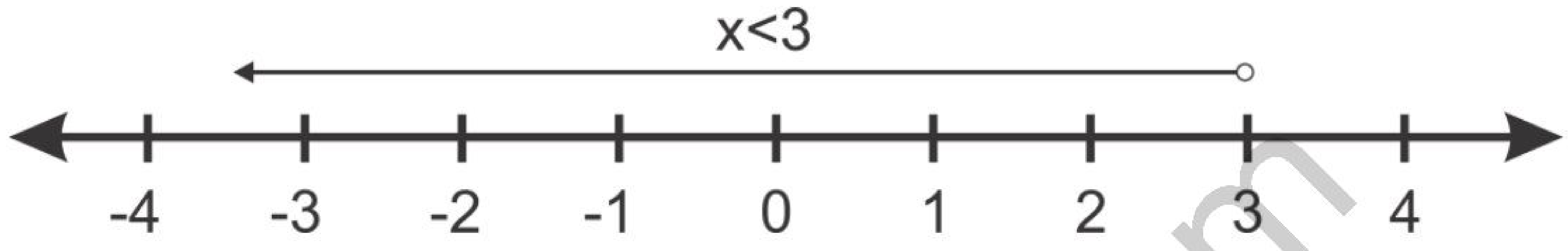
(iii) $\mathbf{3x + 7 < 16}$

$3x < 16 - 7$

$3x < 9$

$x < \frac{\cancel{9}^{3}}{\cancel{3}}$

$x < 3$



(iv) $\mathbf{5(x - 3) \geq 26x - (10x + 4)}$

$5x - 15 \geq 26x - 10x - 4$

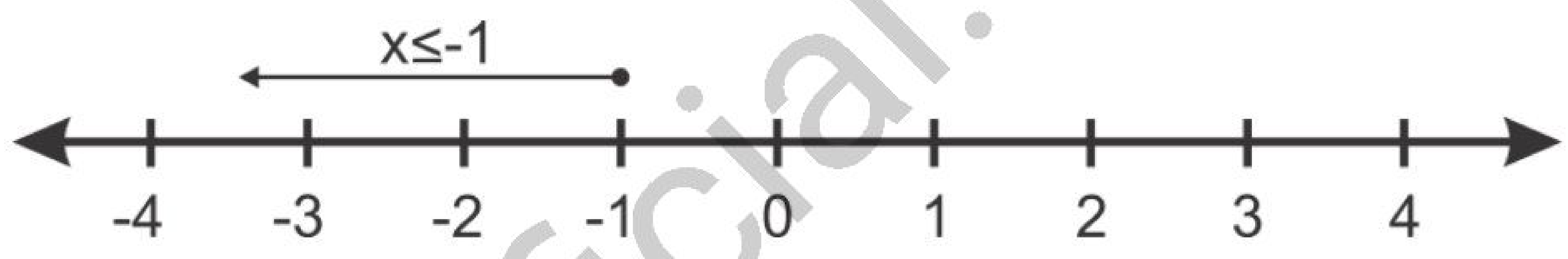
$5x - 15 \geq 16x - 4$

$-15 + 4 \geq 16x - 5x$

$-11 \geq 11x$

$-\frac{^{1}\cancel{11}}{\cancel{11}} \geq x$

$-1 \geq x$ OR $x \leq -1$



Question # 3: Find the solution region of the following linear inequalities:

(i) $\mathbf{3x - 4y \leq 12 \quad ; \quad 3x + 2y \geq 3}$

(a) Associated Equations

$3x - 4y = 12$ ______(A)

$3x + 2y = 3$ _____(B)

(b) x – Intercept

put $y = 0$ in equations (A) and (B)

$3x - 4(0) = 12$

$3x = 12$

$x = \frac{12}{3}$

$x = 4$

P_1 (4,0)

$3x + 2(0) = 3$

$3x = 3$

$x = \frac{3}{3}$

$x = 1$

P_3 (1,0)

(c) y – Intercept

put $x = 0$ in equations (A) and (B)

$3(0) - 4y = 12$

$-4y = 12$

$y = \frac{12}{-4}$

$y = -3$

P_2 $(0, -3)$

$3(0) + 2y = 3$

$2y = 3$

$y = \frac{3}{2}$

P_4 (0,1.5)

(d) Test Point (0,0)

put $x = 0, y = 0$ in given inequalities

$3(0) - 4(0) \leq 12$

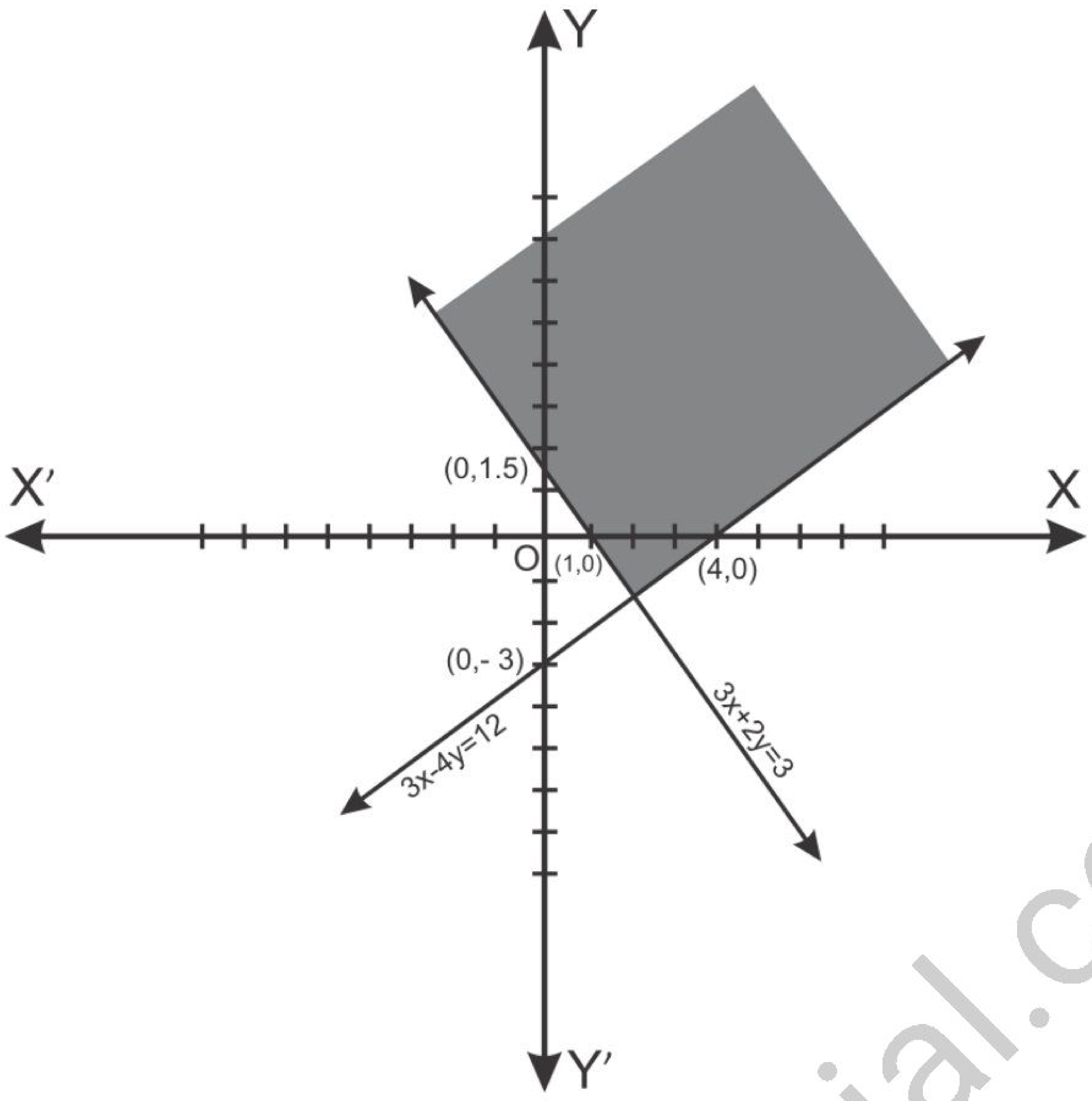
$0 \leq 12$ (True)

Solution region lies towards the origin

$3(0) + 2(0) \geq 3$

$0 \geq 3$ (False)

Solution region lies away from the origin



(ii) $\boldsymbol{2x + y \leq 4 \quad ; \quad x + 2y \leq 6}$

(a) Associated Equations

$2x + y = 4$ ______(A) ‖ $x + 2y = 6$ _____(B)

(b) x – Intercept

put $y = 0$ in equations (A) and (B)

$2x + 0 = 4$
$2x = 4$
$x = \frac{4}{2}$
$x = 2$
$P_1\ (2,0)$

‖

$x + 2(0) = 6$
$x + 0 = 6$
$x = 6$
$P_3\ (6,0)$

(c) y – Intercept

put $x = 0$ in equations (A) and (B)

$2(0) + y = 4$
$y = 4$
$P_2\ (0,4)$

‖

$0 + 2y = 6$
$2y = 6$
$y = \frac{6}{2}$
$y = 3$
$P_4\ (0,3)$

(d) Test Point (0,0)

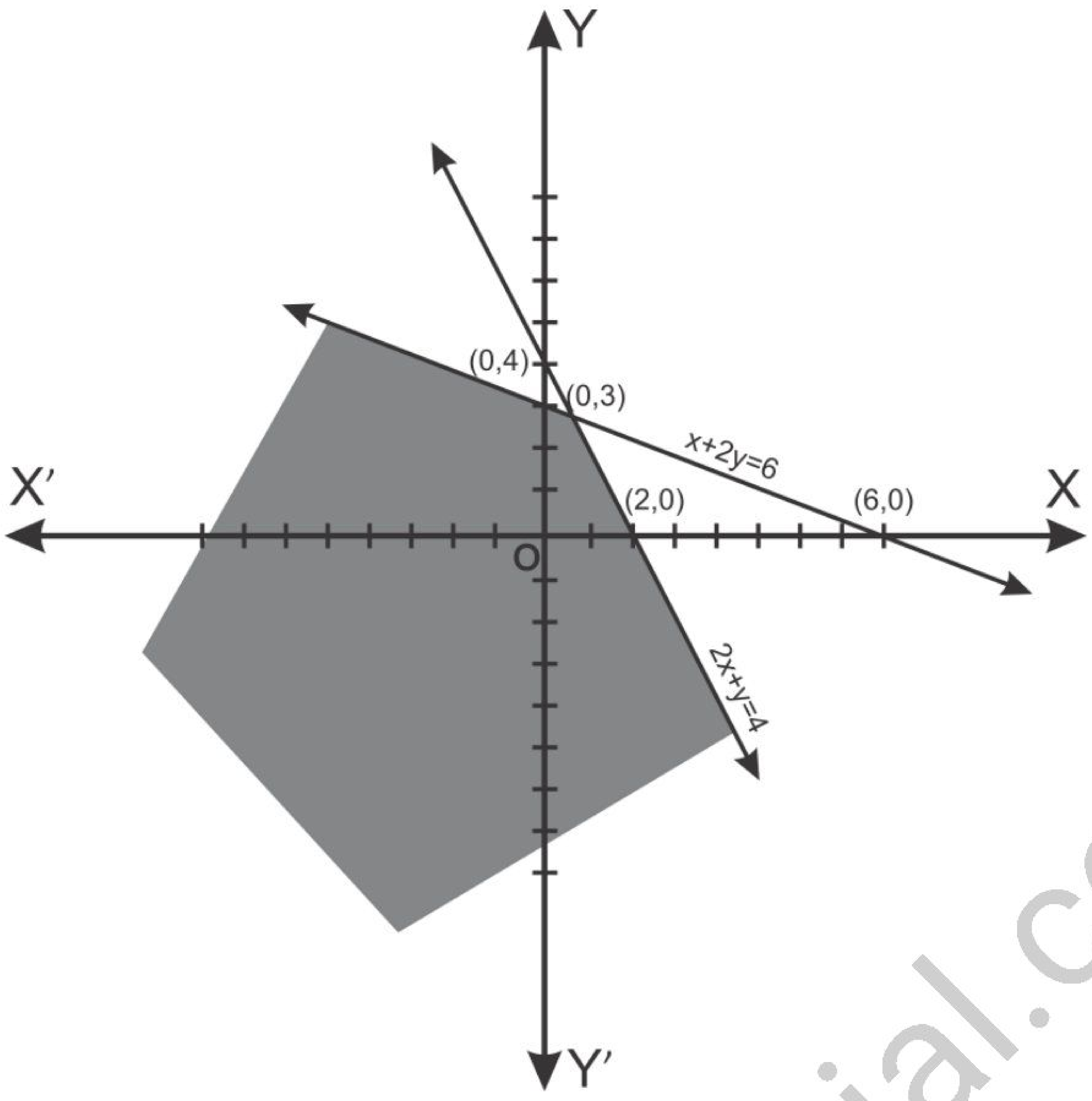
put $x = 0, y = 0$ in given inequalities

$2(0) + 0 \leq 4$
$0 \leq 4$ (True)

‖

$0 + 2(0) \leq 6$
$0 \leq 6$ (True)

Solution regions lie towards the origin



Question # 4: Find the maximum value of $g(x,y) = x + 4y$ subject to constraints:

$$x + y \leq 4 \quad ; \quad x \geq 0; y \geq 0$$

(a) Associated Equations

$x + y = 4$ _______(A)

(b) x – Intercept

put $y = 0$ in equation (A)

$x + 0 = 4$

$x = 4$

$P_1\ (4,0)$

(c) y – Intercept

put $x = 0$ in equation (A)

$0 + y = 4$

$y = 4$

$P_2\ (0,4)$

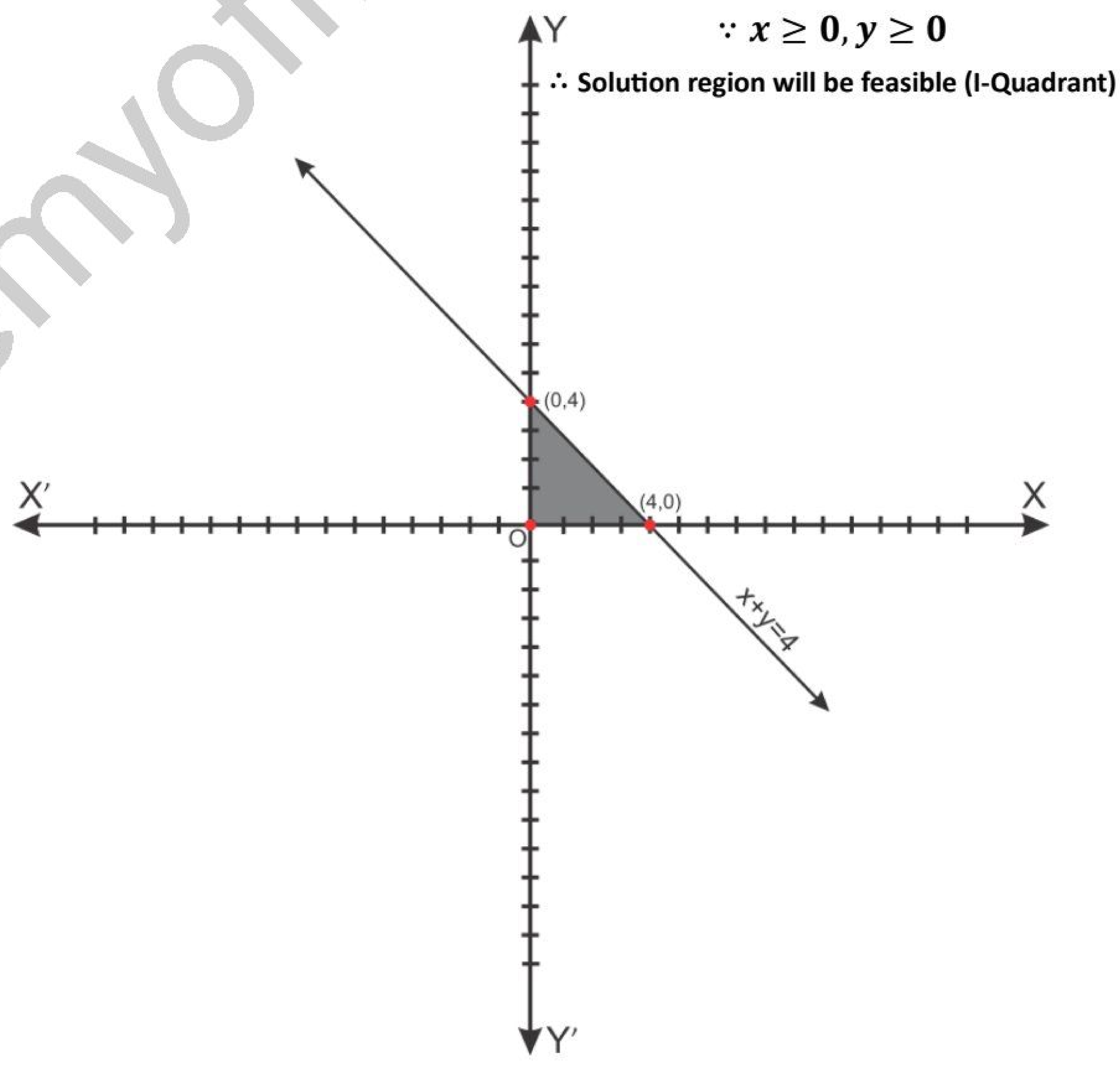
(d) Test Point (0,0)

put $x = 0, y = 0$ in given inequality

$0 + 0 \leq 4$

$0 \leq 4$ (True)

Solution Region lies towards the Origin



(e) Corner Points

$(0,0), (4,0), (0,4)$

$\because g(x,y) = x + 4y$

put $x = 0, y = 0$

$g(0,0) = 0 + 4(0) = 0 + 0 = 0$

put $x = 4, y = 0$

$g(4,0) = 4 + 4(0) = 4 + 0 = 4$

put $x = 0, y = 4$

$g(0,4) = 0 + 4(4) = 0 + 16 = 16$

Hence, $g(x,y)$ is maximized at $(0,4)$

Question # 5: Find the maximum value of $f(x,y) = 3x + 5y$ subject to the constraints:

$$x + 3y \geq 3 \quad ; \quad x + y \geq 2 \quad ; \quad x \geq 0; y \geq 0$$

(a) Associated Equations

$x + 3y = 3$ ______(A) | $x + y = 2$ _____(B)

(b) x – Intercept

put $y = 0$ in equations (A) and (B)

$$x + 3(0) = 3$$
$$x + 0 = 3$$
$$x = 3$$
$P_1\,(3,0)$

$$x + 0 = 2$$
$$x = 2$$
$P_3\,(2,0)$

(c) y – Intercept

put $x = 0$ in equations (A) and (B)

$$0 + 3y = 3$$
$$3y = 3$$
$$y = \frac{3}{3}$$
$$y = 1$$
$P_2\,(0,1)$

$$0 + y = 2$$
$$y = 2$$
$P_4\,(0,2)$

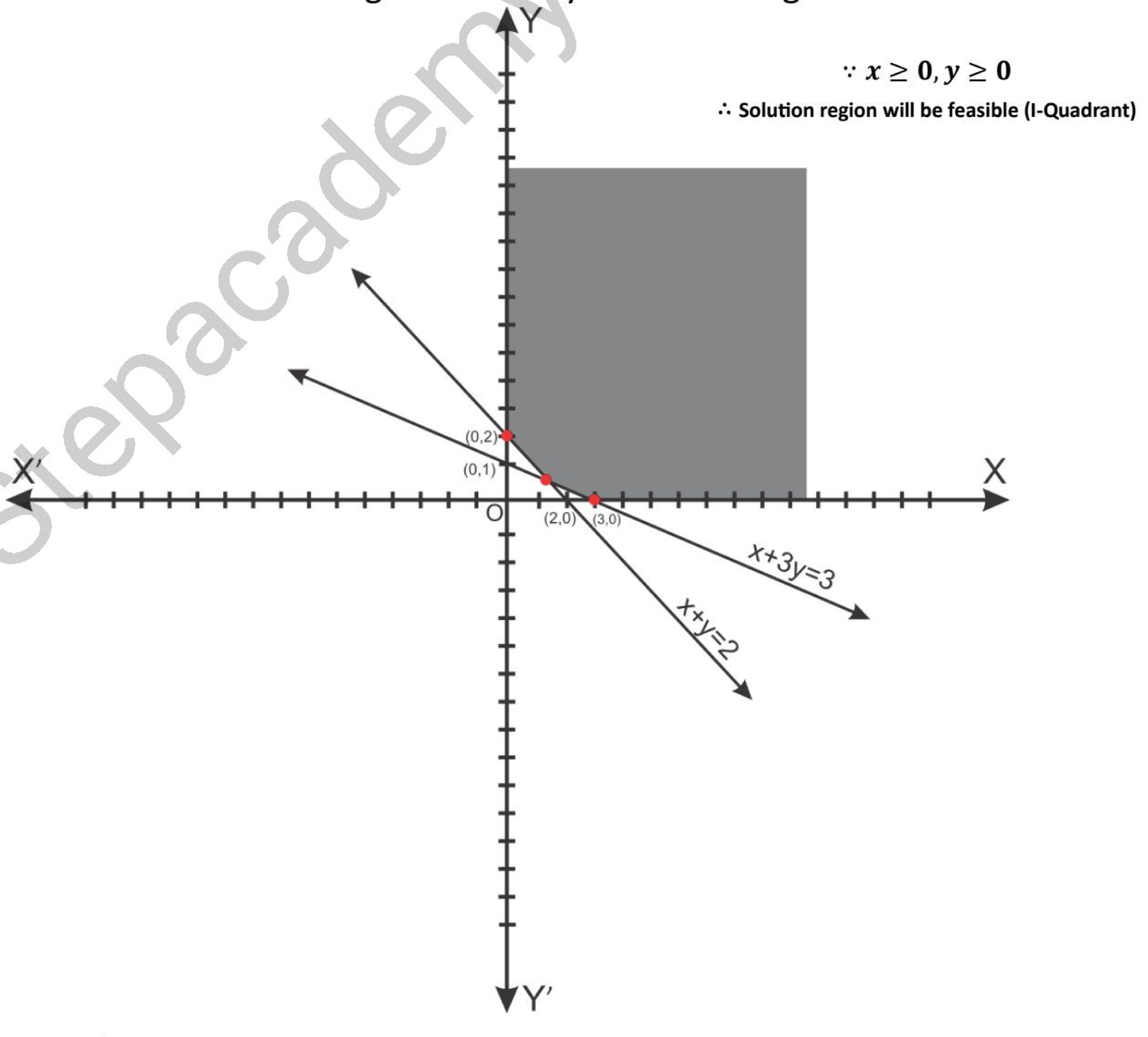
(d) Test Point (0,0)

put $x = 0, y = 0$ in given inequalities

$$0 + 3(0) \geq 3$$
$0 \geq 3$ (True)

$$0 + 0 \geq 2$$
$0 \geq 2$ (True)

Solution Regions lie away from the Origin



(e) Point of Intersection

$$x + 3y = 3 \text{ \_\_\_\_\_\_ (A)}$$
$$x + y = 2 \text{ \_\_\_\_\_\_ (B)}$$

(A) − (B)

$$\cancel{x} + 3y = 3$$
$$\underline{\pm\cancel{x} \pm y = \pm 2}$$
$$2y = 1$$
$$y = \frac{1}{2}$$

Put in equation (B)

$$x + \frac{1}{2} = 2$$
$$x = 2 - \frac{1}{2}$$
$$x = \frac{3}{2}$$

(f) Corner Points

$$(3,0), (0,2), \left(\frac{3}{2}, \frac{1}{2}\right)$$
$$\because f(x, y) = 3x + 5y$$

put $x = 3, y = 0$

$$f(0,0) = 3(3) + 5(0) = 9 + 0 = 9$$

put $x = 0, y = 2$

$$f(5,0) = 3(0) + 5(2) = 0 + 10 = 10$$

put $x = \frac{3}{2}, y = \frac{1}{2}$

$$f(0,7) = 3\left(\frac{3}{2}\right) + 5\left(\frac{1}{2}\right) = \frac{9}{2} + \frac{5}{2} = \frac{14}{2} = 7$$

Hence, $f(x, y)$ is minimized at $\left(\frac{3}{2}, \frac{1}{2}\right)$